NUMB3RS Activity: Follow the Flock
Episode: "In Plain Sight"

Topic: Introduction to Flock Behavior **Grade Level**: 8 - 12
Objective: Use a mathematical model to simulate an aspect of birds flying in a flock
Time: 30 minutes

Introduction

Modeling and simulating nature have been goals of mathematicians for many years. Flock theory refers to modeling the actions of flocking creatures, like birds. In nature, the purpose of a flock is to move as a unit from one point to another. However, the actual structure of the flock is not fixed. The flock must be able to avoid obstacles, yet stay together to reach its goal. The members of the flock obey certain rules: avoid collisions, head in the right general direction, and stay close to neighbors without getting too close.

Models for real flocks are extremely complicated, mainly because they operate in three dimensions. Modern day movies are great places to see simulations that demonstrate how successful these mathematical models are (see "Extensions"). Modeling reality in this sense requires extensive computer programming and more complex mathematics. For the sake of simplicity, this activity uses a simpler version of flock behavior that still conveys the basic concept.

Discuss with Students

In reality, the movement of a flock consists of widely variable shape and structure. Any bird can take the lead, and the others will organize behind it. Most flocks move freely in three dimensions; an exception to this is geese that fly in a V-shaped wedge. A flock must be able to adjust rapidly to things like obstacles or a new bird that takes the lead.

***NUMB3RS* Example** In the episode "In Plain Sight", Charlie tries to figure out how new variables fit into a "flock". The structure of a criminal network changes shape with a new boss, much like a flock of birds reorganizing behind a new leader. In much of the episode Charlie is trying to figure out how an unknown, unforeseen variable fits into the "flock". By understanding individual behavior as it relates to the "flock", he hopes to understand the operations of a criminal network.

The model in this activity uses some simplifying assumptions to make it easier to manage:

- Instead of three dimensions there is one: "front-to-back".
- Each row of the chart represents a step in time.
- The birds (the X's) fly from left to right.
- The far right space is connected to the one on the far left (called "wraparound").
- Birds move ahead according to these rules:
  - If there is at least one empty space in front of a bird, it moves ahead one space.
  - If there is no empty space in front of a bird, it holds its position for one time step.

Example:

	1	2	3	4	5	6	7	8	9	10
Start	**X**		**X**	**X**	**X**			**X**	**X**	**X**
Step 1		X	X	X		X		X	X	X
Step 2	X	X	X		X		X	X	X	

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In Step 1, the birds in positions 1 and 5 move forward (or to your immediate right) because there is room.
In Step 2, birds in positions 4 and 6 move. Also, the bird in position 10 moves to position 1.

The general nature of the flock behavior in this model is similar to traffic on a highway. The birds do not want to be too close to each other, but also do not want to be left behind. Depending on the number of birds and the density of the birds on the grid, you will see different end behavior for the grid.

Student page answers:

1.

Start:					**X**	**X**	**X**		**X**	**X**	**X**	**X**	**X**			
Step 1					X	X		X	X	X	X	X		X		
Step 2					X		X	X	X	X	X		X		X	
Step 3						X	X	X	X	X		X		X		X
Step 4	X					X	X	X	X		X		X		X	
Step 5		X				X	X	X		X		X		X		X
Step 6	X		X			X	X		X		X		X		X	
Step 7		X		X		X		X		X		X		X		X
Step 8	X		X		X		X		X		X		X		X	
Step 9		X		X		X		X		X		X		X		X
Step 10	X		X		X		X		X		X		X		X	

2. *The birds spread out to be one space apart and stay that way.*

3.

Start	**X**		**X**	**X**			**X**	**X**	**X**	**X**		**X**	**X**	**X**		
Step 1		X	X		X		X	X	X		X	X	X		X	
Step 2		X		X		X	X	X		X	X	X		X		X
Step 3	X		X		X	X	X		X	X	X		X		X	
Step 4		X		X	X	X		X	X	X		X		X		X
Step 5	X		X	X	X		X	X	X		X		X		X	
Step 6		X	X	X		X	X	X		X		X		X		X
Step 7	X	X	X		X	X	X		X		X		X		X	
Step 8	X	X		X	X	X		X		X		X		X		X
Step 9	X		X	X	X		X		X		X		X		X	X
Step 10		X	X	X		X		X		X		X		X	X	X
Step 11	X	X	X		X		X		X		X		X	X	X	
Step 12	X	X		X		X		X		X		X	X	X		X
Step 13	X		X		X		X		X		X	X	X		X	X
Step 14		X		X		X		X		X	X	X		X	X	X
Step 15	X		X		X		X		X	X	X		X	X	X	
Step 16		X		X		X		X	X	X		X	X	X		X
Step 17	X		X		X		X	X	X		X	X	X		X	
Step 18		X		X		X	X	X		X	X	X		X		X
Step 19	X		X		X	X	X		X	X	X		X		X	
Step 20		X		X	X	X		X	X	X		X		X		X

4. *The cluster of four becomes a second cluster of three, which appears to move backward through the flock.* **5**. *Answers may vary. Examples: with more birds, it takes a lot longer for a pattern to "settle down"; some groups of birds are unable to spread out because of the available space; a repeating pattern emerges in the spacing of the flock—Step 2's flock appears again in Step 18, starting the pattern over.* **6**. *The extra bird was introduced between steps 5 and 6; the imposter is the leftmost bird in step 6.*

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Name: _____ Date: _____

NUMB3RS Activity: Follow the Flock

Consider a simple model of one aspect of flock theory, namely that birds space themselves in the flock from front to back. In these models, each row of the chart represents a step in time. To save space, assume that the far right space is connected to the far left space (this is called "wraparound"), like the flock is flying in a circle.

