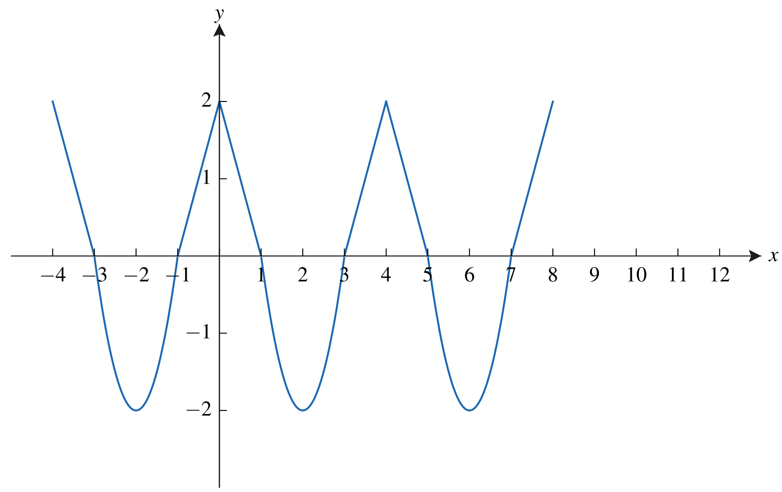
Thursday Night PreCalculus, January 11, 2024

Ride the Wave: Periodic Phenomena

Problems

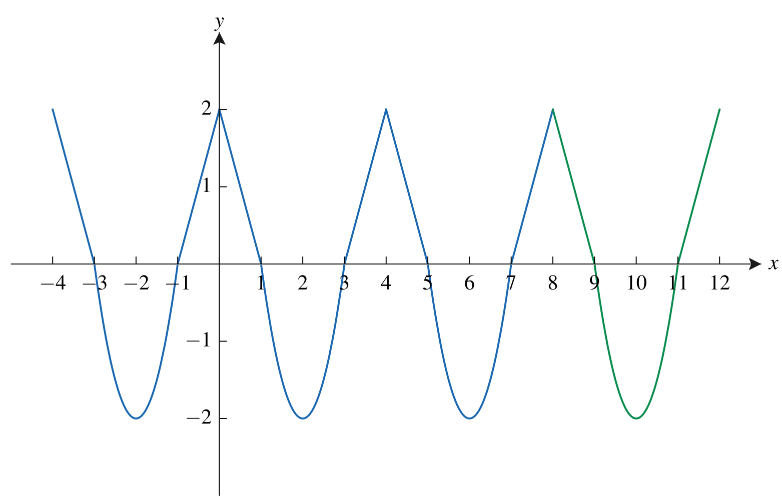
1. The graph of a periodic function f is shown.



(a) What is the period, p, of the function?

$p = 4$

(b) Sketch the next cycle of the given graph.



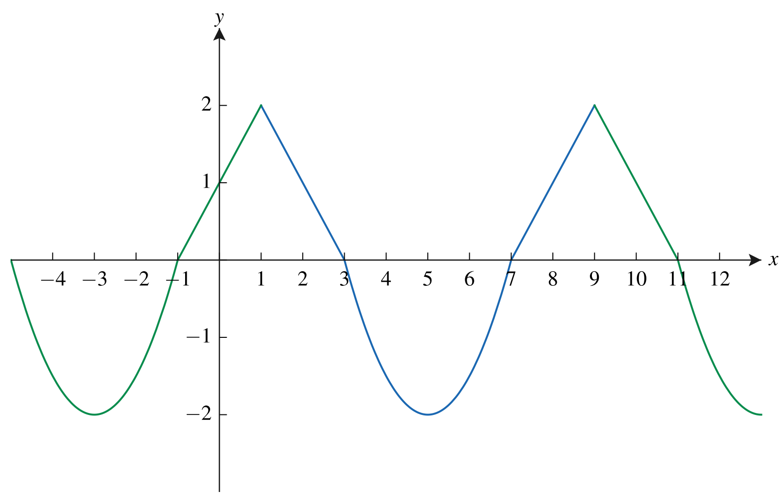
(c) Determine whether each function is periodic. If it is, state the period. If it is not, explain why.

(i) $y = f\left(\frac{1}{2}(x-1)\right)$

$g(x) = f(x-1)$ is an additive transformation of the function f that results in a horizontal translation of the graph of f by 1 unit.

