

Activity 28

Segments Formed by Intersecting Chords, Secants, and Tangents

Objective

- To investigate properties of segments formed when chords, secants, and tangents intersect

Cabri® Jr. Tools



Introduction

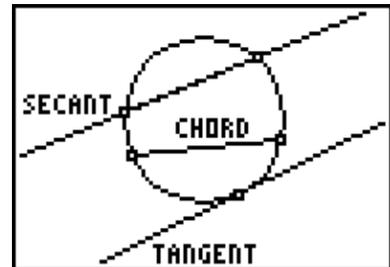
This activity is designed to help you discover several important theorems concerning lengths of segments formed by intersecting chords, secants, and tangents.

This activity makes use of the following definitions:

Chord — a segment with its endpoints on the circle (the diameter is the longest chord of a circle)

Secant — a line that intersects a circle at two points

Tangent line to a circle — a line that intersects a circle at only one point called the *point of tangency*

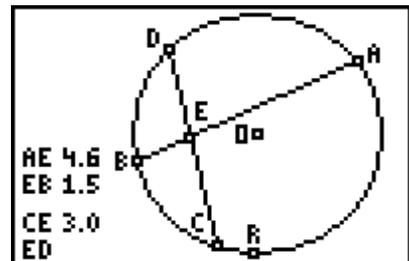


Part I: Chords Intersecting Inside a Circle

Construction

Construct a circle with two intersecting chords.

- A** Draw a circle with a center O and radius point R .
- A** Construct chords \overline{AB} and \overline{CD} that intersect at point E .
- A** Measure the lengths of segments \overline{AE} , \overline{EB} , \overline{CE} and \overline{ED} .



Note: Not all measurements are shown.

Exploration

-   Use the **Calculate** tool to investigate the relationship between the four segments determined by intersecting chords. Move the defining points of the chords and circle to observe the relationship for different length chords.
-   Construct \overline{AC} and \overline{BD} and use several measurement tools to investigate the relationship between the two triangles formed. Move the defining points of the chords and the circle to observe this relationship for different chords.

Questions and Conjectures

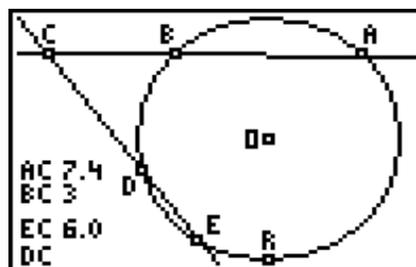
1. Make a conjecture about the relationship between the lengths of segments formed by intersecting chords. Is this relationship true for any pair of chords? Explain your reasoning.
2. Make a conjecture about the relationship between triangles defined by connecting adjacent endpoints of intersecting chords. How does this relationship support your conjecture from Question 1. Explain your reasoning.

Part II: Secants Intersecting Outside the Circle

Construction

Construct a circle with two secants that intersect outside the circle.

-  Clear the previous construction.
-   Draw a circle having a center O and radius point R .
-   Construct secant \overleftrightarrow{AC} that intersects the circle at points A and B .
-   Construct secant \overleftrightarrow{EC} that intersects the circle at points E and D .
-  Measure the lengths of segments \overline{AC} , \overline{BC} , \overline{EC} and \overline{DC}



Note: Not all measurements are shown.

Exploration

-   Use the **Calculate** tool to investigate the relationship between the secant segments \overline{AC} and \overline{EC} and their external segments \overline{BC} and \overline{DC} (respectively). Move the defining points of the secants and the circle to observe the relationship for different secants.

Questions and Conjectures

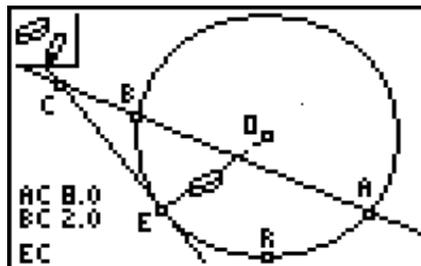
1. Make a conjecture about the relationship between the secant segments and their external segments formed by secants intersecting outside a circle. Is this relationship true for any pair of secants intersecting outside the circle? Explain your reasoning.
2. Constructing chords \overline{AD} and \overline{BE} can be useful when trying to support your conjecture. Explain why this is and be prepared to demonstrate.

Part III: Intersecting Tangents and Secants

Construction

Construct a circle with a tangent and a secant that intersect outside the circle.