The birds (the X's) fly according to the following rules:

- They only fly from left to right (from your perspective).
- If there is at least one space in front (to your right) of a bird, it moves ahead one space.
- If there is no space in front of a bird, it holds position for one time step.

Example:

	1	2	3	4	5	6	7	8	9	10
Start		**X**		**X**	**X**	**X**		**X**		**X**
Step 1	X		X	X	X		X		X	

The birds in positions 2, 6, and 8 move ahead because there is space. Also, the bird in position 10 moves to position 1 (because of the wraparound).

1. Using the rules above and the starting flock below, fill in the rows for each time step. The first two time steps (rows) have been done for you.

Start					**X**	**X**	**X**		**X**	**X**	**X**	**X**	**X**			
Step 1					X	X		X	X	X	X	X		X		
Step 2					X		X	X	X	X	X		X		X	
Step 3																
Step 4																
Step 5																
Step 6																
Step 7																
Step 8																
Step 9																
Step 10																

2. Describe any pattern(s) you see in the spacing of the birds. _____

_____

_____

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3. In the next starting flock, notice that there are more birds for the same amount of flying space. Complete the steps using the same rules as in #1. Again, the first two time steps have been done for you:

Start	**X**		**X**	**X**			**X**	**X**	**X**	**X**		**X**	**X**	**X**		
Step 1		X	X		X		X	X	X		X	X	X		X	
Step 2		X		X		X	X	X		X	X	X		X		X
Step 3																
Step 4																
Step 5																
Step 6																
Step 7																
Step 8																
Step 9																
Step 10																
Step 11																
Step 12																
Step 13																
Step 14																
Step 15																
Step 16																
Step 17																
Step 18																
Step 19																
Step 20																

4. After you have finished the bottom row, look at the entire model. Describe what happens to the "clusters" of four and three birds in the original flock. _____

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5. Compare the two models. How does the second one, with more birds in the same space, compare to the first? _____

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6. Much of Charlie’s role in this episode is dedicated to his trying to figure out how an unknown, unforeseen variable fits into the “flock”. Notice in the flock shown below that there are 10 birds at the beginning, but at step 10 there are 11. Try to determine which bird is the imposter and in which step it joined the flock.

Start		**X**	**X**	**X**			**X**	**X**	**X**			**X**	**X**	**X**	**X**	
Step 1		X	X		X		X	X		X		X	X	X		X
Step 2	X	X		X		X	X		X		X	X	X		X	
Step 3	X		X		X	X		X		X	X	X		X		X
Step 4		X		X	X		X		X	X	X		X		X	X
Step 5	X		X	X		X		X	X	X		X		X		X
Step 6	X	X	X		X		X	X	X		X		X		X	X
Step 7	X	X		X		X	X	X		X		X		X	X	X
Step 8	X		X		X	X	X		X		X		X	X	X	X
Step 9		X		X	X	X		X		X		X	X	X	X	X
Step 10	X		X	X	X		X		X		X	X	X	X	X	

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The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Activity: Modeling a Real Flock

Introduction

Flock theory is an example of mathematics research. Although the original application is given away by the name, the properties of flocks have many applications. The same properties exist in herds of animals, schools of fish, and even fictional warriors in movies. To model realistic behaviors, especially in three dimensions, requires very sophisticated mathematics and a lot of computing power. You can witness examples of simple flock theory in video games, where large numbers of characters or creatures move in the same general direction. These properties are also the basis for such computer programs as "virtual" fish tanks.

Additional Resources

Craig Reynolds of Sony Computer Entertainment America has a web site about flocking creatures he calls "boids". This site gives a lot of mathematical information about how to model the complex behavior of flocking creatures:
http://www.red3d.com/cwr/boids/

This site that has lots of interesting simulations on different topics, including Flock Theory.
http://ccl.northwestern.edu/netlogo/models/community/Flocking%20color or
http://ccl.northwestern.edu/netlogo/models/

For the Student

- Consider the two sample flocks in this activity. Experiment on your own with different percents of the total available flying space filled with birds (#1 was 50% and #3 was 62.5%). Try to generalize what happens to the shape of the flock as it relates to the "density" of the flock.
- This activity used only one dimension, but you can explore two dimensions on your own. Take a strip of graph paper 20-25 squares high, and as long as you like. Draw your own initial flock at one end. Each bird moves forward (to your right) when it can. If it cannot move forward but there is an empty space to its left or right (your up or down), then it moves into one of those spaces.
- Autumn is an excellent time to witness flock theory in its purest form. If you live in a part of the country where birds flock overhead, go outside and witness it for yourself. Notice how a flock of birds maintains its direction, but each individual bird exhibits its own behavior. Reflect on the concepts in this activity, and watch for each of the behaviors in an actual flock.
- Flock behavior appears in many major motion pictures. Revisit some of your favorite movies with a "mathematical eye". Examples include the bat swarms and penguin flocks in "Batman Returns", swarming scarabs in both films of "The Mummy", the wildebeest stampede in "The Lion King", and the orc armies in Lord of the Rings "Return of the King". To get a real feel for how mathematics and technology have improved in the last forty years, watch Alfred Hitchcock's "The Birds" (1963).

Related Topic:

If you have an interest in animation, check out a description of animation techniques from the UCLA Department of Design and Media Arts:
http://classes.design.ucla.edu/Spring02/102/animation.html

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