

## Compound Events – ID: 10136

By Michele Patrick

Time required

45 minutes

## Activity Overview

*In this activity, students are introduced to the Multiplication Rule for both independent and dependent events. Using spinners, they compare probabilities found by looking at the sample space and by using the appropriate formula. They will learn what is meant by a conditional probability and find conditional probabilities, some from a table in a spreadsheet. Last, they will use complements of events to find probabilities of “at least one.”*

## Concepts

- *Independent and dependent events*
- *Conditional probability*
- *Complements*

## Teacher Preparation

- *This activity is designed to be used for students studying Statistics and Probability.*
- *This activity assumes students know the formula for finding the theoretical probability of an event:  $P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$  and that the probability must be between 0 and 1, inclusive.*
- *Students should already be familiar with the Addition Rule for Mutually Exclusive events (even if they do not know it by that name). For example, the probability of randomly rolling either a 1 or a 2 on a die is  $P(1) + P(2) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$ . If you have not covered this topic, you can introduce it before this activity, referring to the sample space of rolling a die while doing so. Most students do not have difficulty with this rule.*
- *The screenshots on pages 2–7 demonstrate expected student results. Refer to the screenshots on pages 8 and 9 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the student .tns file and student worksheet, go to [education.ti.com/exchange](http://education.ti.com/exchange) and enter “10136” in the quick search box.***

## Classroom Management

- *This activity is intended to be mainly **teacher-led**, with breaks for individual student work. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds.*
- *The student worksheet StatAct32\_CompeEvents\_worksheet\_EN helps guide students through the activity and provides a place for students to record their answers.*

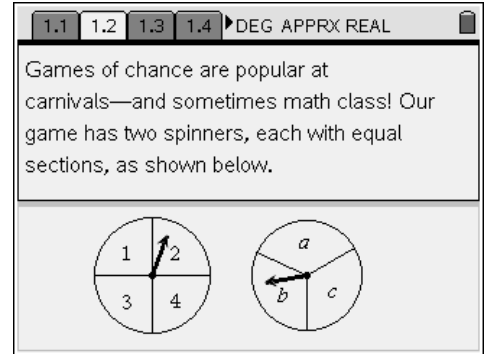
## TI-Nspire™ Applications

*Calculator, Lists & Spreadsheet, Notes*

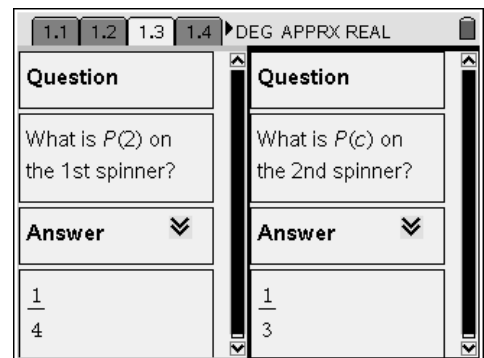
**Problem 1 – Independent events**

The spinners on page 1.2 will be used to answer questions in Problems 1 and 2.

Point out that the sections are equally spaced so the outcomes for each spinner are equally likely.



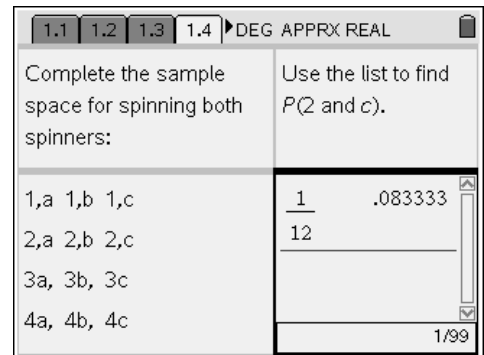
On page 1.3, have students find each indicated probability. These questions can be used to review the basic concepts of probability as needed.



On page 1.4, explain that both spinners will be spun at the same time. Define *independent events*. Events are independent if the occurrence of the first does not affect the probability of the second. Here, the probability of an event on the second spinner is not influenced by what happens on the first.