$h(x) = f\left(\frac{1}{2}(x-1)\right)$ is a multiplicative transformation of the function g that results in a horizontal dilation of the graph of g by a factor of 2.

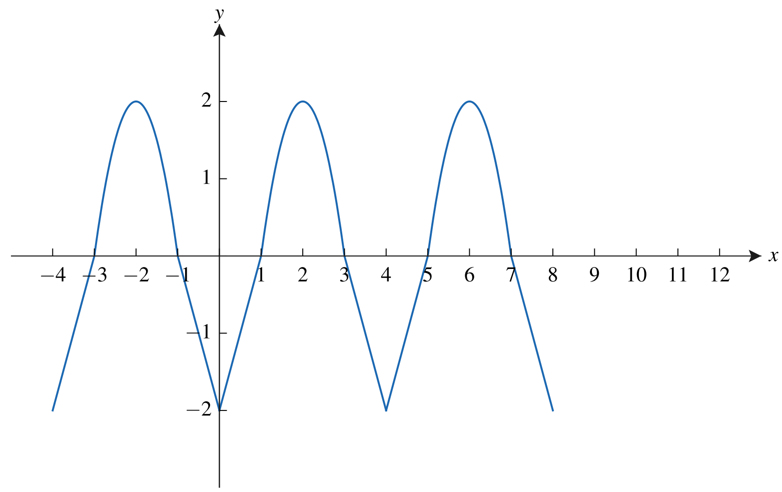
Therefore, the graph of h is periodic with $p = 2 \cdot 4 = 8$.



(ii) $y = -f(x)$

$g(x) = -f(x)$ is a multiplicative transformation of the function f that results in a reflection of the graph of f over the x-axis.

The graph of g is periodic with period $p = 4$.

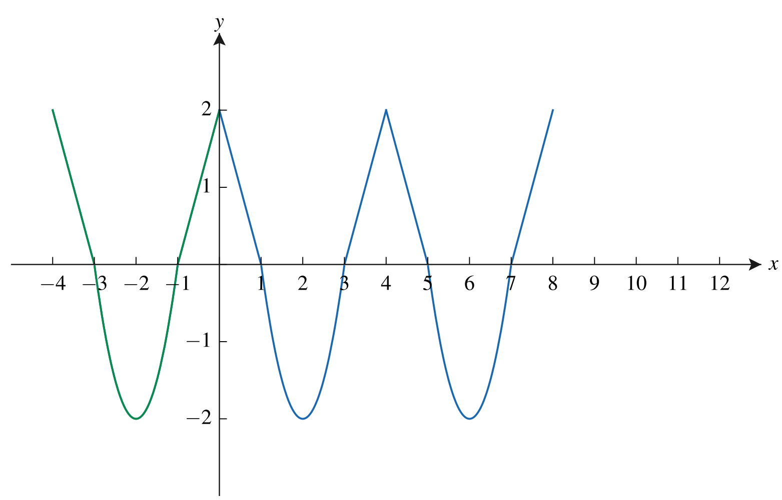


(iii) $y = f(-x)$

$g(x) = f(-x)$ is a multiplicative transformation of the function f that results in a reflection of the graph of f over the y-axis.

The graph of g is periodic with period $p = 4$.

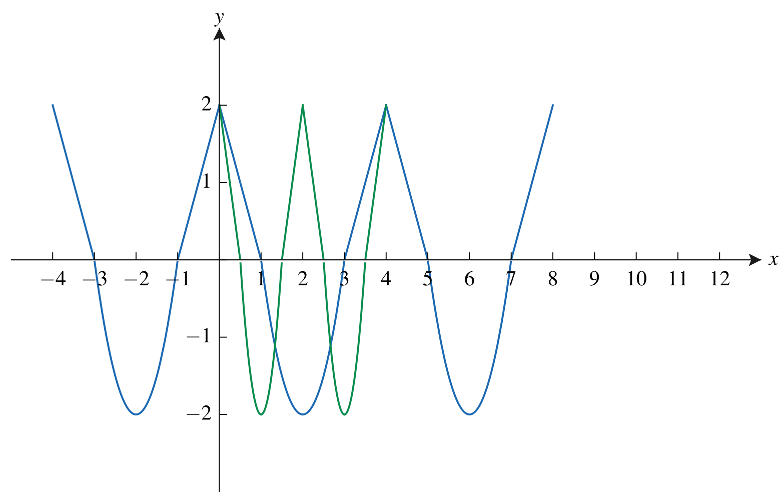
Note that f is an even function: $f(x) = f(-x)$.



