Math Objectives

- Students will be able to classify expressions as the sum of cubes, difference of cubes, or neither.
- Students will be able to make connections between the graph of a cubic polynomial, in the form of a sum or difference of cubes, and its factors.
- Students will be able to factor expressions in the form of a sum or difference of cubes.
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

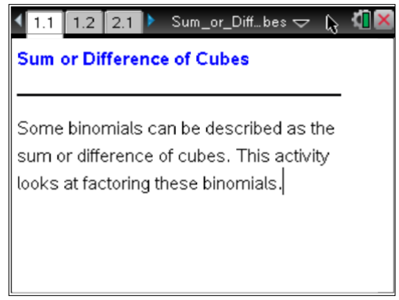
- cube roots
- sum of cubes
- difference of cubes
- linear factor
- quadratic factor

About the Lesson

- This lesson involves factoring expressions that are either the sum of cubes or the difference of cubes.
- Students will:
    - Edit text to find the cube root of terms.
    - Click a slider to identify binomial expressions as the sum of cubes, difference of cubes, or neither.
    - Study the graphs of simple cubic functions, in the form of a sum or difference of cubes, and their linear and quadratic factors.
    - Enter text in a spreadsheet to find the binomial factor of an expression that is the sum or difference of cubes.
    - Enter text in a spreadsheet to find the trinomial factor of an expression which is the sum or difference of cubes.

TI-Nspire™ Navigator™ System

- Use Screen Capture and/or Live Presenter to demonstrate the procedure for using the TI-Nspire document file, to monitor students' progress, and to discuss specific problems.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Edit text
- Enter text in a spreadsheet
- Use a minimized slider

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the entry line by pressing ctrl G.

Lesson Materials:
Student Activity
Sum_or_Difference_of_Cubes_Student.pdf
Sum_or_Difference_of_Cubes_Student.doc

TI-Nspire document
Sum_or_Difference_of_Cubes.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

- Use Quick Poll to access students' understanding of the concepts.

Discussion Points and Possible Answers

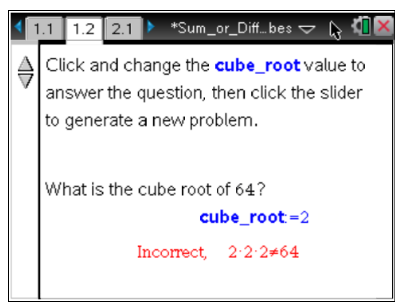
Tech Tip: To edit text on pg 1.2, click on the cube_root value. Remind students to press Enter after they make their change..

Tech Tip: Sometimes students are working with .tns files and they do something "wrong" that you don't know how to fix or don't have the time to fix. A quick fix students can handle themselves is to close the file **without saving** and then to reopen the file.

Teacher Tip: Depending on the expertise of the class, you may want to treat the first two pages of the .tns file as a review. Or you could use the software program and do them as a class to generate discussion on cubes and cube roots.

TI-Nspire Navigator Opportunity: *Screen Capture* and/or *Live Presenter*
See Note 1 at the end of this lesson.

Move to page 1.2.

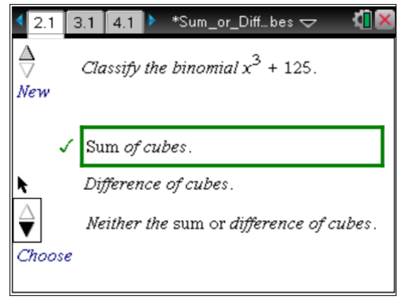


1. In order to factor binomials that are the sum or difference of cubes, you must be able find cube roots. Click on the Cube_root value to edit the cube root of the term shown. Press [del] to erase the current number and enter your answer. A message will tell you if your answer is correct. Click the slider (up or down arrows) to generate a new problem. How can you determine the sign of the cube root?

<u>**Answer:**</u> The sign of the cube root is the same as the sign of the cube. If the cube is positive, you need a positive number to use as a factor three times. If the cube is negative, you need a negative number to use as a factor three times.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

Move to page 2.1.