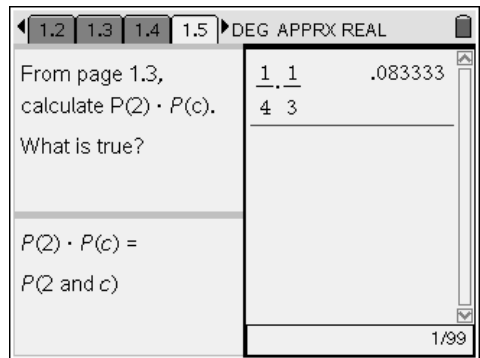
Tell students to complete the list of all the possible outcomes for spinning both spinners. Then, have them use the list to find the probability of the event “2 and c.”

Since one of the 12 outcomes in the sample space is “2 and c,” the probability is  $\frac{1}{12}$ .

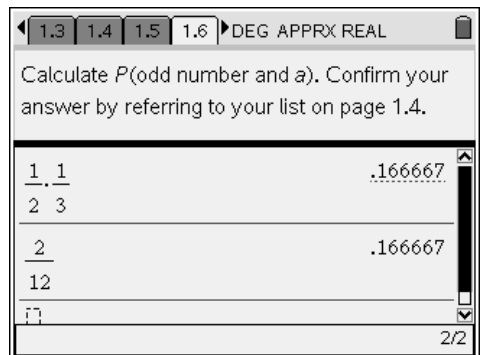


Tell students to multiply the individual probabilities they found for  $P(2)$  and  $P(c)$ . They will see that  $P(2 \text{ and } c) = P(2) \cdot P(c)$ .

Introduce the Multiplication Rule for Independent Events:  $P(x \text{ and } y) = P(x) \cdot P(y)$ .

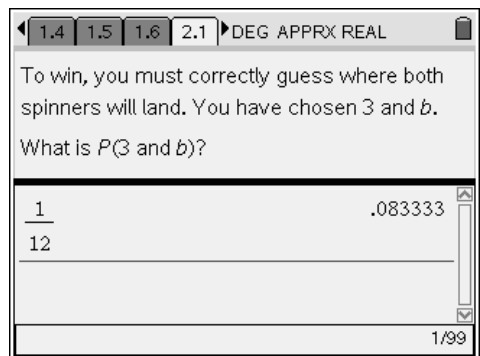


Have students use the rule to calculate the probability asked for on page 1.6. Have them look at the sample space to confirm their answer. (2 out of 12 outcomes are odd and a: 1,a and 3,a).



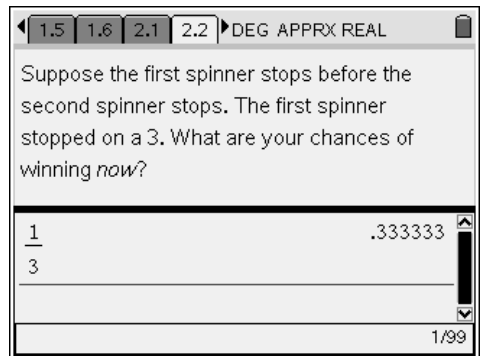
**Problem 2 – Dependent events**

Introduce the scenario on page 2.1 and ask students to find  $P(3 \text{ and } b)$ .



Have students advance to page 2.2 and consider the case where the first spinner stops spinning before the second one stops. At this point, you know if you either lost for sure or still have a chance of winning.

Suppose the first spinner stops on a 3 while the second spinner continues to spin. Have students find the probability of winning now that they know what the first number is. They should say  $\frac{1}{3}$ , because there are three outcomes on the second spinner and one of them,  $b$ , is favorable.



For students unfamiliar with carnival games, you may need to explain briefly how they work. Tell students that all the outcomes are on a table and they place a token on the outcome they think will win. In this case, we are assuming their token is on a square marked “3*b*.” Once the spinners start spinning, they cannot remove or move their token. Their token remains where it is until both spinners have stopped.

Have students advance to page 2.3 and introduce the notation for conditional probability. The vertical line is read “given that,” so  $P(x|y)$  means the probability of  $x$ , given that  $y$  has occurred.

