NUMB3RS Activity: Dial the Phone
Episode: "Brutus"

Topic: Triangular Numbers, Networks **Grade Level:** 9 - 10
Objective: Students will generate a function rule for triangular numbers as they investigate the relationship between the number of telephones in a network and the value of the network.
Time: 15 - 20 minutes
Materials: TI-83/TI-84 Plus graphing calculator

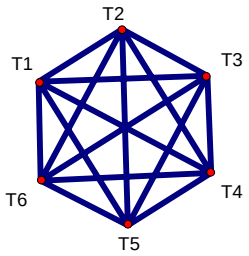
Introduction

In the episode "Brutus," Charlie is trying to trace the sale of a "straw gun." A straw gun is a gun that is purchased legally from a gun shop, transferred to a black market dealer, and then is sold by the black market dealer illegally to another person. The straw gun that was used to murder a senator was one of four guns purchased at a gun shop. Charlie uses a network in an effort to find the black market dealer and trace the remaining three guns before they are used on future targets.

The network Charlie creates consists of all the people involved in the sale and purchase of the straw guns. He explains that this network of people is similar to a network of telephones. In this activity students will create a network of telephones and calculate the value of the network.

Discuss with Students

For this activity the *value of a network* is defined as the number of open lines of communication between the telephones on a network. The network shown below has 6 telephones. The networks we consider for this activity assume that all telephones are connected to each other. The value of this 6-phone network is equal to the number of edges, which is 15.



You may want to talk about the definition of value and discuss practical reasons for stating that the value of a network of telephones increases as more telephones are added. The numbers that emerge as the values are called the triangular numbers; students may have encountered them in other mathematical situations.

You may want to provide more scaffolding for students to as they come up with the formula in question 4. There are many approaches for finding the formula $v(n) = \frac{n(n-1)}{2}$. You may also want to have a discussion about what each factor in the formula represents. Each telephone on a network of n phones is connected to $n - 1$ phones. The product $n(n - 1)$ might appear to count the total number of edges on the graph, but it actually counts each connection twice, once from each end. Dividing this product by 2 accounts for the double counting.

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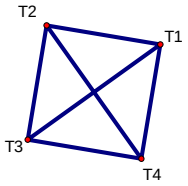
Student Page Answers:

1a.

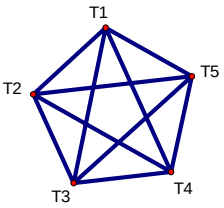


1b. *The value of the network is 6 connections*

2a.

The value of the network is 1 connection.

2b.



The value of the network is 10 connections.

3. *A telephone only has value if it can connect to another telephone. There is no other phone on the network. Therefore, the value of the network is 0 connections.*

4.

# of telephones	**value of network (# of connections)**
1	0
2	1
3	3
4	6
5	10
6	15
7	21
8	28
⋮	
20	190
⋮	
n	$\frac{n(n-1)}{2}$

5. *Let v(n) represent the value of a network of n phones, then* $v(n) = \frac{n(n-1)}{2}$. **5a.** *A power function that closely approximates v(n) is* $Y_2 = \frac{n^2}{2}$ *where a =* $\frac{1}{2}$. **5b.** *Answers will vary. If a =* $\frac{1}{2}$ *then 180,000 connections.* **5c.** *Answers will vary. 720,000 connections for a =* $\frac{1}{2}$ **5d.** *The value of the network grows by a factor of 4 when the number of telephones is doubled. This is true for any acceptable value of a.*

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Name: _____ Date: _____

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In the episode “Brutus,” Charlie is trying to trace the sale of a “straw gun.” A straw gun is a gun that is purchased legally from a gun shop, transferred to a black market dealer, and then sold by the black market dealer illegally to another person. The straw gun that was used to murder a senator was one of four guns purchased at a gun shop. Charlie uses a network in an effort to find the black market dealer and trace the remaining three guns before they are used on future targets.

