EXPLORATIONS

Activity 10

Roots

Concepts/Skills:

Roots, powers of 10, problem solving

Calculator:

TI-30Xa SE or TI-34

Objectives:

Students explore the relationship between the number of digits in a number and the number of digits in the whole number part of the nth root of that number.

Getting Students Involved

Finding patterns is an important part of mathematics. Middle grades students have been finding patterns all their lives.

♦ What strategies do you use to find patterns in numbers?

Look for regularities or regular variations in the patterns.

Making Mathematical Connections

Students will probably know the rule*, If you multiply two powers of 10, the number of zeros in the product is the sum of the number of zeros in the factors*, though they might only know this implicitly. You might want to make the rule explicit.

♦ If you multiply a 3-digit number by a 4-digit number, what can you say about the number of digits in the product? Can it ever be six? Can it be seven? Can it be eight? Can it be nine?

100 x 1000 = 100,000 (a six-digit number).

999 x 9999 = 9,989,001 (a seven-digit number).

Remind students that when you multiply the square root of a number by itself, the result is that number.

♦ If you multiply a whole number by itself, how is the number of digits in the product related to the number of digits in the original number?

Either twice as many or one less than twice as many. For example,
5 x 5 = 25
100 x 100 = 10,000

- If you know the number of digits in a perfect square, what can you say about the number of digits in the square root?

Divide the number of digits in the perfect square by two, and if that number is not a whole number, round up to the next greater whole number.

Carrying Out the Investigation

Use the two transparencies at the end of this activity (pages 85-86) to demonstrate. Two rows of the tables are completed as models for students. Be sure that students understand what *whole number part* means. Although students are asked to write the roots of the numbers, the critical information is the number of digits in the whole number part of each root. **Be sure that students do not waste time writing all of the decimal digits in the roots**.

If students are having trouble seeing the patterns, suggest that they add a column to the table that shows the number of digits in N.

Making Sense of What Happened

Ask students to look for patterns in the tables that will tell them when the number of digits in roots increases.

- What do you notice about N when the number of digits in the whole number part of the root increases from 1 to 2? From 2 to 3? From 3 to 4?
- How can you tell by looking at the number of digits in N how many digits there ought to be in the nth root of N?

Continuing the Investigation

Use the tables to make predictions.

- Will the whole number part of $\sqrt{24}$ be a one-, two-, or three-digit number? Will $\sqrt{24}$ be closer to $\sqrt{16}$ or $\sqrt{25}$?
- Will the whole number part of $\sqrt{450{,}000}$ be a one-, two-, or three-digit number? Will $\sqrt{450{,}000}$ be closer to $\sqrt{360{,}000}$ or $\sqrt{490{,}000}$? Since $\sqrt{360{,}000} = 600$ and $\sqrt{450{,}000} = 700$, what is a reasonable value for $\sqrt{450{,}000}$?
- Will the whole number part of $\sqrt[3]{29{,}000}$ a one-, two-, or three-digit number? Since $\sqrt[3]{27{,}000} = 30$, what is a reasonable value for $\sqrt[3]{29{,}000}$?

Solutions

1.

N	$\sqrt{N}$	Whole number part of $\sqrt{N}$	Number of digits in whole number part of $\sqrt{N}$
1	1	1	1
9	3	3	1
10	3.162...	3	1
99	9.949...	9	1
100	10	10	2
999	31.606...	31	2
1,000	31.622...	31	2
9,999	99.994...	99	2
10,000	100	100	3
99,999	316.226...	316	3
100,000	316.227...	316	3
999,999	999.999...	999	3
1,000,000	1000	1000	4

2. The number of digits in N is twice as great (or one less than twice as great) as the number of digits in the whole number part of $\sqrt{N}$.

3.

N	$\sqrt[3]{N}$	Whole number part of $\sqrt[3]{N}$	Number of digits in whole number part of $\sqrt[3]{N}$
1	1	1	1
9	2.080...	2	1
10	2.154...	2	1
99	4.626...	4	1
100	4.641...	4	1
999	9.996	9	1
1,000	10	10	2
9,999	21.543...	21	2
10,000	21.544...	21	2
99,999	46.415...	46	2
100,000	46.415...	46	2
999,999	99.999...	99	2
1,000,000	100	100	3

4. When the number of digits of N changes from 3 to 4 or from 6 to 7, the number of digits in $\sqrt[3]{N}$ increases by 1.

5.