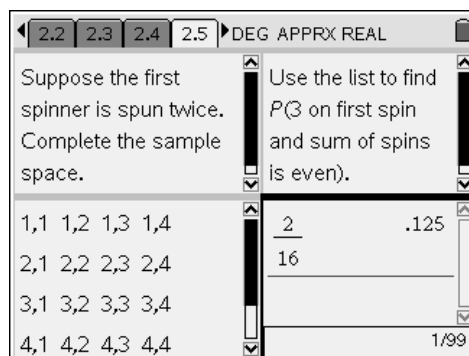
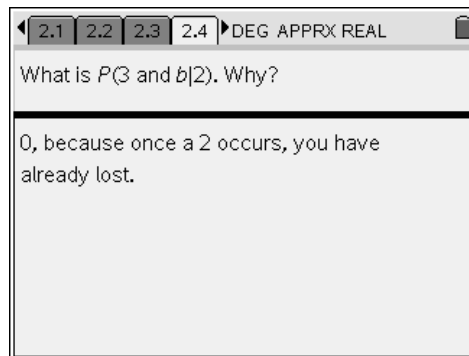
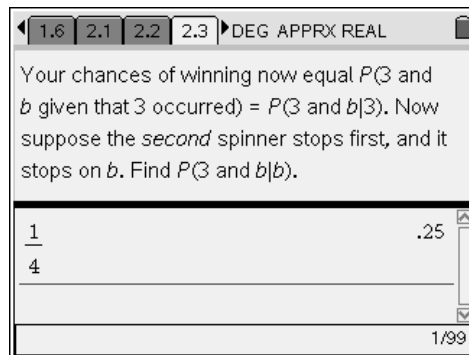
Now have students pretend that the second spinner stops spinning before the first spinner. Tell them to find  $P(3 \text{ and } b|b)$ , the probability of winning knowing that  $b$  has occurred.

To check their understanding of the notation, have students think about the probability on page 2.4.

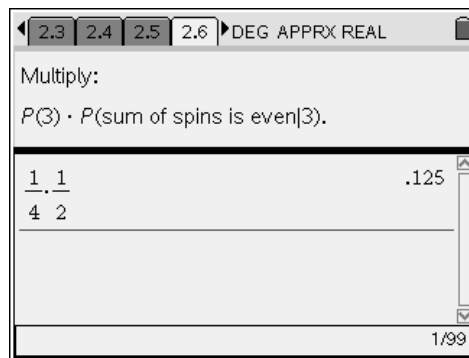
They should say 0. If a 2 occurs on the first spin, there is no chance that they will win when their token is on “3 and  $b$ .”

Have students advance to page 2.5 and explain that this time, the first spinner (numbered 1–4) will be spun twice. Have them complete the list of possible outcomes and then use their list to find the probability of spinning a 3 on the first spin and the sum of the two spins is even.

The probability is  $\frac{2}{16}$  because 2 of the 16 outcomes are favorable (3,1 and 3,3).

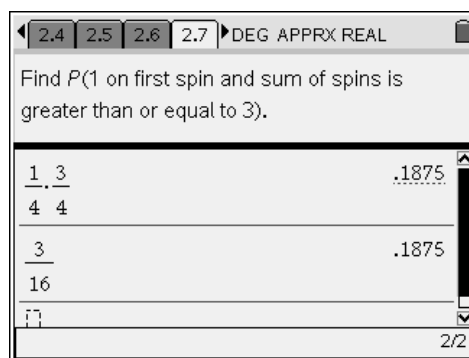


Ask students for  $P(3)$ . It is  $\frac{1}{4}$ . Then ask for the conditional probability:  $P(\text{sum of spins is even} | 3)$ , that is, the probability that the sum is even, given that a 3 has occurred. They can use their list to help come up with the answer of  $\frac{1}{2}$ . Then have them multiply the probabilities on page 2.6 to see what is true.



It is the same as the probability found on the previous page. Introduce the Multiplication Rule for Dependent Events:  
 $P(x \text{ and } y) = P(x) \cdot P(y | x)$ .

Let students practice using the rule by answering the question on page 2.7. Have them look at the sample space to confirm their answer (3 out of 16 outcomes are favorable: 1,2, 1,3 and 1,4).

