

## Investigating the Sine Function

by – Marco A. Gonzalez

### Activity overview

*In this activity, students will use their Nspire handhelds to discover the different attributes of the graph of the sine function. The students will take advantage of the dynamic capabilities of this very unique handheld to explore the amplitude, period, and phase shift of the sine function graph.*

### Concepts

*Graphs of Sine and Cosine Functions (periodic functions), Amplitude of the Sine Function, Periods of the Sine Functions and Phase Shifts of the Sine Function.*

### Teacher preparation

*This activity is designed for use in a Pre-Calculus or Trigonometry classroom. Prior to beginning this activity, students should have prior knowledge of the graphs of periodic functions such as the Sine Function*

### Classroom management tips

*This activity is intended to be a teacher-led activity in which students will make their own discoveries pertaining to the graph of the Sine function. Students should be encouraged to make predictions of how the equation of the graphed function will change after altering the graph of the sine function.*

### TI-Nspire Applications

*Graphs and Geometry*

### Step-by-step directions

*The following step-by-step directions are the same as the student directions.*

#### Part I

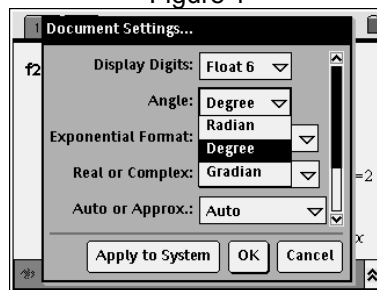
Press the **Home** (🏠) button and open a new **Graphs & Geometry** document.

Now, we need to change the angle settings of the handheld device by pressing the Home button and toggle down to “ $\langle 8 \rangle$ : System Info”, “ $\langle 1 \rangle$ : Document Settings”.

Press (tab) once. Use the “Nav pad” to press down to “**degree**”. Press (enter). Continue to press (tab) until you get to **OK** and press (enter). (See Figure 1)

Enter the function **f1(x) = sin(x)**.

Figure 1



Now, we need to edit the window screen on the handheld.

Press  $\text{MENU}$ ,  $\text{4}$ : **Window**,  $\text{1}$ : **Window Settings**. See **Figure 2** to enter the right window settings for this activity.

Now, let's calculate x-intercepts (zeros) on this graph.

Press  $\text{MENU}$ ,  $\text{6}$ : **Points & Lines**,  $\text{3}$ : **Intersection Point(s)**. Move the cursor with your "Nav pad" to the sine curve and press  $\text{ENTER}$ . Move the cursor to the x-axis and press  $\text{ENTER}$  as well. Press  $\text{ESC}$ . (see Figure 3)

Now we are going to calculate midpoints between the x-intercepts. To do this task, press  $\text{MENU}$ ,  $\text{9}$ : **Construction**, &  $\text{5}$ : **Midpoint**. Scroll the cursor to the 1<sup>st</sup> x-intercept on the screen from left to right (origin) and press  $\text{ENTER}$ . Scroll the cursor to the next x-intercept (@ 90 degrees) and press  $\text{ENTER}$ .

Repeat the process from the middle x-intercept to the far right x-intercept (@ 360 degrees). Press  $\text{ESC}$ .

Now, let's move the label of the sine function to the top left of the screen. To do this, move the cursor with the Nav Pad above the label of the sine function.  $\text{CLICK}$  and hold for 1.7 seconds until the label is "grabbed." Once the label is "grabbed" and by using the Nav Pad, move the label to the top left of the screen. Press  $\text{ENTER}$  to lock the label in place. (see figure 4)

Now, let's calculate a relative Maximum and a relative minimum. This is an extensive process.

Press  $\text{MENU}$ ,  $\text{9}$ : **Construction**,  $\text{1}$ : **Perpendicular**. Move the cursor over the first midpoint (from left to right) until the point is flashing and press  $\text{ENTER}$ . Move the cursor somewhere over the x-axis until it's flashing and press  $\text{ENTER}$ . Repeat the process for the second midpoint. Press  $\text{ESC}$ . See figure 5.