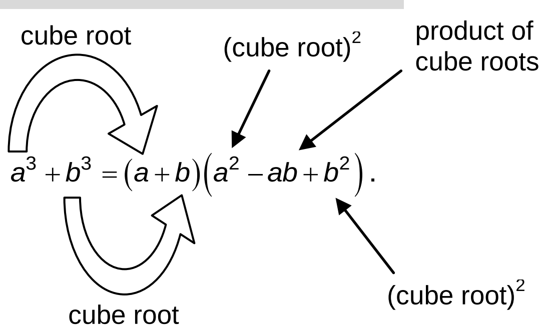
2. Click on the Choose slider to answer the question. Click on the New slider (up or down arrows) to generate a new question. Explain how to determine whether the binomial is the sum of cubes, difference of cubes, or neither.

> **Teacher Tip:** The formula for factoring the sum or difference of cubes will work for non-perfect cubes. For example:
> $$x^3 + 2 = \left(x + \sqrt[3]{2}\right)\left(x^2 - \sqrt[3]{2} \cdot x + \sqrt[3]{4}\right).$$
> However, we usually require perfect cubes in order to classify an expression as the sum or difference of cubes. $x^3 + 2$ would not be called the sum of cubes.

<u>Answer:</u> The binomial's terms should both be perfect cubes. If the sign between the perfect cubes is positive, you have the sum of cubes. If the sign between the perfect cubes is negative, you have the difference of cubes.

> **Teacher Tip:** You may want to do long division for polynomials to demonstrate how the formula is derived.
> $$(a+b)\overline{)a^3 + b^3} = (a+b)\overline{)a^3 + 0a^2b + 0ab^2 + b^3}$$
> You may stimulate discussion by asking questions like: "Why are the first and last terms of the trinomial factor always positive?" "What is the root of a sum or difference of cubes function?" "How do we know that there will always be one root?"

The sum of cubes will factor according to the formula: $a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$.



Notice the sign in the binomial factor is the same as the sign in the original binomial.
Note how the sign between the first two terms of the trinomial factor is the opposite.

Similarly the difference of cubes factors as: $a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$.

Notice the sign in the binomial factor is the same as the sign in the original binomial.
Note how the sign between the first two terms of the trinomial factor is the opposite.

3. a. Is the pattern for factoring the difference of cubes the same as for the sum of cubes? Explain.

Answer: Yes, the pattern is the same. The binomial factor is the difference of the cube roots. The trinomial factor has the first cube root squared, plus the product of the two cube roots, plus the second cube root squared.

Teacher Tip: When expanding with more than one variable, it helps students stay organized if they write the terms in alphabetical order. For example, $a \cdot b^2$ and $b^2 \cdot a$ would both be written as ab^2.

b. Use the Distributive Property to justify that both formulas result in the sum or difference of cubes.

Answer: The formula for the sum of cubes:

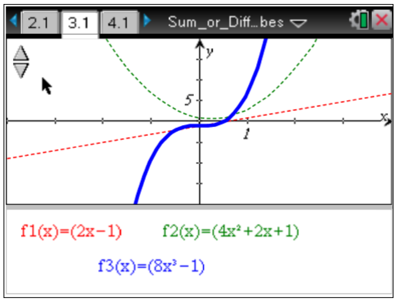
$$\begin{aligned}&(a+b)\left(a^2 - ab + b^2\right)\\ &= a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3\\ &= a^3 + b^3\end{aligned}$$

The formula for the difference of cubes:

$$\begin{aligned}&(a-b)\left(a^2 + ab + b^2\right)\\ &= a^3 + a^2b + ab^2 - a^2b - ab^2 + b^3\\ &= a^3 - b^3\end{aligned}$$

Teacher Tip: Simple cubic polynomial functions in the form of a sum or difference of cubes will have a linear and a quadratic factor. It may help students understand how they are factored by looking at the graph of the factors and their product. Note that only a small part of the cubic function may show in some examples. Changing the viewing window is not really practical here. The *x*-intercept will always be in the viewing window.

Move to page 3.1.



4. Click on the slider (up or down arrows) to generate new graphs. The graphs of the linear and quadratic factors are dotted. The graph of the product (the sum or difference of cubes) is thick. What connection does the graph of the sum or difference of cubes have with its linear factor?

