Mathematical Methods (CAS) 2002 Examination 1 part 1 sample solutions Q 8 to 13

Note: To use *Derive* efficiently, students should be familiar with the 'tick plus equals' and 'tick plus approximately equals' evaluation buttons. These simultaneously 'author' and 'evaluate' expressions exactly and numerically respectively.For example, the 'tick plus equals' or 'author and exact evaluation' button works well for Question 14, while the 'tick plus approximately equals' or 'author and numerical approximation' works well for Question 26. Students should also be familiar with the use of defined functions in the form f(x):= *rule of the function*, such as in the sample solutions for Question 10.

Some questions are conceptual in nature, that is, technology will not be of assistance, for example, Question 7. For other questions, such as Question 1, the facility of *Derive* to quickly produce scaled graphs, and the like, means that such problems could be tackled by inspection of each alternative, although this is not a recommended approach.In many cases students will reason what is likely to be the answer, and then confirm this with *Derive*.

Question 8

Alternatives D and E can be eliminated readily be checking that there is not a constant first or second difference respectively. Alternative B can also be eliminated as the log function should have a decreasing rate of increase, whereas this data has an increasing rate of increase. The closeness of the remaining two functions can be determined by inspection of the corresponding tables of values for x = 0 to 5 (evaluated using approximate mode):

#1: $\mathrm{TAN}\left(\dfrac{x}{2}\right)$

#2: $\mathrm{TABLE}\left(\mathrm{TAN}\left(\dfrac{x}{2}\right), x, 0, 5, 1\right)$

#3: $$\begin{bmatrix} 0 & 0 \\ 1 & 0.5463024898 \\ 2 & 1.557407724 \\ 3 & 14.10141994 \\ 4 & -2.185039863 \\ 5 & -0.7470222972 \end{bmatrix}$$

#4: $e^{x/2}$

#5: $\text{TABLE}(e^{x/2}, x, 0, 5, 1)$

#6: $$\begin{bmatrix} 0 & 1 \\ 1 & 1.64872127 \\ 2 & 2.718281828 \\ 3 & 4.48168907 \\ 4 & 7.389056098 \\ 5 & 12.18249396 \end{bmatrix}$$

Clearly the best alternative is C. For this question, *Derive* could be used to very quickly check the table of values for each alternative.For each of the other rules these are:

#7: $\text{LOG}\left(\frac{x}{2}\right)$

#8: $\text{TABLE}\left(\text{LOG}\left(\frac{x}{2}\right), x, 0, 5, 1\right)$

#9: $$\begin{bmatrix} 0 & -\infty \\ 1 & -0.6931471805 \\ 2 & 0 \\ 3 & 0.4054651081 \\ 4 & 0.6931471805 \\ 5 & 0.9162907318 \end{bmatrix}$$

#10: $\frac{x}{2} + 1$

#11: $\text{TABLE}\left(\frac{x}{2} + 1, x, 0, 5, 1\right)$

#12: $$\begin{bmatrix} 0 & 1 \\ 1 & 1.5 \\ 2 & 2 \\ 3 & 2.5 \\ 4 & 3 \\ 5 & 3.5 \end{bmatrix}$$

#13: $\dfrac{x^2}{2}$

#14: $\text{TABLE}\left(\dfrac{x^2}{2}, x, 0, 5, 1\right)$

#15: $$\begin{bmatrix} 0 & 0 \\ 1 & 0.5 \\ 2 & 2 \\ 3 & 4.5 \\ 4 & 8 \\ 5 & 12.5 \end{bmatrix}$$

Question 9

This is readily done, but students need to note that *linear* factors over *R* are sought, and select the correct field accordingly:

#16: $x^4 + x^3 - 3\cdot x^2 - 3\cdot x$

#17: $x\cdot(x + 1)\cdot(x + \sqrt{3})\cdot(x - \sqrt{3})$

thus, B is the correct alternative.