N	$\sqrt[4]{N}$	Whole number part of $\sqrt[4]{N}$	Number of digits in whole number part of $\sqrt[4]{N}$
1	1	1	1
9	1.732...	1	1
10	1.778...	1	1
99	3.154...	3	1
100	3.162...	3	1
999	5.622...	5	1
1,000	5.623...	5	1
9,999	9.999...	9	1
10,000	10	10	2
99,999	17.782...	17	2
100,000	17.782...	17	2
999,999	31.622...	31	2
1,000,000	31.622...	31	2

6. When the number of digits in N changes from 4 to 5, the number of digits in $\sqrt[4]{N}$ changes from 1 to 2.

7.

N	$\sqrt[5]{N}$	Whole number part of $\sqrt[5]{N}$	Number of digits in whole number part of $\sqrt[5]{N}$
1	1	1	1
9	1.551...	1	1
10	1.584...	1	1
99	2.506...	2	1
100	2.511...	2	1
999	3.980...	3	1
1,000	3.981...	3	1
9,999	6.309...	6	1
10,000	6.309...	6	1
99,999	9.999...	9	1
100,000	10	10	2
999,999	15.848...	15	2
1,000,000	15.848...	15	2

8. When the number of digits in N increases from 5 to 6, the number of digits in the whole number part of $\sqrt[5]{N}$ increases from 1 to 2.

9. Possible answer: The number of digits in a number is less than N times the number of digits in the whole number part of the nth root. Examples will vary.

Activity 10

Roots

Name ______________________

Date ______________________

1. Complete the table.

N	$\sqrt{N}$	Whole number part of $\sqrt{N}$	Number of digits in whole number part of $\sqrt{N}$
1			
9			
10	3.162...	3	1
99			
100			
999			
1,000			
9,999			
10,000			
99,999			
100,000			
999,999			
1,000,000			

2. What patterns do you see between the number of digits in N and the number of digits in the whole number part of $\sqrt{N}$?

____

____

3. Complete the table.

N	$\sqrt[3]{N}$	Whole number part of $\sqrt[3]{N}$	Number of digits in whole number part of $\sqrt[3]{N}$
1			
9			
10			
99			
100			
999			
1,000			
9,999			
10,000			
99,999	46.415...	46	2
100,000			
999,999			
1,000,000			

4. What patterns do you see between the number of digits in N and the number of digits in the whole number part of $\sqrt[3]{N}$?

____

____

5. Complete the table.

N	$\sqrt[4]{N}$	Whole number part of $\sqrt[4]{N}$	Number of digits in whole number part of $\sqrt[4]{N}$
1			
9			
10			
99			
100			
999			
1,000			
9,999			
10,000			
99,999			
100,000			
999,999			
1,000,000			

6. What patterns do you see between the number of digits in N and the number of digits in the whole number part of $\sqrt[4]{N}$?

7. Complete the table.

N	$\sqrt[5]{N}$	Whole number part of $\sqrt[5]{N}$	Number of digits in whole number part of $\sqrt[5]{N}$
1			
9			
10			
99			
100			
999			
1,000			
9,999			
10,000			
99,999			
100,000			
999,999			
1,000,000			

8. What patterns do you see between the number of digits in N and the number of digits in the whole number part of $\sqrt[5]{N}$?

____

____

9. Write a statement that extends the patterns you identified in your answers to questions 2, 4, 6, and 8. Write some examples that would suggest that your statement is true.

____

____

____

____

____

____

Square Roots

N	$\sqrt{N}$	Whole number part of $\sqrt{N}$	Number of digits in whole number part of $\sqrt{N}$
$10^0 = 1$			
9			
$10^1 = 10$			
99			
$10^2 = 100$			
999			
$10^3 = 1{,}000$			
9,999			
$10^4 = 10{,}000$			
99,999			
$10^5 = 100{,}000$			
999,999			
$10^6 = 1{,}000{,}000$			

What patterns are there between the number of digits in N and the number of digits in the whole number part of $\sqrt{N}$?

Cube Roots

N	$\sqrt[3]{N}$	Whole number part of $\sqrt[3]{N}$	Number of digits in whole number part of $\sqrt[3]{N}$
$10^0 = 1$			
9			
$10^1 = 10$			
99			
$10^2 = 100$			
999			
$10^3 = 1,000$			
9,999			
$10^4 = 10,000$			
99,999			
$10^5 = 100,000$			
999,999			
$10^6 = 1,000,000$			

What patterns are there between the number of digits in *N* and the number of digits in the whole number part of $\sqrt[3]{N}$?