Let's calculate the point of intersection between the sine curve and the perpendicular lines. Press  $\text{MENU}$ ,  $\text{6}$ : **Points & Lines**,  $\text{3}$ : **Intersection Point(s)**. Move the cursor over the left perpendicular line

Figure 2

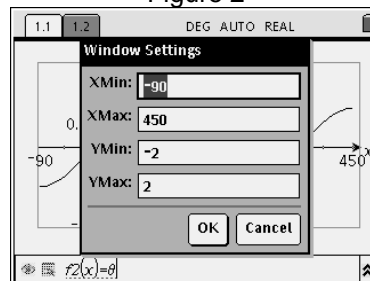


Figure 3

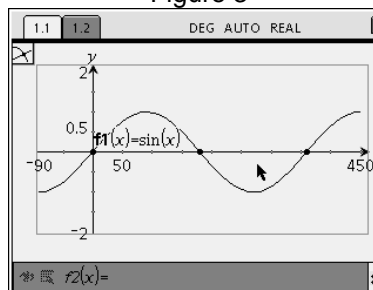


Figure 4

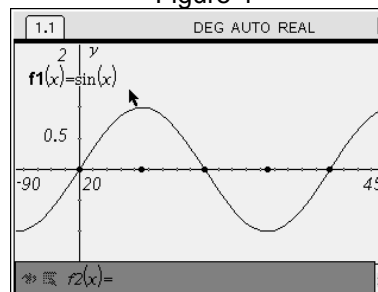
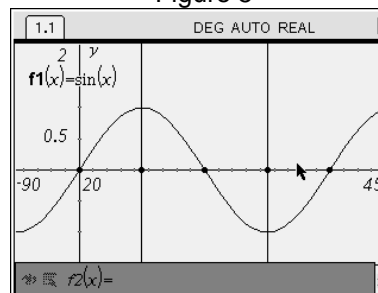


Figure 5



until it is flashing and press  $\text{ENTER}$ . Move the cursor over the sine curve until it is flashing and press  $\text{ENTER}$ . Repeat the process for the perpendicular line to the right. Press  $\text{ESC}$ .

Now, let's hide the perpendicular lines. Move the cursor over one perpendicular line and press  $\text{CTRL}$   $\text{MENU}$  simultaneously. Press  $\text{3}$ : Hide/Show. Repeat the process for the other perpendicular line. (see figure 6)

Now, let's calculate the distance from the first midpoint to the intersecting point on the sine curve. Press  $\text{MENU}$ ,  $\text{7}$ : **Measurement**,  $\text{1}$ : **Length**. Move the cursor to the first midpoint (from left to right) until the point is flashing and press  $\text{ENTER}$ . Now, move the cursor to the point on the curve above until it is flashing and press  $\text{ENTER}$ . A measurement should be appearing in a ghost form. Move the cursor to the upper right hand corner of the screen and press  $\text{ENTER}$  to lock the measurement. Now let's label the point as a Max point. Press  $\text{MENU}$ ,  $\text{1}$ : **Tools**,  $\text{5}$ : **Text** and move the cursor to the left of the measurement we just calculated and press  $\text{ENTER}$ . Press  $\text{OP}$   $\text{M}$   $\text{A}$   $\text{X}$   $\text{=}$ , then  $\text{ENTER}$ . Press  $\text{ESC}$ . Reposition the text if necessary by hovering the cursor above the text and press the  $\text{MOVE}$  button for 1.7 seconds. Move to desired location and press  $\text{MOVE}$ .

Repeat the aforementioned process for the other point on the curve with one exception. Label the point "min" instead. Your window should look like figure 7.