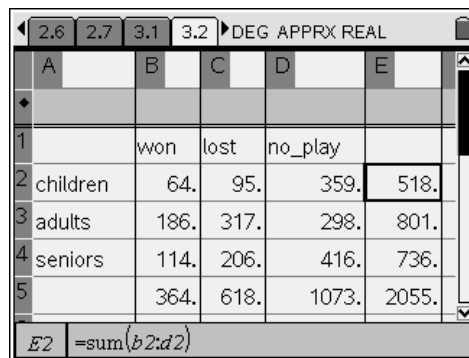


Have students use this example to discuss why the events are dependent. Before the occurrence of the first spin,  $P(\text{sum} \geq 3) = \frac{15}{16}$ .

The occurrence of spinning a 1 limits the sample space.

**Problem 3 – Conditional probabilities from a table**

Have students advance to page 3.2 and use formulas to find the sum of each row and column. For example, to find the sum of the cells B2, C2, and D2, type **=sum(b2:d2)** in cell E2.



Also, have them find the total sum of all the people. One way is to use **=sum(e2:e4)**.

Ask students how they can find the conditional probability  $P(\text{adult}|\text{lost})$ . Show that the sample space is now reduced from 2055 to 618. Then have them find the remaining conditional probabilities on pages 3.3 and 3.4.

Students may find it helpful to highlight certain rows or columns in the chart as they find the probabilities.

	A	B	C	D	E
1		won	lost	no_play	
2	children	64.	95.	359.	518.
3	adults	186.	317.	298.	801.
4	seniors	114.	206.	416.	736.
5		364.	618.	1073.	2055.

Find $P(\text{adult} \text{lost})$ .	Find $P(\text{did not play} \text{child})$ .
$\frac{317}{618}$	$\frac{359}{518}$
.512945	.69305
1/99	1/99

Find:  $P(\text{senior})$ ,  $P(\text{senior}|\text{won})$ , and  $P(\text{won}|\text{senior})$ .

$\frac{114}{364}$	.313187
$\frac{114}{736}$	.154891
1/3	

**Problem 4 – Complements**

Discuss a game where a prize is hidden under one of four identical cups, the cups are quickly moved around, and a player guesses which cup has the prize. Have students find the probabilities of randomly guessing a winning a prize and not winning a prize on page 4.1.

A game has 4 upside-down cups. One cup has a prize under it. If randomly choosing a cup, what is...

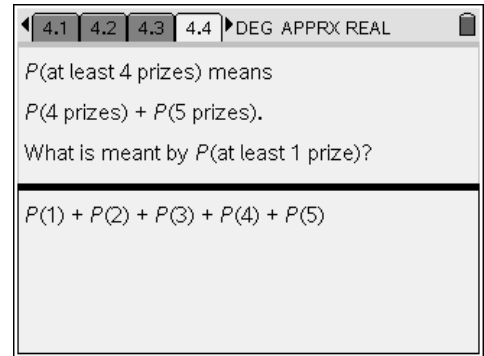
$P(\text{prize})$	$P(\text{no prize})$
$\frac{1}{4}$	$\frac{3}{4}$
.25	.75
1/99	1/99

Use page 4.2 to define and discuss complements.

$P(\text{prize})$  and  $P(\text{no prize})$  are **complements**.  
What is  $P(\text{prize}) + P(\text{no prize})$ ?

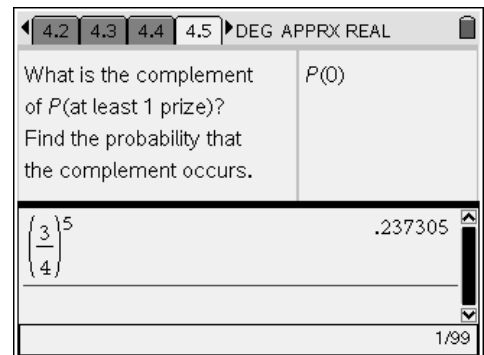
$\frac{1}{4} + \frac{3}{4}$	1.
1/99	

Page 4.3 explains that a child plays the game 5 times. Students should say that the child could win 0, 1, 2, 3, 4, or 5 times. Watch for students who forget to mention 0 times. Have students then advance to page 4.4 to confirm that  $P(\text{at least 1 prize})$  equals  $P(1 \text{ or more prizes})$  equals the sum of the probabilities of 1, 2, 3, 4, and 5 prizes.



