A Random Piece of Pi

Teacher Notes and Answers

7 8 **9** 10 11 12

TI-Nspire Coding Student 60 min

Introduction

Teacher Notes: This activity was inspired by Matt Parker (Stand Up Maths) https://youtu.be/RZBhSi_PwHU . The activity itself is accessible to a range of year levels. For junior levels the activity is left with realisation that π is 'somehow' related to the probability of two randomly selected numbers and the likelihood that they are co-prime. For senior students, Matt Parker's video covers a portion of a proof that is very easy to follow.

In this activity you will explore a pattern related to the occurrence of common factors between two randomly selected numbers. Specifically, the investigation looks at the probability that the two numbers will be co-prime.

Co-Prime: Two numbers are said to be Co-Prime if their highest common factor is 1.

Example:

The numbers 27 and 35 are composite (not prime), however they are co-prime. The highest common factor of 27 and 35 is 1, so they are co-prime.

Question: 1.

Identify which of the following pairs of numbers are co-prime:

a)	(25, 60) GCD = 5	b)	(39, 100) Co-prime	c)	(45, 64) Co-prime
d)	(98, 56) GCD = 14	e)	(23, 115) GCD = 23	f)	(18, 29) Co-prime

Teacher Notes: Students should be encouraged to work through question 1 without a calculator. Note that Part (e) provides a clue that the presence of one prime number in the pair does not necessarily mean the numbers are co-prime. It is also worthwhile discussing how the prime factorisation of a number can be used to quickly identify the highest common factor. Part (b) for example: $39 = 3 \times 13$ and $100 = 2^2 \times 5^2$. From the prime factorisation we can see that 39 and 100 will not have any factors in common.

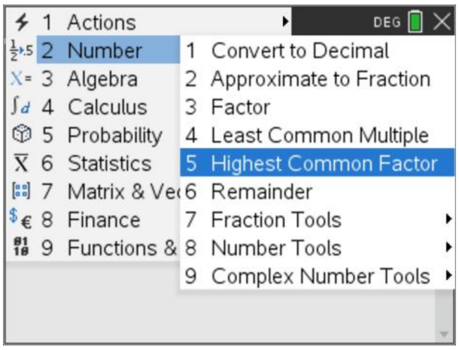
Useful Commands

GCD:

GCD = Greatest Common Divisor, also known as the Highest Common Factor. This command can be used to identify the highest common factor of any two numbers.

menu > **Number** > **Highest Common Factor**

Note: The screen shown has the language set to English (UK). If your calculator is set to English (US) Greatest Common Divisor will be displayed.



Use the GCD command to check your answers to Question 1.

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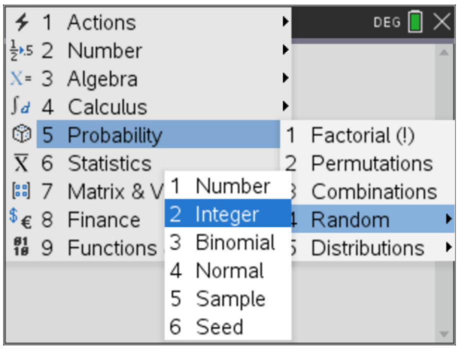
RandInt:

The random Integer command can generate random whole numbers between two specified values.

menu > **Probability** > **Random** > **Integer**

Try: randint(1,100)

Press enter several times to see what sort of numbers are produced.



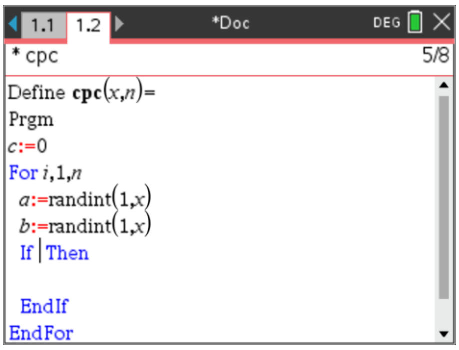
Teacher Notes: Unlike many of the other coding activities, this one leaves students to complete the necessary coding components, in other words, there is less scaffolding provided here.

