Asymptotes and Zeros – ID: 9301

Time required
45 minutes

Topic: Rational & Radical Functions & Equations

- *Graph any rational function and identify its singularities and asymptotes.*

- *Factor the denominator of a rational function to locate its singularities.*

- *Approximate the solutions to rational equations and inequalities by graphing.*

Activity Overview

In this activity, students relate the graph of a rational function to the graphs of the polynomial functions of its numerator and denominator. Students graph these polynomials one at a time and identify their y-intercepts and zeros. These features of the graph are connected with the standard and factored forms of the equation. Next students graph the rational function, and connect its behavior with that of the polynomial functions. Finally, students view the numerator, denominator and rational functions on the same graph. Using the handheld's manual manipulation functions, students can manipulate the graphs of the numerator and denominator functions and see the effect on the rational function.

Teacher Preparation

- *This activity is designed to be used in an Algebra 2 or Pre-calculus classroom.*

- *Prior to beginning this activity, students should have an introduction to polynomial functions, their graphs, and the concept of degree. This activity may serve as an introduction to rational functions.*

Classroom Management

- *This activity is intended to be mainly **teacher-led** with brief periods of independents student work.*

- *This worksheet helps guide students through the activity and provides a place for them to record their answers.*

TI-84 Plus Applications
None

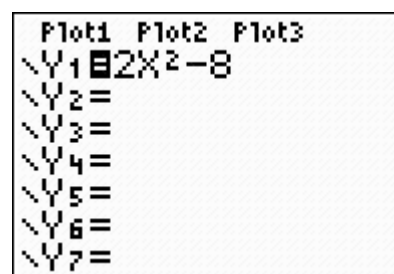## Asymptotes and Zeros of Rational Functions
ID: 9301

ACTIVITY OVERVIEW:
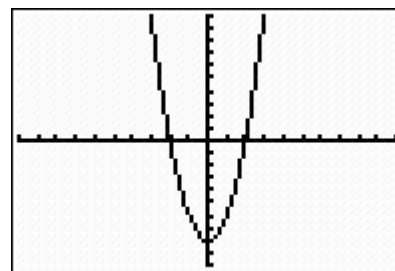
In this activity we will

- *graph polynomial functions and examine their zeros and y-intercepts*

- *analyze how the zeros and y-intercepts of the numerator and denominator affect the graph of a rational function*

A rational function is the quotient of two polynomial functions where the polynomial function in the denominator is of degree 1 or higher. To understand the behavior of rational functions better, let's examine the polynomial functions that make them up.
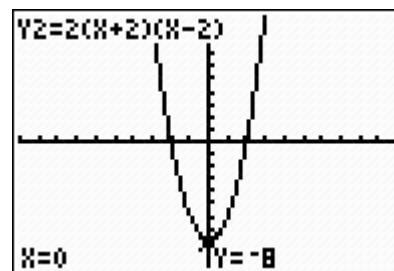

Press ! and enter the polynomial function shown. Later this function will become the *numerator* of a rational function.

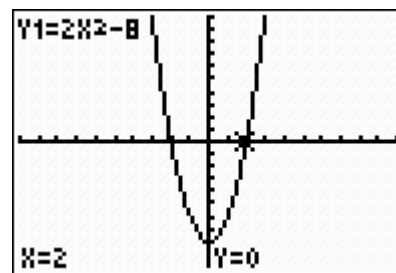Press %. Examine the graph. Where are the zeros? What is the *y*-intercept?

Use the trace feature to examine the graph closer. Find the *y*-intercept. Press $and type in 0 (for *x* = 0) e. What indication did the equation give of what the *y*-intercept would be? Record this value.

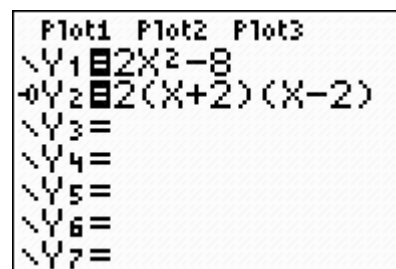*y*-intercept for *numerator* is *y* = _____

Continue to trace to find the zeros. Record.
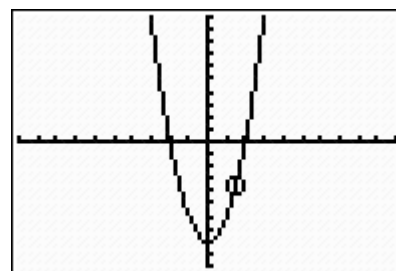
The *numerator* is zero at $x =$ _____

Press !. In **Y2** enter the factored form for $y = 2x^2 - 8$.
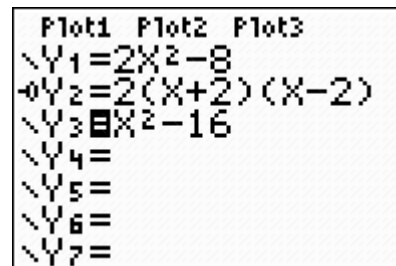
Left arrow to the left of **Y2** and press e multiple times to change the format of the graph to be a circle that leaves a trail.