   Answer: The graphs of the linear factor and the sum or difference same *x*-intercept.

5. Will the sum or difference of cubes function always cross the *x*-axis? How do you know?

   Answer: The cubic function either comes from $-\infty$ and goes to ∞ or vice versa. It must cross the *x*-axis at least once.

6. a. How does the graph of the quadratic factor of the sum or difference of cubes show that it is not factorable?

   Answer: The graph of the quadratic function does not intersect the *x*-axis. Therefore, it has no real roots. Students might also be encouraged to use the discriminant studied earlier to determine that there are no real roots.

   b. Prove algebraically that the trinomial factors as shown above $\left(a^2 - ab + b^2\right.$ and $\left.a^2 + ab + b^2\right)$ are not factorable.

   Answer: Using the quadratic root formula: $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where $(a = 1, b = \pm b, c = b^2)$

$$\text{root } a = \frac{b \pm \sqrt{(-b)^2 - 4b^2}}{2} = \frac{b \pm \sqrt{-3b^2}}{2} \quad \text{or} \quad \text{root } a = \frac{-b \pm \sqrt{b^2 - 4b^2}}{2} = \frac{-b \pm \sqrt{-3b^2}}{2}$$

   In both cases we have the square root of a negative number. Therefore, neither trinomial is factorable.

> **Teacher Tip:** You may want to assign different problems to students or possibly have students work in pairs and share their results.

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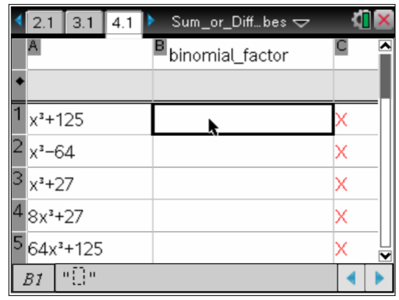
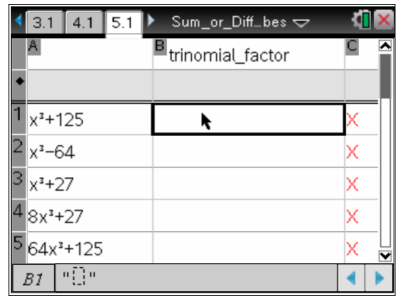


Enter the binomial factor, in **quotes,** into Column B in the spreadsheet. Press [enter] and a check mark will indicate when your answer is correct. It automatically moves to the next entry.

Move to page 5.1.



Enter the trinomial factor, in **quotes,** into Column B in the spreadsheet. Press [enter] and a check mark will indicate when your answer is correct. It automatically moves to the next entry.

7. Andrew correctly finds that the first factor of the sum of cubes is $(3xy+2z)$. What is the second factor? What is the expanded form?

   <u>Answer:</u> The second factor is $\left(9x^2y^2-6xyz+4z^2\right)$. The expanded form is $27x^3y^3+8z^3$.

8. Alex correctly finds that the second factor of the sum of cubes is $\left(4a^2-10abc+25b^2c^2\right)$. What is the first factor? What is the expanded form?

   <u>Answer:</u> The first factor is $(2a+5bc)$ and the expanded form is $8a^3+125b^3c^3$.

9. Sometimes the sum or difference of cubes can be factored further.

   a. Can x^6-64 be considered the difference of cubes? Explain and factor accordingly.

   <u>Answer:</u> Yes, it is the difference of cubes. $x^6-64=\left(x^2\right)^3-(4)^3$

$$\begin{aligned} x^6-64 &= \left(x^2\right)^3-(4)^3 \\ &= \left(x^2-4\right)\left(x^4+4x^2+16\right) \\ &= (x-2)(x+2)\left(x^4+4x^2+16\right) \end{aligned}$$

Teacher Tip: Students may not expect the second factor in the difference of cubes to factor further. This question shows that if the difference of cubes is not a simple cubic polynomial, it is possible that one or both factors can be simplified further. You may stimulate discussion by asking the question: “What other powers or numbers are both perfect squares and perfect cubes?” (The number 1 is often overlooked by students as both a perfect square and a perfect cube. The power x^{12} could be expressed as $\left(x^4\right)^3$ or as $\left(x^6\right)^2$, for example.) You may want to deal with the sum or difference of cubes with common factors as well. “Factor $5x^3+40$.” (It factors to $5\left(x^3+8\right)=5(x+2)\left(x^2-2x+4\right)$.)

b. Can x^6-64 be considered the difference of squares? Explain and factor accordingly.