(iv) $y = f(2x)$

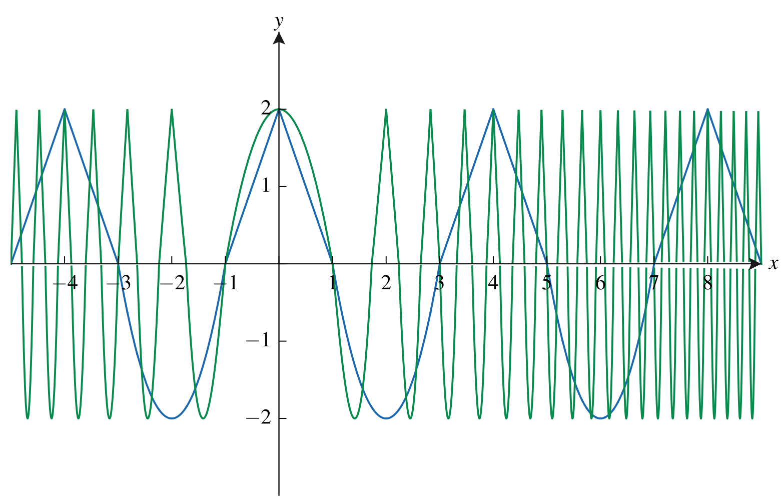
$g(x) = f(2x)$ is a multiplicative transformation of the function f that results in a horizontal dilation of the graph of f by a factor of $\frac{1}{2}$.

Therefore, the graph of g is periodic with period $p = \frac{1}{2} \cdot 4 = 2$.

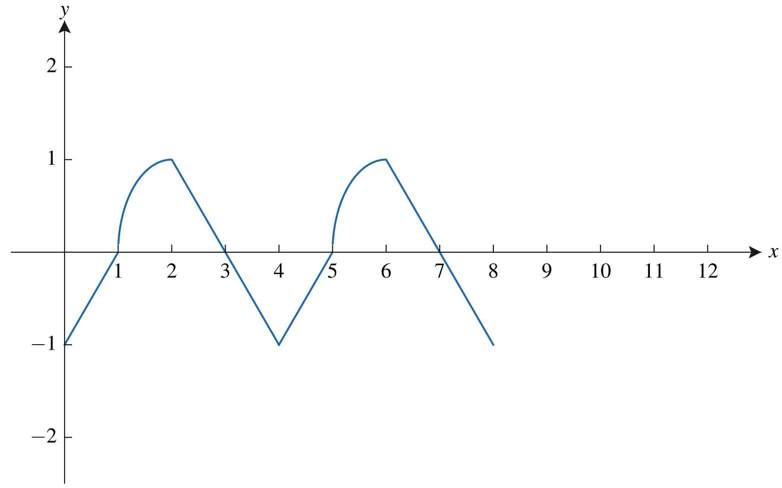


(v) $y = f(x^2)$

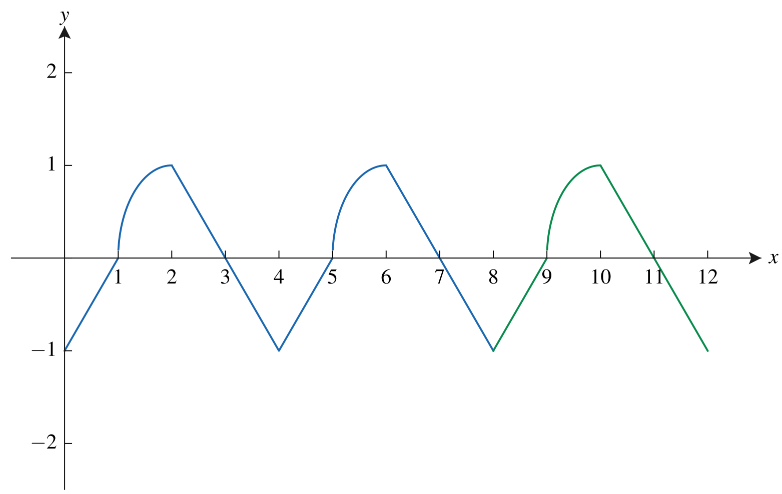
$g(x) = f(x^2)$ is neither an additive nor multiplicative transformation of the function f.



