When does 1 + 1 + 1 = 1?

Teachers Teaching with Technology™
Professional Development from Texas Instruments

Teacher Notes & Answers

7 8 **9** **10** 11 12

TI-Nspire™ | Investigation | Student | 120 min

Introduction

The equation title is derived from a largely avoidable, real life anomaly where three litres of water are required to produce one litre of bottled water.[1] In a world where water is a precious commodity it seems preposterous to waste water in this way. While it takes a lot more water (per litre of product) to produce soft-drinks and alcoholic beverages, these are not readily available from a tap. Drinkable water is available from a tap, where the water is less accessible, re-usable containers can be filled with water. Petrol prices are often in the news, yet bottled water is often more expensive per litre than petrol.

Of even greater concern than the unbalanced water equation is the enormous waste generated by single use plastic water bottles. While many of these bottles contain a label promoting: 'recyclable', the reality is that most of these bottles are not recycled. In 1999 the NOAA[2] removed 125 tonnes of plastic from the Midway atoll. Where is the Midway atoll? It is in the middle of the Pacific Ocean, it does not contain human inhabitants; the plastic pollution on the shores of the island has been washed up from the ocean. Birds on the island are dying from starvation as they consume indigestible plastic making them feel full where in fact they are nutrient poor. In 2004 the USA put more than 60 million plastic bottles into landfill, every day! While it is wonderful that many of us are choosing a healthier option as our beverage of choice, we are doing serious long term damage to our environment and ultimately ourselves.

The mathematical purpose behind this activity is for students to collect and analyse data, extrapolate the information, accurately use scientific and engineering notation and to convert between units of measurement. The activity helps provide purpose for the calculations in addition to a much stronger message aiming to make students more aware of the environmental impact of single use plastic items such as water bottles. Australia has already started on a plastic minimisation journey by reducing the use of plastic shopping bags. While the absence of plastic bags at supermarket registers is initially inconvenient, as habits form we will all be better off for their demise. In some countries, take-away food is not served in disposable containers; instead the consumer brings their own re-usable container, again, a habit that helps our environment.

This activity does not contain a specific student hand out sheet. The resources and questions provided in this document are designed as a guide for teachers.

One way to introduce this activity is to have students watch a short video documentary. There are many suitable videos on YouTube that can be found by searching "Plastic Oceans".

Midway Atoll: https://www.youtube.com/watch?v=IsJqMmuFWO4

Catalyst (ABC): https://www.youtube.com/watch?v=cwTDvqaqPIM

[1] Source: http://waterfootprint.org – Calculations include those associated with the manufacturing of plastic bottles.

[2] NOAA: National Oceanic and Atmospheric Administration

Authors: Peter Fox

Fermi Question

Fermi questions are named after Enrico Fermi, a physicist that was able to make approximate yet reasonably accurate calculations with little or no actual data. In this part of the investigation students are be required to:

Estimate the quantity of single use plastic water bottles discarded in Australia each year.

Options for estimating this quantity could include but not limited to:

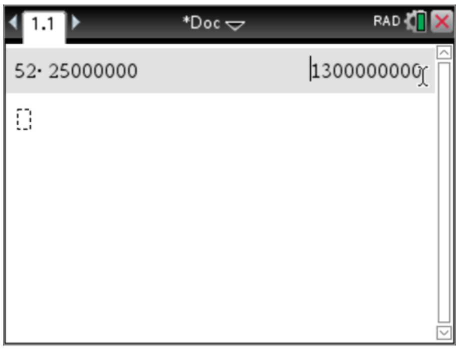
- Survey of students / classes to help quantify the number of water bottles consumed per person.
- Monitoring sales of single use water bottles in a supermarket or shop over a set time as a proportion of the total number of shoppers.

Regardless of the method used to obtain the sample data, students should extrapolate the figures to represent the Australian population (approximately 25 Million people in 2018) and the corresponding number of water bottles discarded per year.

Sample:

If each person on average disposes of one water bottle per week then the total number of bottles would be of the order:

52 x 25,000,000 = 1,300,000,000.