Now, let's calculate the distance from the origin to the point where the sine curve completes one "cycle". Press  $\text{MENU}$ ,  $\text{7}$ : **Measurement**,  $\text{1}$ : **Length**. Move the cursor to the origin until the point flashes and press  $\text{ENTER}$ . Move the cursor to the third x-intercept from left to right until it flashes and press  $\text{ENTER}$ . Move the measurement to the bottom right hand side of the screen. Press  $\text{ESC}$ . Label this measurement as the period of this curve. Press  $\text{MENU}$ ,  $\text{1}$ : **Tools**,  $\text{5}$ : **Text** and move the cursor to the left of the measurement we just calculated and press  $\text{ENTER}$ . Enter the desired label and press  $\text{ENTER}$  followed by  $\text{ESC}$ . (see figure 8)

All of these measurements are dynamic, which

Figure 6

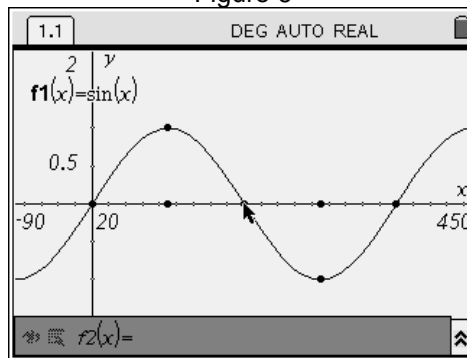


Figure 7

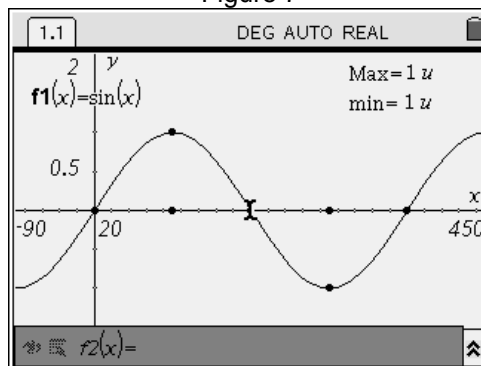
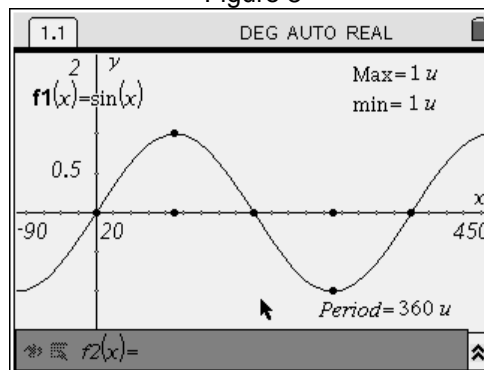


Figure 8



means that they will change accordingly if the relative Maximum or the relative minimum points are translated. Move the cursor to the relative minimum point until it flashes. Press and hold the  $\text{2nd}$  button for 1.7 seconds. Move the point in several directions and watch the measurements adjust accordingly. (see figure 9)

### Discovery Questions

1. Which number represents the amplitude of the function?
2. Which number is responsible for altering the period of the function?

### Part II.

Add a new Graphs & Geometry page. Press  $\text{2nd}$  and toggle to “**Graphs & Geometry**” and press  $\text{enter}$ .

Change window settings to match the figure 10. Press  $\text{menu}$ ,  $\text{4}$ : **Window**,  $\text{1}$ : **Window Settings**.

Now, enter the function  $f2(x)=\sin(2x) + 2$  and press  $\text{enter}$ . Move the label to the upper left hand side of the screen. Move the cursor to the label and press the  $\text{2nd}$  button for 1.7 seconds. Move the label to the desired location and press  $\text{2nd}$ .

Since this sine curve has been translated up 2 units, let's graph the horizontal axis. Enter the function  $f3(x)=2$  and press  $\text{enter}$ . Now, let's change the horizontal axis from a solid line to a dashed line. Move the cursor to the line and press  $\text{ctrl}$  and  $\text{menu}$  simultaneously. Now press  $\text{2}$ : **Attributes**. Press the “**Nav pad**” down once. Press the “**Nav pad**” to the right once and press  $\text{2nd}$ .