Writing the Program

The start of a co-prime counting program (cpc) is shown opposite.

In the program x represents the size of the random numbers. Theoretically this should be infinite, however very larger numbers will slow program execution. Consider the size of x in relation to the number of samples, 'n'.

For example sampling 1000 pairs of random numbers to check if they are co-prime, x should be more than 100 as this limit would provide for only 9900 possible pairs of random numbers.



Finish writing the co-prime counting program so that you can explore the proportion of randomly selected pairs of numbers are co-prime. Your program needs to:

- Test and record when two randomly selected numbers are co-prime.
- Calculate the proportion of the numbers tested that are co-prime.
- Provide the opportunity to vary the number of tests conducted (n) and the magnitude of the numbers being generated (x)

Question: 2.

Testing your program is an important step in any coding exercise. Explain how you tested your program to make sure it was working as expected.

Answer: Answers will vary, but should include the use of either the "disp" (display) command or possibly lists to ensure that only co-prime numbers were counted. Students may include program listing and sample outputs.

Question: 3.

Run your program several times using cpc(100,100), cpc(100,1000) and cpc(100,10000). Record the proportion of co-primes in each sample.

Answer: Answers will vary since we are using random numbers, however as the sample size increases the variation in the proportion will decrease.
Samples:

- cpc(100,100) = 0.53, 0.55, 0.58, 0.60, 0.61…

- cpc(100,1000) = 0.58, 0.67, 0.62, 0.64 …
- cpc(100,10000) = 0.64, 0.61, 0.58, 0.66 …

Question: 4.

Run your program several times using cpc(1000,100), cpc(1000,1000) and cpc(1000,10000). Record the proportion of co-primes in each sample.

Note: cpc(1000,10000) will take a while to run!

Answer: Answers will vary since we are using random numbers, however as the sample size increases the variation in the proportion will decrease.
Samples:

- cpc(1000,100) = 0.611, 0.633, 0.594, 0.612, 0.596…
- cpc(1000,1000) = 0.629, 0.601, 0.608, 0.597 …
- cpc(1000,10000) = 0.604, 0.587, 0.603, 0.625 …

Question: 5.

For each of the proportions computed in the previous question, calculate:

$$\sqrt{\frac{6}{p}}$$ where p represents the proportion.

Answer: Answers will vary, however they should typically be around 3.15.

Question: 6.

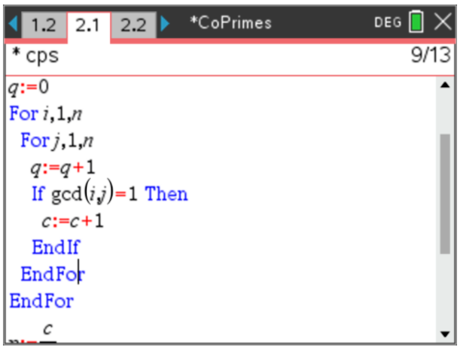
Add the calculation from Question 5 to your program and display the result. Run your program several times using the following: cpc(1000,1000). Calculate the average of the results and discuss.

Answer: As above, answers will vary, however the average should get very close to 3.14. By now students should see that the result is approaching π.

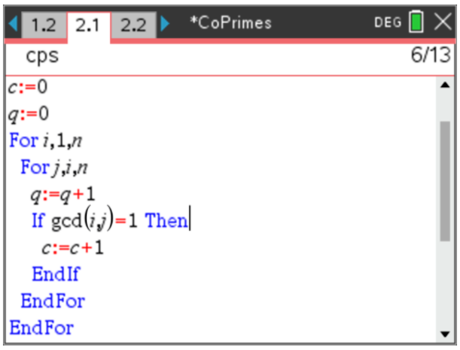
Teacher Notes: An additional option is to have students write a second program to count all the co-prime pairs for numbers up to 100 or up to 'n' and also consider 'repeats'.

Samples:

This program includes repeats, both GCD(10,30) and GCD(30,10) will be included in the count.



This program does not include repeats such as GCD(10,30) can occur but GCD(30,10) will not.



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