

Activity 12

The World Population: Logistic Model



The population of the world continues to increase. In order for us to prepare adequately for the next century, accurate projections for the population are needed.






The rate of population growth is jointly proportional to the current population and the availability for new population growth. Population approaches an upper bound as the number of years increase and a lower bound as the number of years decrease. One form of the population

$$\text{model is } f(x) = \frac{a}{1 + e^{-b(x-c)}} + d$$

where x is the year and $f(x)$ is the population. This model is called a *logistic* model. In this activity, the logistic model for the growth of the world's population will be developed.

Exploration



1. Open a new TI InterActive! document. Title this report **World Population Growth: The Logistic Model**. Add your name and the date to this document.
2. Select Math Box  and define $f(x) = \frac{a}{1 + e^{-b(x-c)}} + d$. In the next Math Boxes, define $a := 1$, $b := 1$, $c := 0$, and $d := 0$.
3. Select List  to open the Data Editor. Double-click on the name **L1** and change the name to **Years**. Click OK. Double-click on the name **L2** and change the name to **Population**. Click OK.

4. Click on the Web Browser icon  to open TI InterActive!'s Internet Browser. Click on the Data Sites icon . Under the **Activity Book Links** category, click *TI InterActive! Math for High School*. Choose **Activity 12: World Population**.
5. Once the page has loaded in the browser, click on **Select**, then **Extract** to download the data into the open Data Editor.
6. Highlight and copy the numerical data for the Years and Population from 1800 to this year. Click into cell 1 of the **Years** column and paste the copied data.
7. Double-click **L3**. Change the name from **L3** to **Time**. Enter the formula **Years-1900** to look at the years since 1900. Click on Save to Document .
8. Select Graph . Enter **f(x)** in $y_1(x)$. Click on the Stat Plots tab. Enter **Time** in the first field and **Population** in the second field.
9. Click on **Format**. Set up the window with x: [-100, 100], Xscale = 10, and y: [0, 10], Yscale = 1. Click on Apply. Click on the Labels tab. Title this graph **World Population**. Label the X-Axis **Years since 1900** and the Y-Axis **Population in Billions**. Click on OK and then Save to Document .

Analysis

Record the answers to the following questions in the report. Be sure to write in complete sentences when answering these questions.

1. The horizontal shift, c , would be the year when the rate of change in the population data appears to be the greatest. Using the data from 1800 to now, predict a value for c . Scroll up to the top of the document and redefine c as this value.
2. An estimate for the maximum population would be twice the population at the year you selected for c . Using your value for c , predict a value for the maximum population. Redefine a as this value.
3. The vertical shift, d , can be approximated by the lower limiting value. Redefine d as the estimate of the lower limiting value.
4. Select another data point (time, population) on the graph other than $(c, f(c))$ and solve for b . For example, if $(0, 1.65)$ is the other point, in a math box type **solve(f(0) = 1.65, b)**. Redefine b as this value.

5. The maximum population is the value of $a + d$. Redefine a to be the difference between the current value of a and d . For example, $a = a - d$.
6. In Math Boxes, use this function to predict the population in the years 2020, 2030, and 2040 by evaluating the function, $f(x)$. Discuss the accuracy of these predictions based upon the collected data.
7. Double-click on the data. Highlight and copy the original numerical data for the Years and Population from 1800 through 2050 as predicted by the United Nations. Click in cell 1 of **Years** and paste the copied data. Close the Data Editor.
8. Double-click on the graph and click on . Set up the window with x : $[-100, 150]$, Xscale: 10, and y : $[0, 10]$, Yscale: 1. Click on OK and then . How well does your function model the population from the United Nations? Explain.
9. According to your model, what appears to be the maximum population that the Earth can sustain? Does this population seem reasonable to you? Explain.
10. Save this document as **worldpop_logistic.tii**. Print a copy of this document.

Teacher Notes






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The World Population: The Logistic Model

Objective

- ◆ Students will develop an understanding of the logistic function and use it to model real world data

Applicable TI InterActive! Functions

- ◆ Solve $\text{solve}(\text{equation}, \text{variable})$
- ◆ Define $\text{function_name} := \text{function}$
- ◆ List Editor 
- ◆ Graph 
- ◆ Web Browser 

Problem

Students will collect world population data from the Internet, find a logistic model for the data, and use their model to predict future populations.


Pre-Activity


Investigate the parameters a , b , c , and d of the function $f(x) = \frac{a}{1 + e^{-b(x-c)}} + d$.