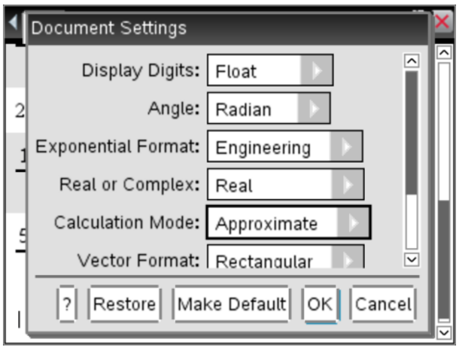


Students will find this amount difficult to read in the absence of commas separating 1000's.

There are two options for expressing large numbers on most calculators:

Scientific

Engineering



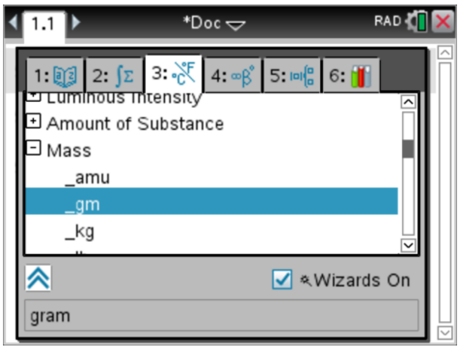
Scientific notation will express a number such as 15,000 as 1.5 x 10^4 whereas Engineering will express this as 15 x 10^3. The advantage of Engineering format is that it applies to multiples of 1,000 aligning nicely with prefixes such as 'kilo' and 'mega'. On TI-Nspire these options can be set in the "Document Settings".

The next section of the investigation works with units of mass and volume. The mass of a single plastic bottle may be difficult to determine as each bottle has a relatively small mass. Indeed, the mass of each bottle has been reduced significantly over the past decade.

Estimate the total mass of waste from single use plastic water bottles.

Students will need to determine ways to measure the mass of a single bottle. Students could use the scales in the science department or collectively weigh many bottles and determining the mass of a single bottle. Students should consider whether all the bottles need to be exactly the same or does a random sample of these bottles also reflect the distribution of bottles found in the waste?

Note that the catalogue feature of TI-Nspire also includes options for unit calculations. Enter a quantity such as 1000_gm and the calculator will automatically write the answer in kilograms. 1_kg.

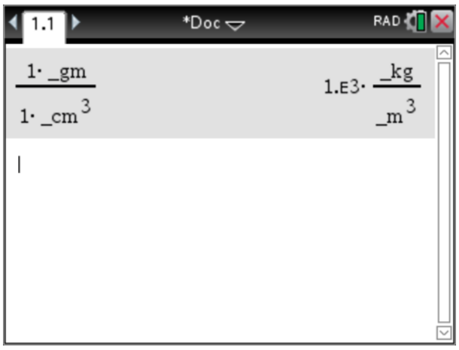


Authors: Peter Fox

A challenging question for students to calculate is the volume of waste.

Estimate the total volume of waste from single use plastic water bottles.

This question assumes that the plastic bottles will be squashed, but by how much? Students could try and cram as many squashed bottles into a box and then calculate the volume attributed per bottle. Another option is to imagine the bottles were completely destroyed and work out the corresponding volume of plastic. This measurement could be done experimentally or by combining the previous calculation for the mass of plastic bottles with the density for PET type plastic used in most drink bottles: 1.38gm/cm^3.



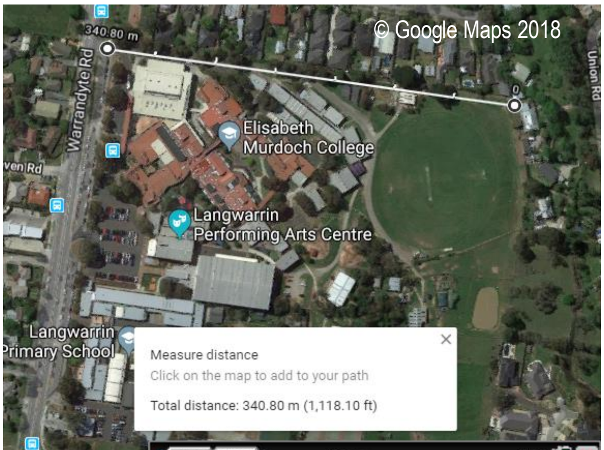
Students can use their calculators to help with unit conversions also.