Now, calculate the x-intercepts, relative Maximum, relative minimum, and period following the same procedures as in Part I. Label all of your measurements. (see figure 11)

Figure 9

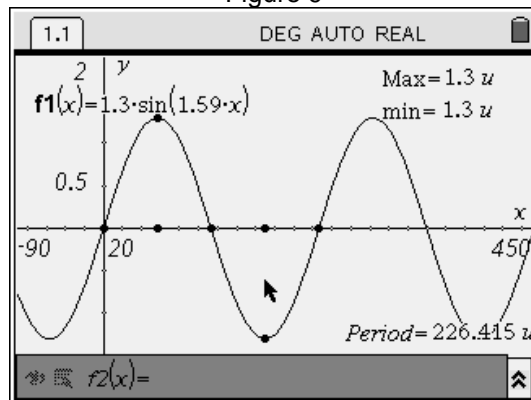


Figure 10

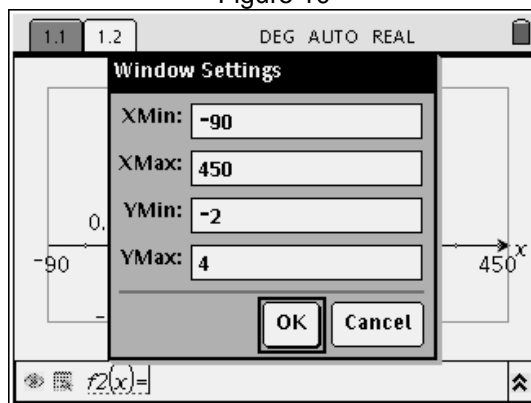
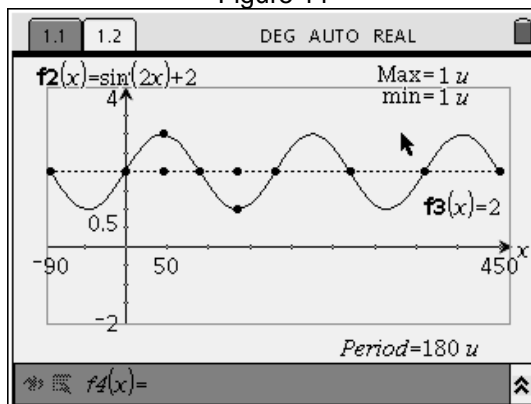



Figure 11

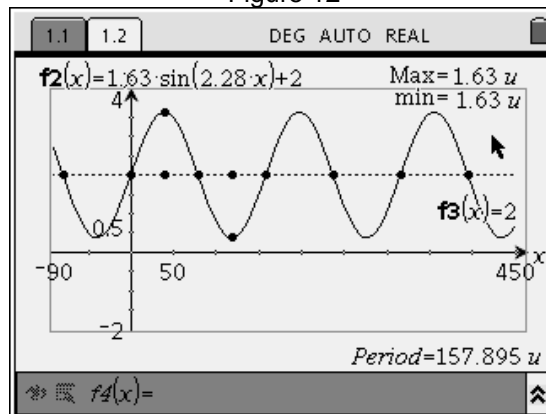


Now  and grab one of the relative Maximum or the relative minimum and move the point in several directions and watch the measurements change accordingly. (see figure 12)

### Discovery Question

Did you notice any changes in the equation of the sine function? Describe the changes.

Figure 12



Assessment and evaluation *(NOTE: this section is separate from the student step-by-step directions.)*

- An independent practice handout (attached after student document) can be used as an evaluation tool.

Activity extensions

- The student should explore the attributes of the cosine function by following the same procedure in this activity.
- The student should explore the attributes of the tangent function.
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## Student TI-Nspire Document

**Part I**

Press the **Home** (⏠) button and open a new **Graphs & Geometry** document.

Now, we need to change the angle settings of the handheld device by pressing the Home button and toggle down to “(8): System Info”, “(1): Document Settings”.

Press (tab) once. Use the “Nav pad” to press down to “**degree**”. Press (enter). Continue to press (tab) until you get to **OK** and press (enter). (See Figure 1)

Enter the function **f1(x) = sin(x)**.

Now, we need to edit the window screen on the handheld.

Press (menu), (4): **Window**, (1): **Window Settings**. See **Figure 2** to enter the right window settings for this activity.

Now, let's calculate x-intercepts (zeros) on this graph.