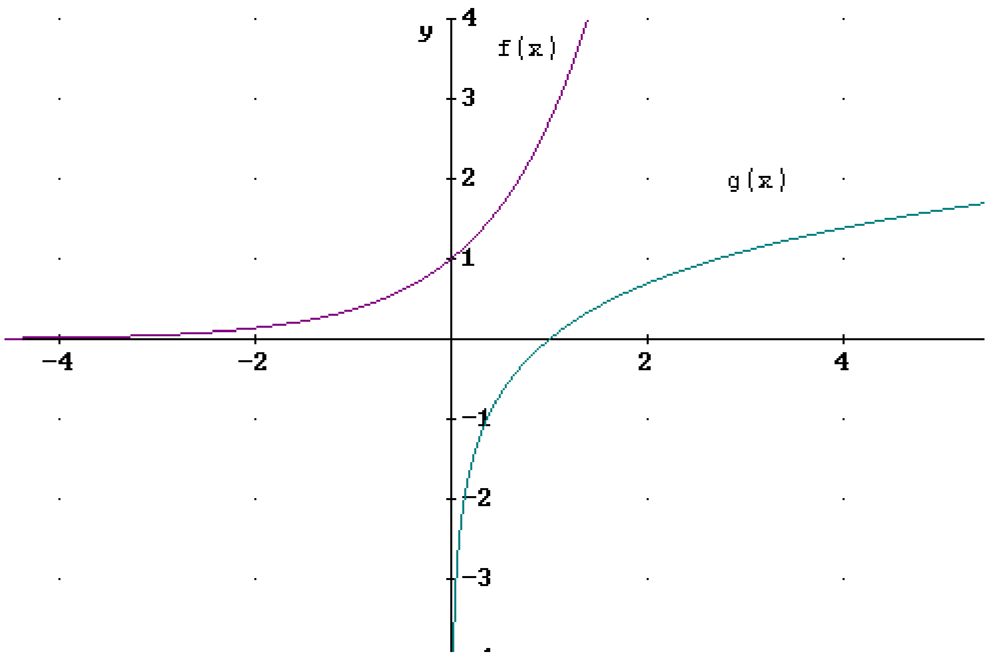
Question 10

This question can be answered directly by noting that the question asks 'which *one* of the following statements is true' and recognising C as a correct statement, from the definition of inverse functions.

If the function rules are defined, each alternative can be tested in turn, although this is much less efficient, and results require interpretation:

#18: $f(x) := e^{x}$

#19: $g(x) := LOG(x)$



these graphs can be used to eliminate alternatives A and B.

#20: $f(g(x))$

#21: x

this evaluation confirms the correctness of alternative C. No further work is required to answer the question, however the following evaluations are provided to illustrate how each of the other alternatives could be eliminated:

#22: $f(x) \cdot g(x)$

#23: $e^{x} \cdot LN(x)$

students should be able to see that if x = 1, for example, the product will be e^1*0 which is 0, and thus not equal to 1, hence alternative D is not correct. To eliminate E, evaluate g(f(x)) to see it is not the same as 1/x (this should be known from the definition of inverse functions anyway):