Based on the average mass of a 600mL PET bottle (approximately 14g) and the figures provided as samples here, the typical volume of landfill attributed to the disposal of single use drink bottles would be:

$$52 \times 25{,}000{,}000 \times 14 \div 1.38 \approx 13{,}000\text{m}^3$$

Students are not likely to have a full understanding of this volume. It can be compared to the volume of a classroom: "How many classrooms would these plastic bottles fill?" A recent promotion relating to disposable coffee cups showed a Melbourne tram filled with cups, the amount consumed every 30 minutes!

https://www.youtube.com/watch?v=zF9Rd8Cw-Xc



The tram filled with coffee cups created a great deal of attention both directly and via media. In this example the coffee cups were not squashed down flat or compacted into a solid mass, however the message was the same, giving people a volume to which they could relate.

To help students get an understanding of the volume of plastic it can be turned into a height in metres above the school ground. Google Maps can be used to estimate the area of the school. A right mouse click on Google Maps allows you to measure a distance. For irregular shaped grounds a scaled map could be placed as an image into a TI-Nspire document. Include a measurement on the Google map image and use it to scale the image in the document (as shown). The scale in the top right corner of TI-Nspire can be changed to suit the map. Draw a polygon drawn around the boundary and the corresponding area measured can be displayed. If the units are set on the scale the area will be displayed correctly.

Example:

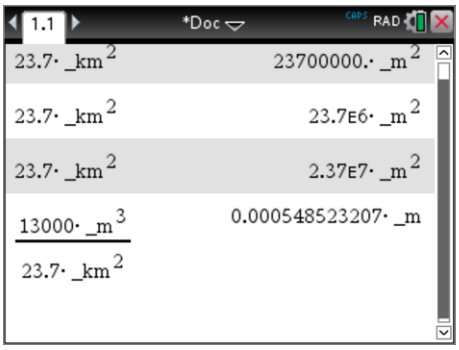
The school shown opposite is located in Langwarrin, a southern suburb of Melbourne. The length of the school grounds is 340m (shown) and the width 190m. The area is therefore approximately: 64,600m^2. Using a polygon around the school's boundaries obtains a slightly larger measurement of 65,200m^2 due to the curvature of one of the boundaries.

Authors: Peter Fox

Imagine if the plastic bottles were shredded and compacted, the entire school grounds could be covered by a solid piece of plastic approximately 20cm thick. The mathematics problem being explored here relates to exploring a fixed volume (the amount of plastic) and variable box dimensions. In this situation the 'base' area of the box is defined by the area of the suburb. The mixture of units adds another level of complexity to this problem.

$$\frac{\text{Volume of plastic}}{\text{School area}} = \frac{13000\ m^3}{64200\ m^3} \approx 0.20\ m$$

Changing the base area (mathematical context) can be achieved by considering the notion of covering an entire suburb (practical context). The area of Langwarrin is given as 23.7km^2. The calculator can be used to convert this value automatically. The calculations shown here include the conversion for 'Normal', 'Engineering' and 'Scientific' format for the area of the suburb of Langwarrin.



Normal: 23700000 m^2

Engineering: 23.7 x 10^6 m^2

Scientific: 23.7 x 10^7 m^3

The thickness of plastic that would cover the entire suburb of Langwarrin is therefore:

$$\frac{\text{Volume of plastic}}{\text{School area}} = \frac{13000\ m^3}{23700000\ m^2} \approx 0.00055\ m$$

This thickness does not sound particularly impressive, unless you use a comparable entity. Cling wrap is approximately 12μm. ($\mu = 10^{-6}$) The entire suburb of Langwarrin could be covered by approximately 46 sheets of cling wrap!

All of these calculations represent different ways to think about and visualise the amount of plastic that is generated every year just for single use, disposable plastic drink bottles. From this small sample of calculations it is clear that students will work through numerous measurement conversions and a range of different exponential representations that can include negative exponents for very small values such as the thickness of cling wrap.

Using the array of information collected in this activity, students could present their findings as a poster with supporting calculations and data on a separate page. Videos are also a creative way of presenting findings; imagine pretending to cover an entire suburb with 45 layers of cling wrap, perhaps by accelerating the video animation. The video production brings out the A in STEAM but also helps students remember the entire experience.

Authors: Peter Fox

Authors: Peter Fox