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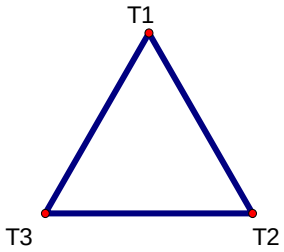
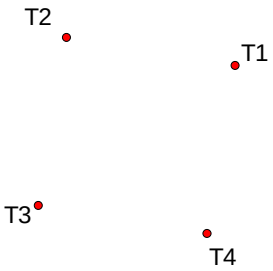
Creating Networks

Suppose there is a network of three telephones and each telephone is connected to each of the other telephones. A picture of the network is shown below. The vertices (T1, T2, T3) on the graph below represent telephones. Each edge that connects two vertices represents an open line of communication between the two phones. This network has three lines of communication. If we define the **value** of the network as the number of open lines of communication, then the value of the network below is three connections.



1. A network consists of four telephones. Assume each telephone is connected to any other telephone on the network.



a. Draw in all possible lines of communication.

b. Determine the value of the network.

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2. Draw in networks for the following number of phones, and determine the value of each network.

a. 2 phones

b. 5 phones

value of 2-phone network = _____ value of 5-phone network = _____

3. Charlie states that a network with only one telephone is fundamentally useless. Explain why this is true.

Metcalfe's Law

4. Fill in the table below using the value calculations you made above. Determine the missing entries by looking for patterns or drawing networks. Assume that each phone in a network is connected to all the other phones in that network.

# of telephones in network	value of network (# of connections)
1	
2	
3	3
4	
5	
6	
7	
8	
⋮	
20	
⋮	
n	

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Amita suggests that Charlie use **Metcalfe's Law** to find the four missing straw guns. Metcalfe's Law states that the value of a telephone network is proportional to the square of the number of telephones on the network. This law is a general rule that people use to make mental approximations for the value of a network and does not exactly match the formula you developed in question 4.

5. Let $v(n)$ represent the formula you developed in the last row of the table in question 4, which calculates the value of a network with n telephones. Metcalfe's Law states $v(n) \approx ax^2$.

a. Put the formula for $v(n)$ in Y_1 of your graphing calculator. Under Y_2 put any function of the form $Y_2 = ax^2$. Find a value for a that closely matches the graphs of Y_1 and Y_2.

b. Use the rule you generated in part a to calculate the value of a network with 600 telephones.

c. Calculate the value of the network if the number of telephones in the network increases to 1,200.

d. In general, explain how the value of the network changes when the number of telephones in the network doubles.

In this activity you used the number of connections in a network as a measure of value and studied how this value changes as more telephones are added. Charlie uses different measures of value for the network of people involved in the sale and purchase of straw guns as he searches for the black market dealer who sold the straw guns. He still uses Metcalfe's Law to narrow the list of black market dealers but with different measures of value.

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The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Networks

- Metcalfe's Law is used as a general rule for determining how the value of a network grows as the number of users increase. For example if the number of users in a network grows by a factor of 3, then the value of the network grows by a factor of 9. Explain how Metcalfe's Law matches this rule.

- Use a search engine on the Internet to read more about Metcalfe's Law. Some sites claim the general rule overestimates the value of a network while others say it underestimates the value. Investigate the controversy over the accuracy of Metcalfe's Law in determining the value of a network. Describe at least three points of disagreement associated with this rule.

Handshake Problem

A related problem that generates the same set of answers as the value of a network is the Handshake problem. The numbers generated are called "triangular numbers." Look at the Web sites below to learn more about the Handshake problem.

- A lesson plan for the Handshake problem is located at **http://illuminations.nctm.org/LessonDetail.aspx?ID=L630**.

- An interactive web activity showing the network and table representations of the Handshake problem can be viewed at **http://illuminations.nctm.org/ActivityDetail.aspx?ID=126**.

- Go to **http://mathforum.org/library/drmath/view/56157.html** to see alternative solutions to the Handshake problem. This classic problem has a familiar solution.

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