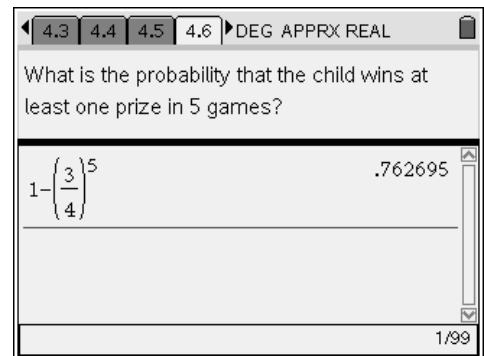
Have students move to page 4.5 and give the complement of  $P(\text{at least one prize})$ . Help them see that the complement is  $P(0)$ . Then have them find  $P(0)$ . Students will use the Multiplication Rule for Independent Events.  $P(0)$  is the probability of zero wins. Every game was a loss,  $P(\text{no prize})$ , so

$$P(0) = \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \left(\frac{3}{4}\right)^5$$

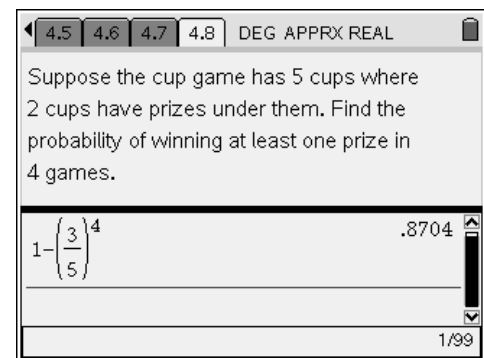
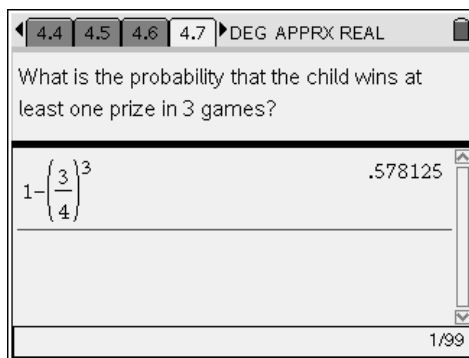


Have students use the definition of complements to now find  $P(\text{at least one prize in 5 games})$ .

If students are confused, say, “*The probability of no prize in 5 games was about 23.7%. The complement of that, the probability of at least one prize, is 100% minus that.*”



Have students check their understanding by answering the questions on pages 4.7 and 4.8 independently. Review the answers when everyone is finished.



### Compound Events – ID: 10136

(Student)TI-Nspire File: *StatAct32\_CompEvents\_EN.tns*

1.1 1.2 1.3 1.4 ▶ DEG APPRX REAL

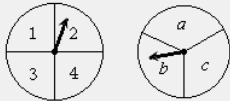
**COMPOUND EVENTS**

**Statistics**

Independent/dependent events,  
conditional probability, complements

1.1 1.2 1.3 1.4 ▶ DEG APPRX REAL

Games of chance are popular at carnivals—and sometimes math class! Our game has two spinners, each with equal sections, as shown below.



1.1 1.2 1.3 1.4 ▶ DEG APPRX REAL

<b>Question</b>	<b>Question</b>
What is $P(2)$ on the 1st spinner?	What is $P(c)$ on the 2nd spinner?
<b>Answer</b> ▼	<b>Answer</b> ▼

1.1 1.2 1.3 1.4 ▶ DEG APPRX REAL

Complete the sample space for spinning both spinners:	Use the list to find $P(2)$ and $c$ .
1,a 1,b	

0/99

1.2 1.3 1.4 1.5 ▶ DEG APPRX REAL

From page 1.3, calculate  $P(2) \cdot P(c)$ . What is true?

0/99

1.3 1.4 1.5 1.6 ▶ DEG APPRX REAL

Calculate  $P(\text{odd number and } a)$ . Confirm your answer by referring to your list on page 1.4.