Press %. If the circle traces over the original graph of **Y1** then you know that you have the correct factored form in **Y2**. How does the factored form relate to the zeros of the function?
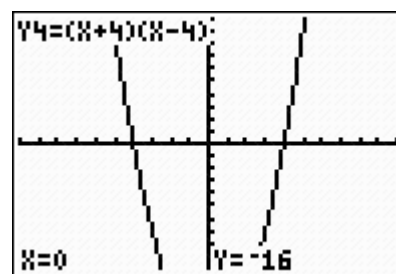
Press !. Move to the equals sign for **Y1** and press e to turn the equation off. Do the same for **Y2**. Enter the function $y = x^2 - 16$ into **Y3**. Later this will become the *denominator* of our rational function.
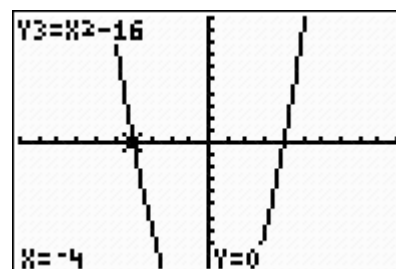
Press %. Press $and type in 0 (for $x = 0$) e. What indication did the equation give of what the *y*-intercept would be? Record this value.

y-intercept for *denominator* is $y =$ _____

Continue to trace to find the zeros. Record.
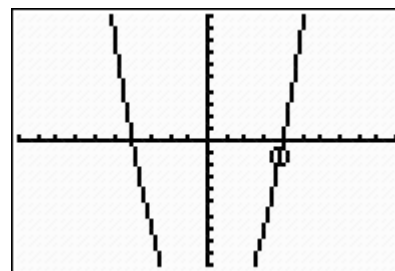
The *denominator* is zero at $x =$ _____

Press !. In **Y4** enter the factored form for $y = x^2 - 16$. Left arrow to the left of **Y4** and press e multiple times to change the format of the graph to be a circle that leaves a trail.

```
Plot1  Plot2  Plot3
\Y1=2X²-8
•Y2=2(X+2)(X-2)
\Y3❏X²-16
•Y4❏(X+4)(X-4)
\Y5=
\Y6=
\Y7=
```

Press %. If the circle traces over the original graph of **Y3** then you know that you have the correct factored form in **Y4**. How does the factored form relate to the zeros of the function?
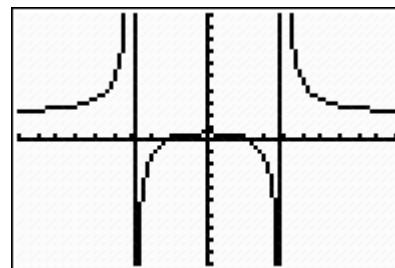
Press !. Turn the equations **Y3** and **Y4** off. Enter the rational function $y = (2x^2 - 8)/(x^2 - 16)$ into **Y5**.
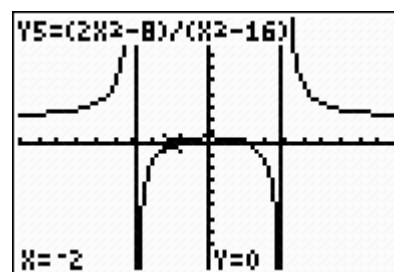
```
Plot1  Plot2  Plot3
\Y1=2X²-8
•Y2=2(X+2)(X-2)
\Y3=X²-16
•Y4=(X+4)(X-4)
\Y5❏(2X²-8)/(X²-
16)■
\Y6=
```

Press %. On cursory inspection, where do you see interesting things happening on this graph? Is this a graph of a function?
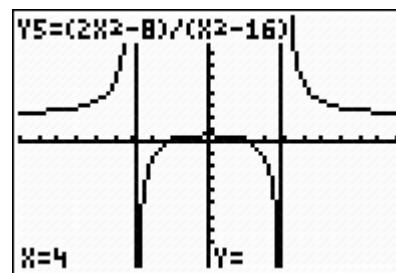
Press $. Examine the behavior of the graph at some of the interesting values of *x*. Where do zeros of this function occur? The zeros of this function occur at the same locations as the zeros of the *numerator*. Why is this true?
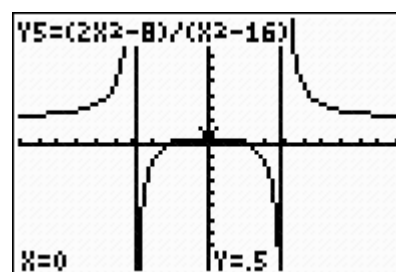
```
Y5=(2X²-8)/(X²-16)
```

```
X=-2          Y=0
```

There appear to be some vertical lines on the graph. Where do these appear? While still in trace mode, type in 4 for $x = 4$. What is the y-value at this point? What about when $x = -4$?

The things that appear to be vertical lines are not…they are vertical *asymptotes* of the function. They occur because y is not defined for these values of x. Recall that 4 and –4 were the zeros of the *denominator*. What happens when the denominator of a fraction is zero?
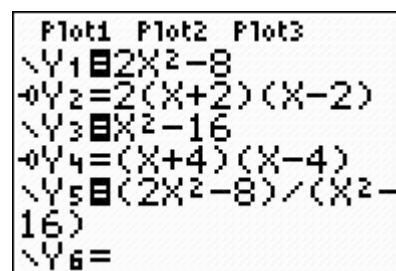
Lastly, look for the *y*-intercept. What is the value of y when x is 0?

Recall that the *y*-intercept of the *numerator* was –8 and the *y*-intercept of the *denominator* was –16. What is the quotient of these two values?

Press !. Turn the equations **Y1** and **Y3** back on.

Press %. If you look closely you can see that the zeros of the *numerator*'s parabola intersect the zeros of the rational function and that the zeros of the *denominator*'s parabola appear to cross the vertical asymptotes at $x = 4$ and $x = -4$. [Note: Asymptotes are normally invisible.]

Try this investigation with another set of polynomial functions.