Year 10 Mathematics Problem Solver

Challenge Problem : Fraction Machine Investigation

MARKING SCHEME

a) Define a function in your calculator that simulates this machine. Write the calculator command below
Define(f1=(1-f)/(1+f))   (M1)

They need to have a different variable here: f1 or fr or similar, not f.

b) Feed in $\dfrac{3}{5}$. What is the result? Write the calculator command and the result below.

fr/f=3/5   (M1)
Answer given above = ¼ so no marks for that.

c) What results if $\dfrac{1}{4}$ is fed into the machine?

fr/f=1/4   output = 3/5   (A1)

d) Suppose $\dfrac{2}{3}$ was fed into the machine. The result is fed back into the machine. The new result is fed back into the machine and so on for 1000 processes altogether. What will the final result be?

fr/f=2/3   output=1/5 (odd number of times)
fr/f=1/5   output=2/3 (even number of times)   (A1) for 2/3
Since 1000 is an even number, the output will be 2/3   (R1) for valid reasoning

e) What is the result if the general fraction $\dfrac{a}{b}$ is fed into the machine?

fr/f=a/b   output=$\dfrac{-(a-b)}{a+b}=\dfrac{b-a}{a+b}$ (A1)

What would be the result after one million processes?

$\dfrac{a}{b}$ (A1)

What can you conclude about the number of processes and the resulting fraction?

Even: a/b   (R1)

Odd $\dfrac{-(a-b)}{a+b} = \dfrac{b-a}{a+b}$ (R1)

f) Investigate what happens when we feed a negative fraction into the machine. Select a few negative fractions and use your calculator to produce the answer. Write all steps below.

$$-\frac{2}{3} \to 5, \quad -\frac{3}{5} \to 4, \quad -\frac{1}{7} \to \frac{4}{3}$$

(M2) for a few examples shown

g) What is the result if the general negative fraction $-\dfrac{a}{b}$ is fed into the machine?

fr/ f=-a/b   output=$\dfrac{-(a+b)}{a-b}$ (A1)

h) Assume that we want to get a whole number as the output of the machine. For example: when $f = -\dfrac{3}{5}$, the machine returns the number 4. Can you come up with the general rule for the fraction f which will produce a whole number as an output?

When $a-b=1$ the output is the whole number   (R1)

Also, when $\dfrac{even}{2}$ the result is the whole number   (R1)

NOTE: Saying that it happens when the fraction is negative results in no marks for this part.

They may have other valid reasoning – use your judgment.

TOTAL = 14 marks