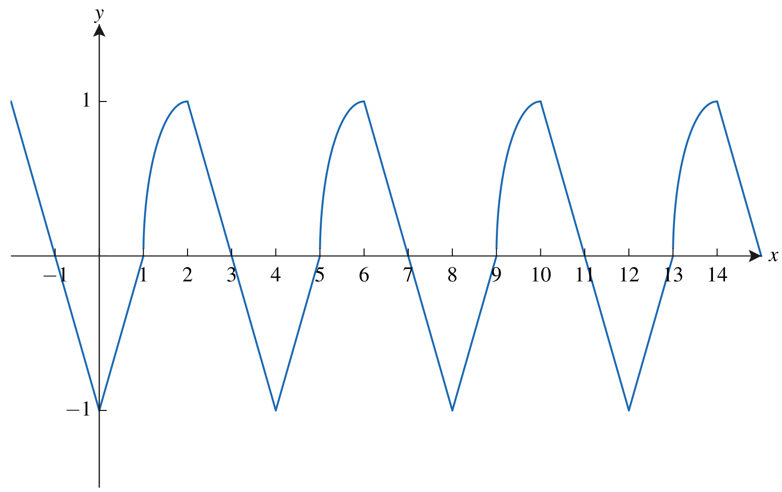
2. The graph of a periodic function f is shown below.



(a) Sketch another cycle of the function on the interval [8, 12].

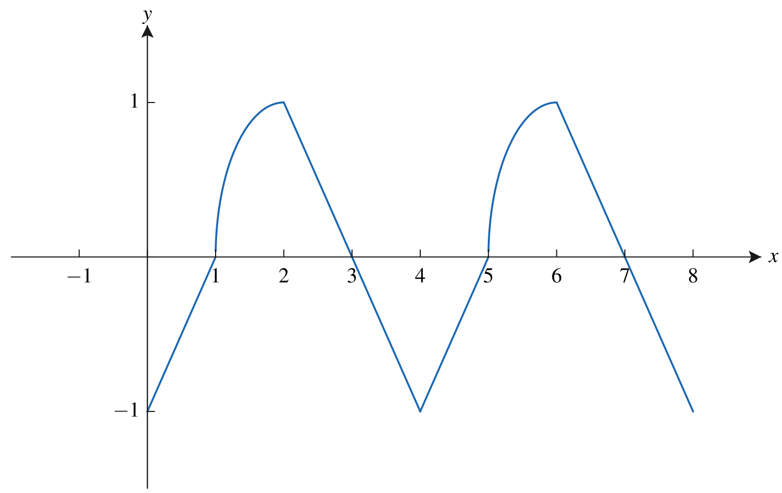


(b) Find $f(14)$ and $f(-1)$.



$f(14) = 1$ $\qquad$ $f(-1) = 0$

(c) Find the open intervals for $0 \leq x \leq 8$ on which the function is increasing and concave down.

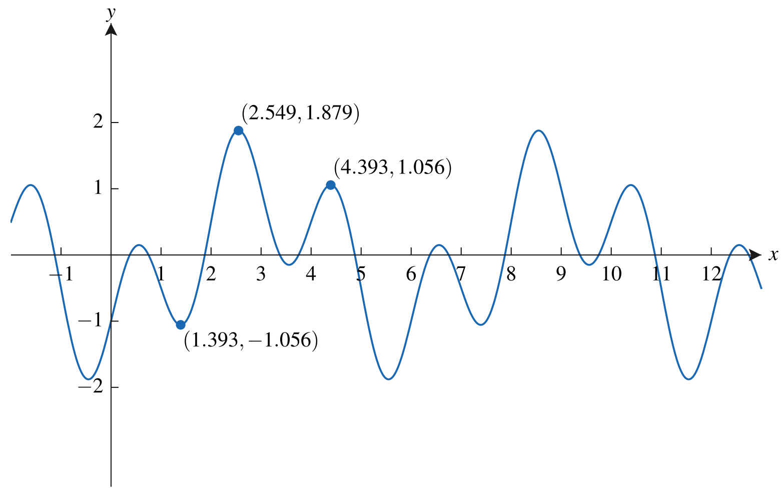


The function f is increasing and concave down on the intervals $(1, 2)$ and $(5, 6)$.

