Great Expectations

Student Activity

7 8 9 10 **11** **12**

TI-Nspire | Investigation | Student | 120 min

Introduction

Sometimes things just don't live up to their 'expectations'. In this activity you will explore three special dice and determine if they live up to their expectations. These dice may be regular in terms of their shape and form; however each die does not contain the numbers one through to six. The numbers on each die are listed below:

- Red: {3, 3, 3, 3, 3, 6}
- Blue: {2, 2, 2, 5, 5, 5}
- Green: {1, 4, 4, 4, 4, 4}

A simple game is played between two people. Each player selects their own die: Red, Blue or Green. The dice are rolled, the player that rolls the highest number wins.

Question: 1.

Determine the Expected value for each die: Red, Blue and Green.

Question: 2.

Based on your calculations from Question 1, do you think the game is fair?

Playing the Game

Open the TI-Nspire file: Great Expectations

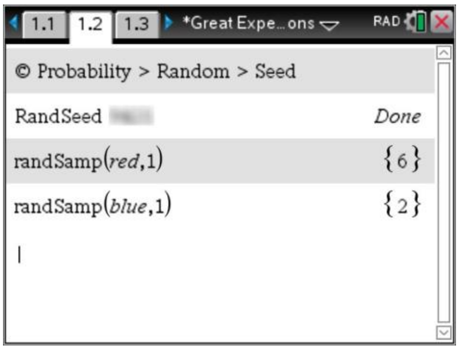
Navigate to page 1.2 and seed the random number generator using a four digit number of your choosing.

Probability > **Random** > **Seed**

Enter your own, unique four (or more) digit number for the random seed. This ensures your results will be unique.

Select a friend to play against and ask them to choose a dice colour. Once your friend has selected their dice, choose a different colour for yourself.

It is time to start playing. A dice roll can be simulated by taking a random sample:



Probability > **Random** > **Sample**

Use the VAR key to enter your selected dice colour followed by a comma and a one. The one represents the quantity of samples to be taken.

Compare your randomly generated number with your friends, remember highest number wins!

In the example shown: Red wins.

Author: P. Fox

Question: 3.

Play 10 games and record the results below.

Game	#1	#2	#3	#4	#5
Colour:					
Colour:					
Winner					
Game	#6	#7	#8	#9	#10
Colour:					
Colour:					
Winner					

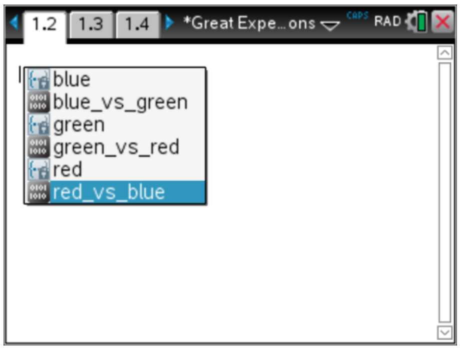
Question: 4.

Based on your relatively small sample of 10 games, do you think the game is fair? Discuss.

It is possible to simulate 100's of games all at once. There are three programs:

- Red_vs_Blue
- Blue_vs_Green
- Green_vs_Red

Red_vs_Blue will play a nominated number of games between the red dice and the blue and return the total number of games won by each, similarly for Blue_vs_Green and Green_vs_Red.



Question: 5.

Simulate 100 games of Red vs Blue and record the overall results. From the results decide whether the red or blue dice provides a greater chance of winning.

Question: 6.

Simulate 100 games of Blue vs Green and record the overall results. From the results decide whether the blue or green dice provides a greater chance of winning.

Question: 7.

Based on your results to date; is Green more or less likely to beat Red? Discuss.

Question: 8.

Simulate 100 games of Green vs Red and record the overall results. Discuss the outcome.

Question: 9.

Draw probability tree diagrams for each game: Red vs Blue, Blue vs Green and Green vs Red. Use your tree diagrams to explain the results you obtained for Questions 5, 6 and 8.

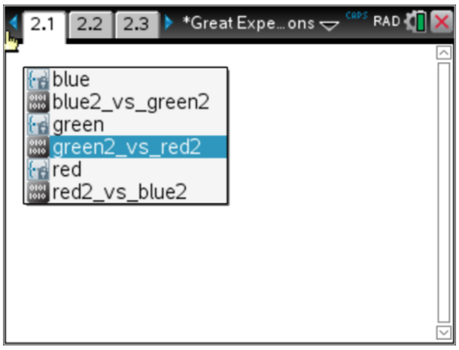
Question: 10.

Explain why the 'expected' value does not help determine which dice is likely to win.

Author: P. Fox

Doubling Up

Navigate to page 2.1



In problem 2 each selected dice is rolled twice and the sum of the results is used to determine the winner. The names of the simulation programs have changed to reflect the changing conditions.

- Red2_vs_Blue2
- Blue2_vs_Green2
- Green2_vs_Red2

Question: 11.

Use the programs to determine approximately probabilities for Blue vs Green, Green vs Red and Red vs Blue games.

Question: 12.

Use probability to determine the theoretical probabilities for each combination and discuss the results in relation to the original order of the probabilities for single rolls.

Extension

The dice used in this initial investigation are called Grime dice after a very talented and entertaining mathematician Dr. James Grime. James has a passion for communicating many of the wonders hidden within mathematics, search for him on YouTube or visit his website: http://singingbanana.com/ to find out more about non-transitive dice and other mathematical curiosities.

Similar dice sets have been created by other mathematicians. Efron dice for example are named after the American statistician Brad Efron.

- Blue = {3, 3, 3, 3, 3, 3}
- Red = {0, 0, 4, 4, 4, 4}
- Green = {1, 1, 1, 5, 5, 5}
- Magenta = {2, 2, 2, 2, 6, 6}

Determine your own investigation into this set of non-transitive dice. There are many options to explore here:

- Determine the order, with four dice this means extra possibilities
- Doubles (As per the original investigation)
- Picking two dice and using the sum
- Picking two dice and using the product
- Consider a three player game and consider scenarios such as: "Your opponents pick the Blue and Green dice, which die should you choose now to give you the best chance of winning?"

Author: P. Fox