Press (menu), (6): **Points & Lines**, (3): **Intersection Point(s)**. Move the cursor with your “Nav pad” to the sine curve and press (enter). Move the cursor to the x-axis and press (enter) as well. Press (esc). (see Figure 3)

Now we are going to calculate midpoints between the x-intercepts. To do this task, press (menu), (9): **Construction**, & (5): **Midpoint**. Scroll the cursor to the 1<sup>st</sup> x-intercept on the screen from left to right (origin) and press (enter). Scroll the cursor to the next x-intercept (@ 90 degrees) and press (enter).

Repeat the process from the middle x-intercept to the far right x-intercept (@ 360 degrees). Press (esc).

Now, let's move the label of the sine function to the top left of the screen. To do this, move the cursor with the Nav Pad above the label of the sine function. (⏏) and hold for 1.7 seconds until the label is “grabbed.” Once the label is “grabbed” and

Figure 1



Figure 2

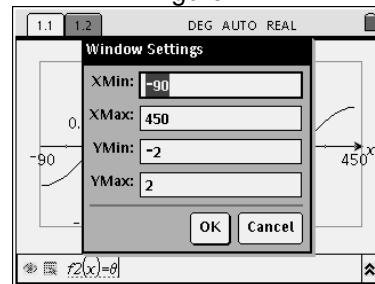


Figure 3

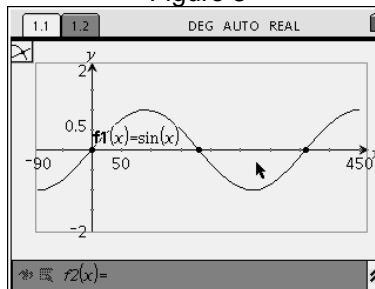


Figure 4

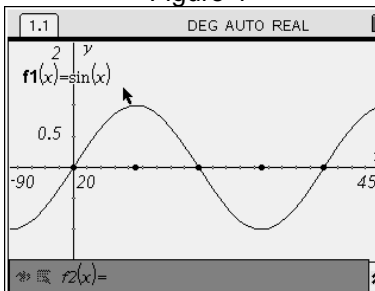


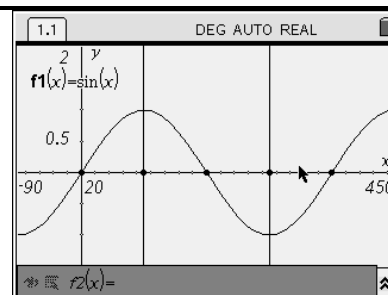
Figure 5

by using the Nav Pad, move the label to the top left of the screen. Press  $\text{2nd} \rightarrow \text{enter}$  to lock the label in place. (see figure 4)

Now, let's calculate a relative Maximum and a relative minimum. This is an extensive process.

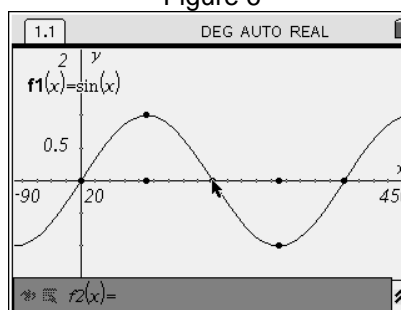
Press  $\text{2nd} \rightarrow \text{9}$ : **Construction**,  $\text{1}$ :

**Perpendicular.** Move the cursor over the first midpoint (from left to right) until the point is flashing and press  $\text{2nd} \rightarrow \text{enter}$ . Move the cursor somewhere over the x-axis until it's flashing and press  $\text{2nd} \rightarrow \text{enter}$ . Repeat the process for the second midpoint. Press  $\text{esc}$ . See figure 5.



Let's calculate the point of intersection between the sine curve and the perpendicular lines. Press  $\text{2nd} \rightarrow \text{6}$ : **Points & Lines**,  $\text{3}$ : **Intersection Point(s)**. Move the cursor over the left perpendicular line until it is flashing and press  $\text{2nd} \rightarrow \text{enter}$ . Move the cursor over the sine curve until it is flashing and press  $\text{2nd} \rightarrow \text{enter}$ . Repeat the process for the perpendicular line to the right. Press  $\text{esc}$ .