-  Clear the previous construction.
-  **A** Draw a circle having a center O and radius point R .
-  **A** Construct a radius \overline{OE} .
-  **A** Construct a line perpendicular to radius \overline{OE} passing through and defined by point E .
-  **A** Construct a secant \overline{AC} that intersects the circle at points A and B and intersects the line perpendicular to \overline{OE} at point C .
-  Measure the lengths of segments \overline{AC} , \overline{BC} , and \overline{EC} .
-  Hide radius \overline{OE} .



Note: Not all measurements are shown.

Exploration

-   Use the **Calculate** tool to investigate the relationship between the secant segment \overline{AC} , its external segment \overline{BC} , and the tangent segment \overline{EC} . Move the defining points of the secant, the tangent, and the circle to observe the relationship for different secants and tangents.
-  Move point A so that it coincides with point B , thus making \overline{AC} a tangent to the circle. Investigate the relationship between the two tangent segments \overline{AC} and \overline{EC} .

Questions and Conjectures

1. Make a conjecture about the relationship between a secant segment, its external segment, and a tangent segment formed by the intersection of a secant and a tangent at a point outside a circle.
2. Constructing chords \overline{AE} and \overline{BE} can be useful when trying to support your conjecture. Explain why this is and be prepared to demonstrate.

3. In Part II you could have created the intersection of a secant and a tangent by moving point E to coincide with point D . Is the relationship you found in this exploration simply a special case of the relationship you found in Part II? Explain your reasoning.
4. Make a conjecture about the relationship of two tangent segments defined by the intersection of two tangent lines to the same circle. Justify your conjecture.

Teacher Notes



Activity 28

Segments Formed by Intersecting Chords, Secants, and Tangents

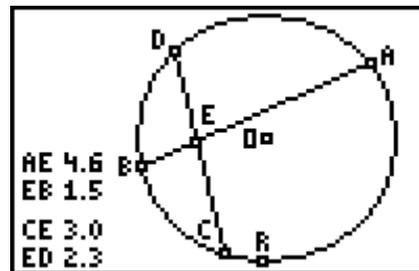
Part I: Chords Intersecting Inside a Circle

Answers to Questions and Conjectures

1. Make a conjecture about the relationship between the lengths of segments formed by intersecting chords. Is this relationship true for any pair of chords? Explain your reasoning.

When two (or more) chords intersect at a point inside a circle, the products of the lengths of the chords are equal. For this example, $AE \times EB = CE \times ED$.

This relationship can also be expressed as a ratio: $\frac{AE}{ED} = \frac{CE}{EB}$. This relationship may be difficult for students to see due to the accuracy in length measurement with the Cabri[®] Jr. application. (See "A Specific Cabri Jr. Issue" on page vi.)



Objective

- To investigate properties of segments formed when chords, secants, and tangents intersect

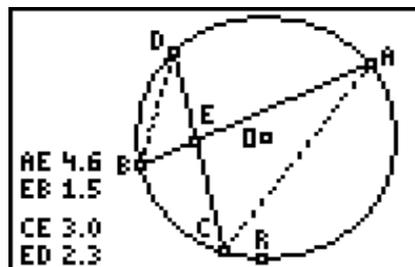
Cabri[®] Jr. Tools



2. Make a conjecture about the relationship between triangles defined by connecting adjacent endpoints of intersecting chords. How does this relationship support your conjecture from Question 1. Explain your reasoning.

It can be shown that the triangles formed are similar since the triangles have vertical angles that are congruent ($\angle AEC$ and $\angle BED$), and that there are two pairs of inscribed angles that intercept the same arc ($\angle ACD$ and $\angle ABD$ intercept \widehat{AD} and $\angle CAB$ and $\angle BDC$ intercept \widehat{CB}). Corresponding sides of similar triangles are proportional,

$$\text{thus } \frac{AE}{ED} = \frac{CE}{EB}$$

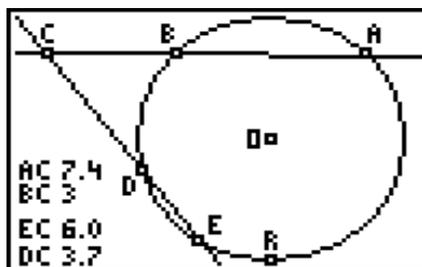


Part II: Secants Intersecting Outside the Circle

Answers to Questions and Conjectures

1. Make a conjecture about the relationship between the secant segments and their external segments formed by secants intersecting outside a circle. Is this relationship true for any pair of secants intersecting outside the circle? Explain your reasoning.