(d) Find the open intervals for $0 \leq x \leq 8$ on which the function is decreasing and concave up.

There are no open intervals for $0 \leq x \leq 8$ on which the function f is decreasing and concave up.

3. The graph of a periodic function f is shown.



(a) Write an expression for a function g that is a horizontal translation of the graph of f which would be the exact same graph as that of f.

$g(x) = f(x + 6)$

(b) Using the period of f, find the number of complete cycles of the graph of f in the xy-plane on the interval $0 \leq x \leq 350$.

f is periodic with period $p = 6$.

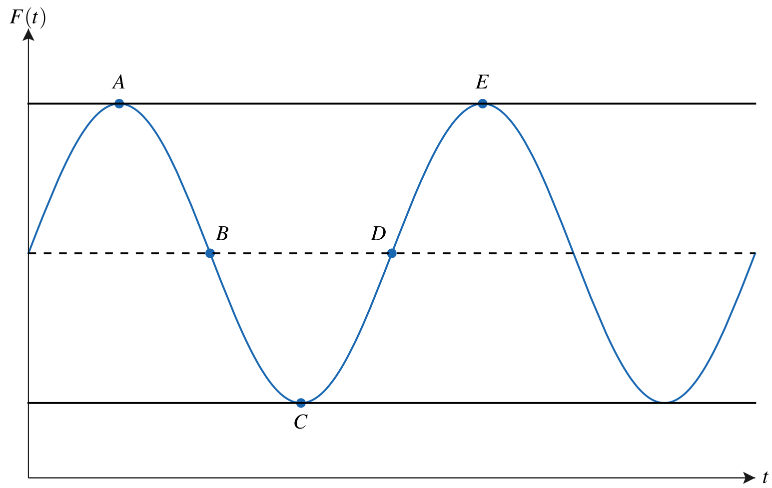
$$\frac{350}{6} = 58.333$$

Number of complete cycles $= 58$.

4. The blades of a large industrial fan spin in a clockwise direction and rotate at a rate of 10 revolutions per second. Let the point P be the tip of the blade that is straight up at time $t = 0$. Point A is 75 inches from the floor. Each blade has length 14 inches from the center.

Let the periodic function F model the distance between point A and the floor, in inches, as a function of time t, in seconds.

(a) Use the given information to find possible coordinates $(t, F(t))$ of the points A, B, C, D, and E on the graph below.



$$10 \text{ rev per sec} \quad \Rightarrow \quad 1 \text{ rev per } \frac{1}{10} \text{ sec}$$

$A: \ (0, 75)$

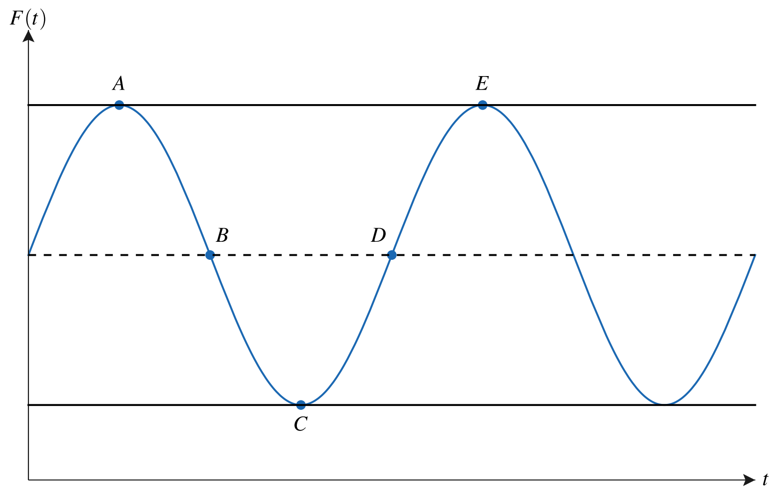
$E: \ \left(\frac{1}{10}, 75\right)$ or $(0.1, 75)$

$C: \ \left(\frac{1}{20}, 47\right)$ or $(0.05, 47)$

$B: \ \left(\frac{1}{40}, 61\right)$ or $(0.025, 61)$

$D: \ \left(\frac{3}{40}, 61\right)$ or $(0.075, 61)$

(b)



Use the graph of $y = F(t)$ and the points A, B, C, D, and E to find a time interval on which the graph of F is increasing and concave down.

