



Graphing Calculator Investigation

A Preview of Lesson 11-4

Limits

You may have noticed that in some geometric sequences, the later the term in the sequence, the closer the value is to 0. Another way to describe this is that as n increases, a_n approaches 0. The value that the terms of a sequence approach, in this case 0, is called the **limit** of the sequence. Other types of infinite sequences may also have limits. If the terms of a sequence do not approach a unique value, we say that the limit of the sequence does not exist.

Find the limit of the geometric sequence $1, \frac{1}{3}, \frac{1}{9}, \dots$

Step 1 Enter the sequence.

- The formula for this sequence is $a_n = \left(\frac{1}{3}\right)^{n-1}$.
- Position the cursor on L1 in the **STAT** EDIT Edit ... screen and enter the formula seq(N,N,1,10,1). This generates the values 1, 2, ..., 10 of the index N.
- Position the cursor on L2 and enter the formula seq((1/3)^(N-1),N,1,10,1). This generates the first ten terms of the sequence.

KEYSTROKES: Review sequences in the Graphing Calculator Investigation on page 585.

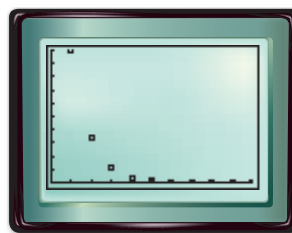


Notice that as n increases, the terms of the given sequence get closer and closer to 0. If you scroll down, you can see that for $n \geq 8$ the terms are so close to 0 that the calculator expresses them in scientific notation. This suggests that the limit of the sequence is 0.

Step 2 Graph the sequence.

- Use a STAT PLOT to graph the sequence. Use L1 as the Xlist and L2 as the Ylist.

KEYSTROKES: Review STAT PLOTS on page 87.



[0, 10] scl: 1 by [0, 1] scl: 0.1

The graph also shows that, as n increases, the terms approach 0. In fact, for $n \geq 6$, the marks appear to lie on the horizontal axis. This strongly suggests that the limit of the sequence is 0.

Exercises

Use a graphing calculator to find the limit, if it exists, of each sequence.

1. $a_n = \left(\frac{1}{2}\right)^n$

2. $a_n = \left(-\frac{1}{2}\right)^n$

3. $a_n = 4^n$

4. $a_n = \frac{1}{n^2}$

5. $a_n = \frac{2^n}{2^n + 1}$

6. $a_n = \frac{n^2}{n + 1}$