1. Students should open a new TI InterActive! document. They should insert a

Math Box and use the Math Palette to define $f(x) = \frac{a}{1 + e^{-b(x-c)}} + d$. The

Math Palette will allow student to enter a fraction and will allow the e 's exponent to be written in superscript. Students should then define each of the parameters of this function as follows: $a = 1$, $b = 1$, $c = 0$, and $d = 0$.

2. Students should click on Graph  and enter $f(x)$ as $y1: (x)$ and then

Save to Document . Students should then double click on $a = 1$ and adjust a 's value. For example, let $a = 3$. Click out of the math box and note the change in the graph. Students should continue adjusting the value of a (positive and negative numbers) until they can make a statement about the effect a has on this graph.

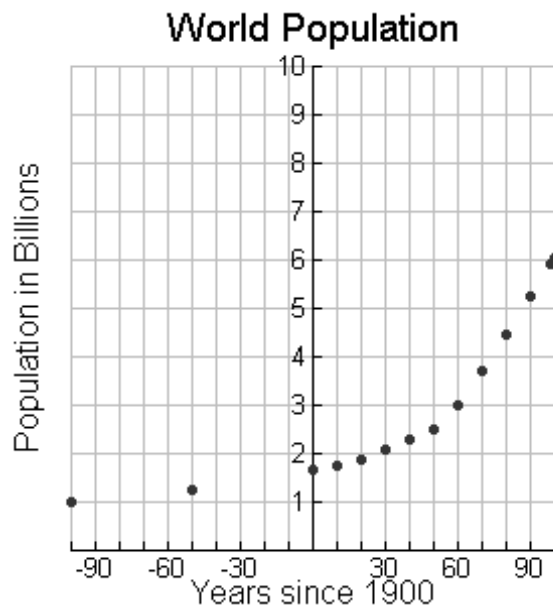
3. Students should double-click on $b = 1$ and adjust b 's value. For example, let $b = 1/2$. Click out of the Math Box and note the change in the graph. Students should continue adjusting the value of b (positive and negative numbers) until they can make a statement about the effect b has on this graph.

- Students should double-click on $c = 0$ and adjust c 's value. For example, let $c = 3$. Click out of the Math Box and note the change in the graph. Students should continue adjusting the value of c (positive and negative numbers) until they can make a statement about the effect c has on this graph.
- Students should double-click on $d = 0$ and adjust d 's value. For example, let $d = 3$. Click out of the Math Box and note the change in the graph. Students should continue adjusting the value of d (positive and negative numbers) until they can make a statement about the effect d has on this graph.

Exploration

Steps 1 through 9 are details for the students to set up the problem and extract the data from the Internet. The following solutions are based on data from the United Nations, Population Division, Department of Economic and Social Affairs posted on September 1, 1999 (Date Site **Activity 12: World Population**). When students have completed step 9, their graph should appear as shown.

- through 9.



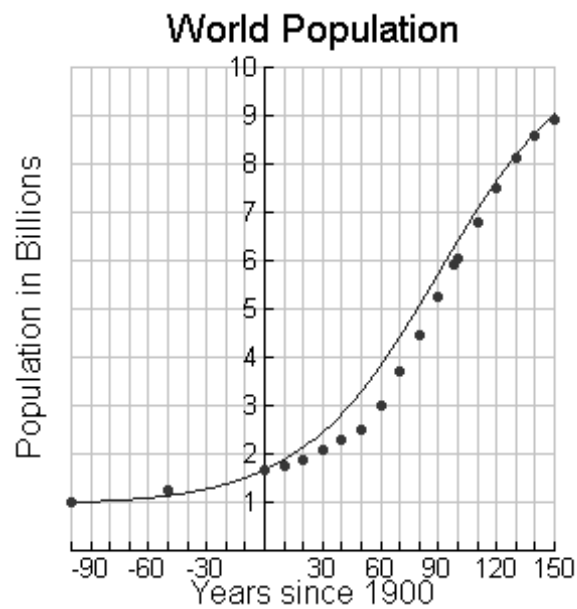
Analysis

- Answers will vary. For one solution, start with $c = 90$.
- Answers will vary. For one solution, let $a = 10.54$.
- Answers will vary. For one solution, let $d = 0.95$.
- Answers will vary. For one solution, select $(60, 3.02)$. One method of solving for b is to enter **solve(f(0) = 1.65, b)** in a Math Box. $b = 0.028$.
- Answers will vary. For one solution, let $a = 9.59$.

6. Answers will vary. Solutions below are based on $f(x) = \frac{9.59}{1 + e^{-0.028(x-90)}} + 0.95$.

Year	Years since 1900	Population in Billions
2020	120	7.648
2030	130	8.181
2040	140	8.643

7. through 8.



8. Answers will vary.
9. Answers will vary.