Figure 6



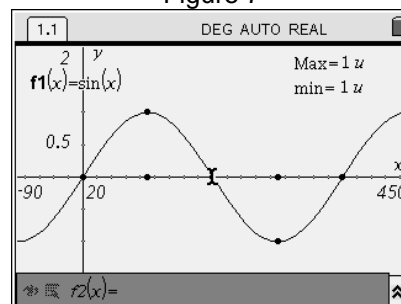
Now, let's hide the perpendicular lines. Move the cursor over one perpendicular line and press  $\text{ctrl} \rightarrow \text{2nd} \rightarrow \text{menu}$  simultaneously. Press  $\text{3}$ : **Hide/Show**. Repeat the process for the other perpendicular line. (see figure 6)

Now, let's calculate the distance from the first midpoint to the intersecting point on the sine curve.

Press  $\text{2nd} \rightarrow \text{7}$ : **Measurement**,  $\text{1}$ : **Length**.

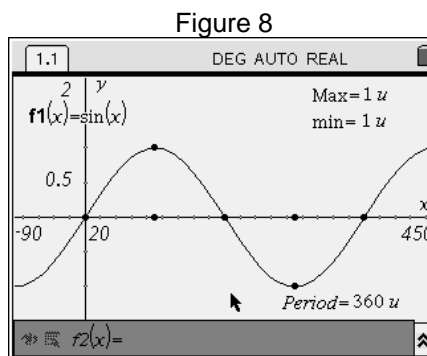
Move the cursor to the first midpoint (from left to right) until the point is flashing and press  $\text{2nd} \rightarrow \text{enter}$ . Now, move the cursor to the point on the curve above until it is flashing and press  $\text{2nd} \rightarrow \text{enter}$ . A measurement should be appearing in a ghost form. Move the cursor to the upper right hand corner of the screen and press  $\text{2nd} \rightarrow \text{enter}$  to lock the measurement. Now let's label the point as a Max point. Press  $\text{2nd} \rightarrow \text{1}$ : **Tools**,  $\text{5}$ : **Text** and move the cursor to the left of the measurement we just calculated and press  $\text{2nd} \rightarrow \text{enter}$ . Press  $\text{2nd} \rightarrow \text{M} \rightarrow \text{A} \rightarrow \text{X} \rightarrow \text{}$ , then  $\text{2nd} \rightarrow \text{enter}$ . Press  $\text{esc}$ . Reposition the text if necessary by hovering the cursor above the text and press the  $\text{2nd} \rightarrow \text{right arrow}$  button for 1.7 seconds. Move to desired location and press  $\text{2nd} \rightarrow \text{right arrow}$ .

Figure 7

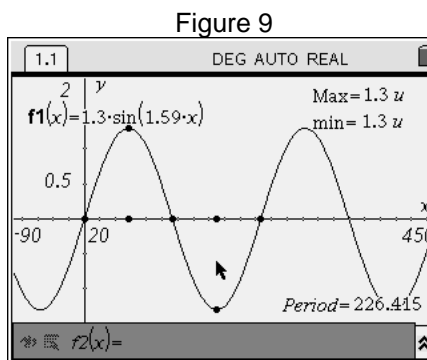


Repeat the aforementioned process for the other point on the curve with one exception. Label the point "min" instead. Your window should look like figure 7.

Now, let's calculate the distance from the origin to the point where the sine curve completes one "cycle". Press  $\text{MENU}$ ,  $\text{7}$ : **Measurement**,  $\text{1}$ : **Length**. Move the cursor to the origin until the point flashes and press  $\text{ENTER}$ . Move the cursor to the third x-intercept from left to right until it flashes and press  $\text{ENTER}$ . Move the measurement to the bottom right hand side of the screen. Press  $\text{ESC}$ . Label this measurement as the period of this curve. Press  $\text{MENU}$ ,  $\text{1}$ : **Tools**,  $\text{5}$ : **Text** and move the cursor to the left of the measurement we just calculated and press  $\text{ENTER}$ . Enter the desired label and press  $\text{ENTER}$  followed by  $\text{ESC}$ . (see figure 8)