The graph of F is increasing and concave down on the interval from D to E, that is,

on the interval $\left(\frac{3}{40}, \frac{1}{10}\right)$

(c) Find a time interval on which the graph of F is decreasing and concave down.

The graph of F is decreasing and concave down on the interval from A to B, that is,

on the interval $\left(0, \frac{1}{40}\right)$.

Overtime Problems

1. The table gives values for the amount of a certain substance, in grams, on certain days. The data are modeled by an exponential function f given by $f(t) = 3e^{r \cdot t}$, where t is the number of days since day 0. The constant r is defined as the continuous growth rate of this model. Based on the table, what is the value of r?

Day	0	5
Amount (grams)	3	19.563

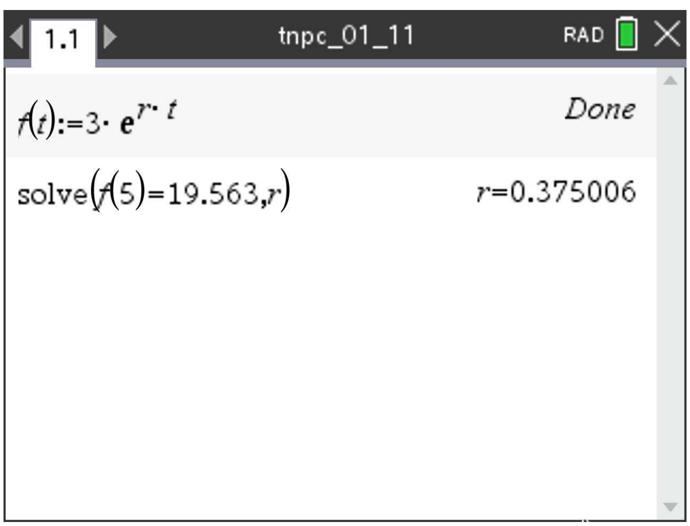
$$f(0) = 3 \cdot e^{r \cdot 0} = 3 \cdot e^0 = 3 \cdot 1 = 3$$

$$f(5) = 3 \cdot e^{r \cdot 5} = 19.563$$

$$e^{5r} = \frac{19.563}{3} = 6.521$$

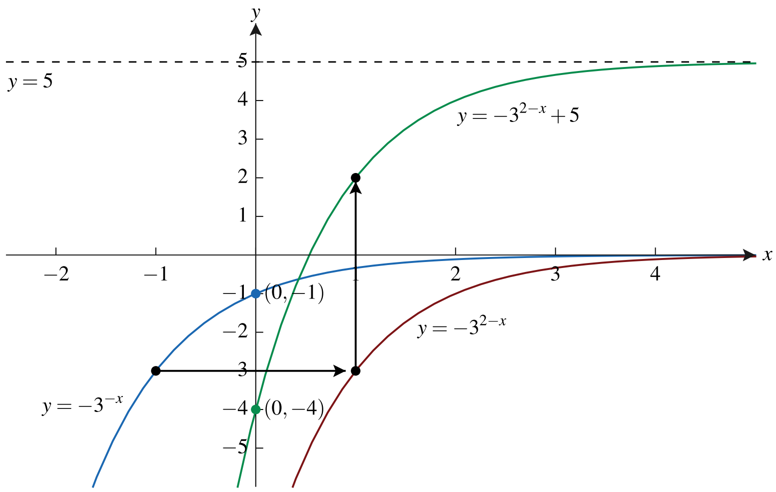
$$5r = \ln(6.521) = 1.875$$

$$r = \frac{1.875}{5} = 0.375$$



2. Let the functions f and g be defined by $f(x) = -3^{-x}$ and $g(x) = -3^{2-x} + 5$.

Sketch a graph of the function f, then use the graph to obtain the graph of g. Label the asymptote and y-intercept of each graph. Find the domain and range of g.



Domain g : $(-\infty, \infty)$

Range g : $(-\infty, 5)$