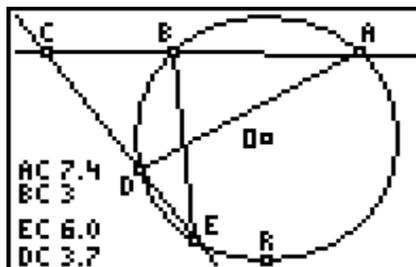
When two (or more) secants intersect at a point outside a circle, the product of the lengths of the secant segment and the corresponding external secant segment will be equal. For this example, $AC \times BC = EC \times DC$. This relationship can also be expressed as a ratio: $\frac{AC}{EC} = \frac{DC}{BC}$.



2. Constructing chords \overline{AD} and \overline{BE} can be useful when trying to support your conjecture. Explain why this is and be prepared to demonstrate.

It can be shown that constructing chords \overline{AD} and \overline{BE} produce similar triangles ($\triangle ADC$ and $\triangle ECB$) since the triangles have $\angle BCD$ in common and that there are two angles that intercept the same arc (angles $\angle DAB$ and $\angle BED$ intercept \widehat{BD}). Corresponding sides of similar triangles are proportional, thus

$$\frac{AC}{EC} = \frac{DC}{BC}$$

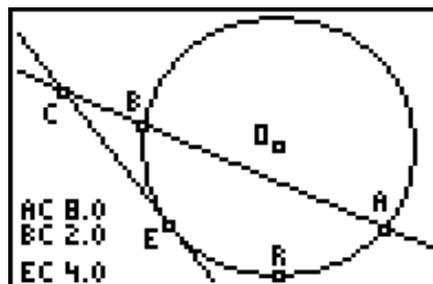


Part III: Intersecting Tangents and Secants

Answers to Questions and Conjectures

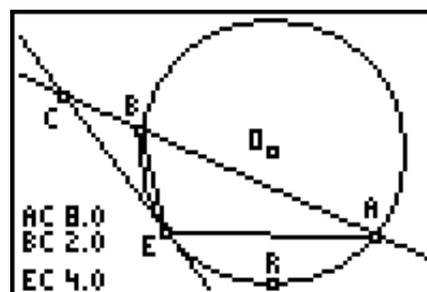
1. Make a conjecture about the relationship between a secant segment, its external secant segment, and a tangent segment formed by the intersection of a secant and a tangent at a point outside a circle.

When a secant intersects a tangent at a point outside a circle, the product of the lengths of the secant segment and the corresponding external secant segment will equal the square of the length of the tangent segment. For this example, $AC \times BC = EC \times EC$. This relationship can also be expressed as a ratio: $\frac{AC}{EC} = \frac{EC}{BC}$.



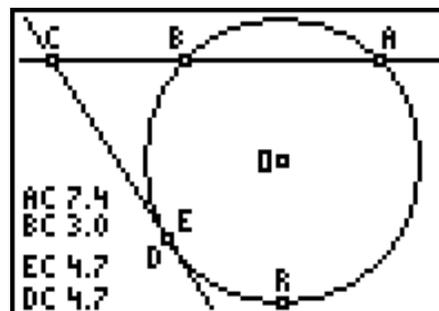
2. Constructing chords \overline{AE} and \overline{BE} can be useful when trying to support your conjecture. Explain why this is and be prepared to demonstrate.

It can be shown that the construction of chords \overline{AE} and \overline{BE} produce similar triangles ($\triangle AEC$ and $\triangle EBC$) since the triangles have $\angle BCE$ in common and there are two angles that are both $1/2 m\angle BOE$ ($\angle EAB$ and $\angle BEC$). Corresponding sides of similar triangles are proportional, thus $\frac{AC}{EC} = \frac{EC}{BC}$.



3. In Part II you could have created the intersection of a secant and a tangent by moving point E to coincide with point D . Is the relationship you found in this exploration simply a special case of the relationship you found in Part II? Explain your reasoning.

As you move point E closer to point D , the difference in the lengths of segments \overline{EC} and \overline{DC} gets smaller. When points E and D coincide, you can see the length of segment \overline{EC} equals the length of segment \overline{DC} . Replacing EC for DC in the relationship $AC \times BC = EC \times DC$ gives the relationship found in Question 1.



4. Make a conjecture about the relationship of two tangent segments defined by the intersection of two tangent lines to the same circle. Justify your conjecture.

The tangent segments would be congruent. This can be shown by constructing radii \overline{OE} and \overline{OA} and segment \overline{OC} . This would produce two congruent right triangles ($\triangle CEO$ and $\triangle CAO$) by hypotenuse-leg since radii \overline{OE} and \overline{OA} are perpendicular to the tangent segments, \overline{OE} and \overline{OA} are radii of the same circle, and the triangles share side \overline{OC} . \overline{CE} and \overline{CA} are congruent since they are corresponding sides of congruent triangles.