#24: $g(f(x))$

#25: x

Question 11

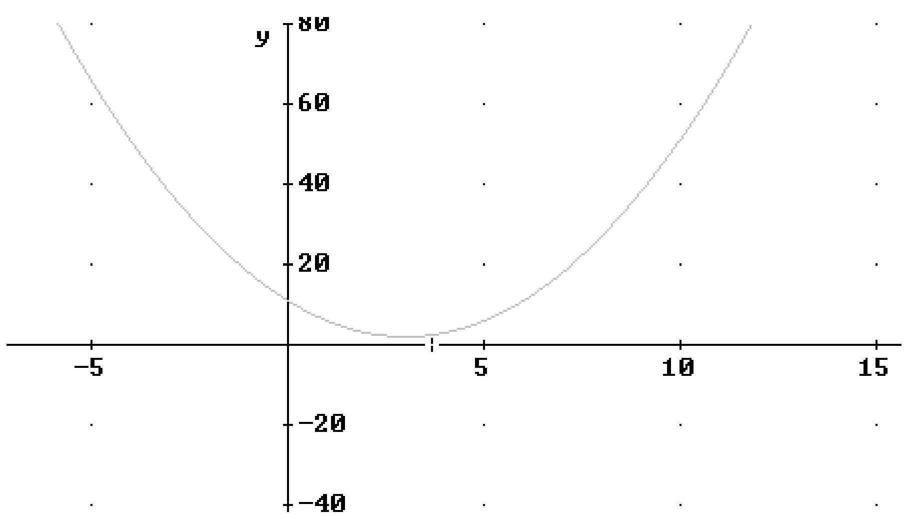
This should be known from familiarity with the functions and their graphs, D is the only function which is m – 1 over its natural (implied or maximal domain). This can be tackled graphically, but such an approach should not be necessary in this case.

Question 12

Students should know that for f to have an inverse function, the set A must be a subset of R such that the function f is one to one over A. Thus, any interval which is a subset of the interval from (and possibly including)the x value of the vertex (x = 3) to its left or from (and possibly including)the x value of the vertex (x = 3)to its right will do. D is the only correct alternative. Each alternative could be checked visually against the graph of f(x) over a suitably large domain (note *Derive* automatically expands the rule):

#26: $(x - 3)^2 + 2$

#27: $x^2 - 6\cdot x + 11$



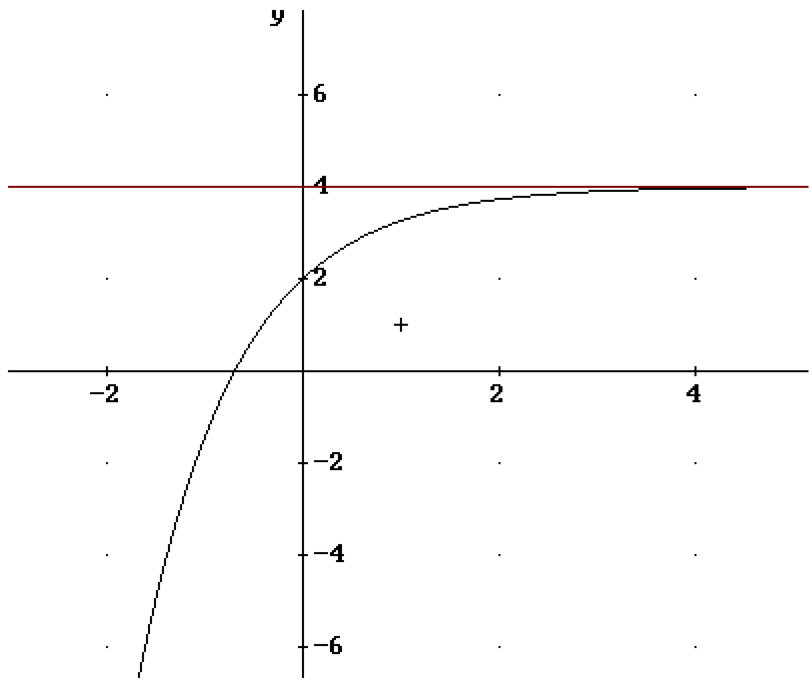
Question 13

This is another conceptual question that can be readily answered. As x increases, f is asymtotic to y = 4, thus its gradient will be asymptotic to y = 0. The graph in alternative C is the only possible graph for f′ with this behaviour.

A more cumbersome and less efficient approach is to model the graph of f with a known function, and consider the graph of its derivative. This approach would require students to quickly be able to identify a suitable function (which may not be readily done, or even possible, if an unscaled graph were to be used) and carry out he corresponding graphical analysis, focussing on features relevant to the problem. For example, try f(x) = −2e^(−x) +4.

#28: $f(x) :\!= -2\cdot e^{-x} + 4$

#29: 4



#30: $f'(x)$

#31: $2\cdot e^{-x}$