Answer: Yes, it is the difference of squares. $x^6-64=\left(x^3\right)^2-(8)^2$

$$\begin{aligned} x^6-64 &= \left(x^3\right)^2-(8)^2 \\ &= \left(x^3-8\right)\left(x^3+8\right) \\ &= (x-2)\left(x^2+2x+4\right)(x+2)\left(x^2-2x+4\right) \\ &= (x-2)(x+2)\left(x^2+2x+4\right)\left(x^2-2x+4\right) \end{aligned}$$

c. Explain the different factored forms for part 9a and part 9b.

Answer: Both answers are the same. Looking at the expression as the difference of squares allowed us to factor more fully.

$$(x-2)(x+2)\left(x^4+4x^2+16\right)\stackrel{?}{=}(x-2)(x+2)\left(x^2+2x+4\right)\left(x^2-2x+4\right)$$

$$\cancel{(x-2)}\cancel{(x+2)}\left(x^4+4x^2+16\right)\stackrel{?}{=}\cancel{(x-2)}\cancel{(x+2)}\left(x^2+2x+4\right)\left(x^2-2x+4\right)$$

$$\left(x^4+4x^2+16\right)\stackrel{?}{=}\left(x^2+2x+4\right)\left(x^2-2x+4\right)$$

$$\left(x^4+4x^2+16\right)\stackrel{?}{=}x^4\cancel{-2x^3}+\cancel{4x^2}+\cancel{2x^3}\cancel{-4x^2}+\cancel{8x}+4x^2\cancel{-8x}+16$$

$$\left(x^4+4x^2+16\right)\stackrel{?}{=}\left(x^4+4x^2+16\right)$$

TI-Nspire Navigator Opportunity: ***Quick Poll***
See Note 3 at the end of this lesson.

Wrap Up

At the end of the discussion, students should understand:

- How to identify a binomial in the form of a sum or difference of cubes.
- How to factor a binomial in the form of a sum or difference of cubes.

Assessment

Sample Questions:

1. Which of the following is not a perfect cube: $343x^3, a^3b, -64, -216c^9$.

   a. $343x^3$

   b. a^3b

   c. -64

   d. $-216c^9$

2. If a difference of cubes has one factor equal to $(2x-3yz)$, the other factor is:

   a. $(4x^2-6xyz+9y^2z^2)$

   b. $(4x^2-6xyz-9y^2z^2)$

   c. $(4x^2+6xyz-9y^2z^2)$

   d. $(4x^2+6xyz+9y^2z^2)$

3. If a difference of cubes has one factor equal to $(2x-3yz)$, the expanded form is:

   a. $8x^3-27y^3z^3$

   b. $8x^3+27y^3z^3$

   c. $4x^3-9y^3z^3$

   d. $4x^3+9y^3z^3$

4. $3x^3-24$ can be factored to:

   a. $3(x+2)(x^2-2x+4)$

   b. $3(x-2)(x^2+2x+4)$

   c. $3(x-2)(9x^2+6x+4)$

   d. $3(x-2)(x^2+6x+4)$

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Note 1

Questions 1–4, *Screen Capture* and/or *Live Presenter:* You can demonstrate the procedure for using the TI-Nspire document file, monitor student progress, and display specific questions for class discussion.

Note 2

Question 1, *Quick Poll:* In addition to the results from page 1.2, you may want to use *Quick Poll* to ask students questions that use variables. For example, "What is the cube root of x^6? (or x^9 or $8x^3$)" This can encourage lots of discussion.

Note 3

End of Lesson, *Quick Poll:* You can use *Quick Poll* to assess students' understanding of the concepts of this lesson. Some sample questions are included above.