All of these measurements are dynamic, which means that they will change accordingly if the relative Maximum or the relative minimum points are translated. Move the cursor to the relative minimum point until it flashes. Press and hold the  $\text{ZOOM}$  button for 1.7 seconds. Move the point in several directions and watch the measurements adjust accordingly. (see figure 9)



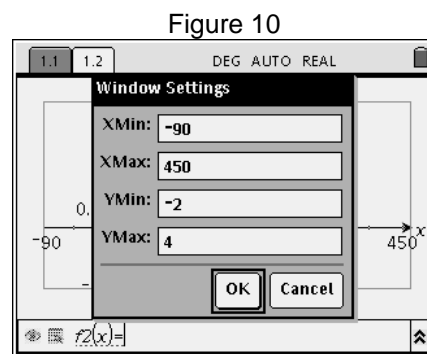
### Discovery Questions

1. Which number represents the amplitude of the function?
2. Which number is responsible for altering the period of the function?

### Part II.

Add a new Graphs & Geometry page. Press  $\text{2ND}$  and toggle to "Graphs & Geometry" and press  $\text{ENTER}$ .

Change window settings to match the figure 10. Press  $\text{MENU}$ ,  $\text{4}$ : **Window**,  $\text{1}$ : **Window Settings**.



Now, enter the function  $f2(x) = \sin(2x) + 2$  and press  $\text{ENTER}$ . Move the label to the upper left hand side of the screen. Move the cursor to the label and press the  $\text{ZOOM}$  button for 1.7 seconds. Move the label to the desired location and press  $\text{ZOOM}$ .

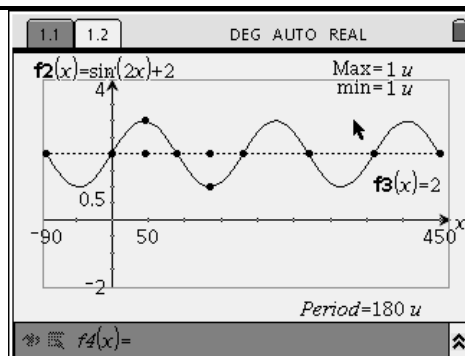
Since this sine curve has been translated up 2 units, let's graph the horizontal axis. Enter the function  $f3(x) = 2$  and press  $\text{ENTER}$ . Now, let's change the horizontal axis from a solid line to a dashed

Figure 11



line. Move the cursor to the line and press  $\text{ctrl}$  and  $\text{menu}$  simultaneously. Now press  $\langle 2 \rangle$ :  
**Attributes.** Press the “Nav pad” down once. Press the “Nav pad” to the right once and press  $\langle \text{arrow} \rangle$ .

Now, calculate the x-intercepts, relative Maximum, relative minimum, and period following the same procedures as in Part I. Label all of your measurements. (see figure 11)

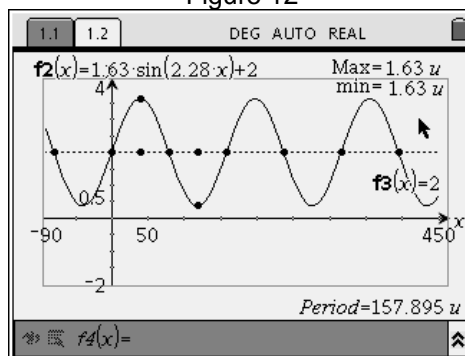


Now  $\langle \text{arrow} \rangle$  and grab one of the relative Maximum or the relative minimum and move the point in several directions and watch the measurements change accordingly. (see figure 12)

### Discovery Question

Did you notice any changes in the equation of the sine function? Describe the changes.

Figure 12



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Student Independent Practice

*Directions: Compared to the parent function  $f(x) = \sin(x)$ , please describe the amplitude and the period in each of the sine functions below. Sketch the graph below.*

1.  $f(x) = 4 \sin(2x)$

2.  $f(x) = -2 \sin\left(\frac{1}{2}x\right)$

3.  $f(x) = 3 \sin(90x)$

4.  $f(x) = \left(\frac{1}{3}\right) \sin\left(\frac{1}{4}x\right)$