0/99

1.4 1.5 1.6 2.1 ▶ DEG APPRX REAL

To win, you must correctly guess where both spinners will land. You have chosen 3 and  $b$ . What is  $P(3 \text{ and } b)$ ?

0/99

1.5 1.6 2.1 2.2 ▶ DEG APPRX REAL

Suppose the first spinner stops before the second spinner stops. The first spinner stopped on a 3. What are your chances of winning *now*?

0/99

1.6 2.1 2.2 2.3 ▶ DEG APPRX REAL

Your chances of winning now equal  $P(3 \text{ and } b \text{ given that } 3 \text{ occurred}) = P(3 \text{ and } b|3)$ . Now suppose the *second* spinner stops first, and it stops on  $b$ . Find  $P(3 \text{ and } b|b)$ .

0/99

2.1 2.2 2.3 2.4 ▶ DEG APPRX REAL

What is  $P(3 \text{ and } b|2)$ . Why?

2.2 2.3 2.4 2.5 ▶ DEG APPRX REAL

Suppose the first spinner is spun twice. Complete the sample space.	Use the list to find $P(3)$ on first spin and sum of spins is even).
1,1 1,2	

0/99

2.3 2.4 2.5 2.6 ▶ DEG APPRX REAL

Multiply:  
 $P(3) \cdot P(\text{sum of spins is even}|3)$ .

0/99



2.4 2.5 2.6 2.7 ▸ DEG APPRX REAL

Find  $P(1 \text{ on first spin and sum of spins is greater than or equal to } 3)$ .

0/99

2.5 2.6 2.7 3.1 ▸ DEG APPRX REAL

All who attended a carnival on the first day received a ticket to play a spinner game for free. The types of tickets and whether they won, lost, or did not play the game are shown on the next page.

Use formulas to find the sum of each row and each column.

2.6 2.7 3.1 3.2 ▸ DEG APPRX REAL

	A	B	C	D	E
1		won	lost	no_play	
2	children	64.	95.	359.	
3	adults	186.	317.	298.	
4	seniors	114.	206.	416.	
5					

4/7

2.7 3.1 3.2 3.3 ▸ DEG APPRX REAL

Find  $P(\text{adult}|\text{lost})$ . Find  $P(\text{did not play}|\text{child})$ .

0/99 0/99

3.1 3.2 3.3 3.4 ▸ DEG APPRX REAL

Find:  $P(\text{senior})$ ,  $P(\text{senior}|\text{won})$ , and  $P(\text{won}|\text{senior})$ .

0/99

3.2 3.3 3.4 4.1 ▸ DEG APPRX REAL

A game has 4 upside-down cups. One cup has a prize under it. If randomly choosing a cup, what is...

$P(\text{prize})$        $P(\text{no prize})$

0/99 0/99

3.3 3.4 4.1 4.2 ▸ DEG APPRX REAL

$P(\text{prize})$  and  $P(\text{no prize})$  are **complements**. What is  $P(\text{prize}) + P(\text{no prize})$ ?

0/99

3.4 4.1 4.2 4.3 ▸ DEG APPRX REAL

Suppose a child plays the cup game 5 times. List all the possible number of times the child could win a prize.

4.1 4.2 4.3 4.4 ▸ DEG APPRX REAL

$P(\text{at least 4 prizes})$  means  $P(4 \text{ prizes}) + P(5 \text{ prizes})$ . What is meant by  $P(\text{at least 1 prize})$ ?

4.2 4.3 4.4 4.5 ▸ DEG APPRX REAL

What is the complement of  $P(\text{at least 1 prize})$ ? Find the probability that the complement occurs.

0/99

4.3 4.4 4.5 4.6 ▸ DEG APPRX REAL

What is the probability that the child wins at least one prize in 5 games?

0/99

4.4 4.5 4.6 4.7 ▸ DEG APPRX REAL

What is the probability that the child wins at least one prize in 3 games?

0/99

4.5 4.6 4.7 4.8 ▸ DEG APPRX REAL

Suppose the cup game has 5 cups where 2 cups have prizes under them. Find the probability of winning at least one prize in 4 games.

0/99