# Estimating Slope of a Tangent Line

ID: 13324

Time required
*15 minutes*

## Activity Overview

*In this activity, students will look at the graph of $y = x^2$ with a secant through two points, p and q. Point p will be locked at (1, 1) and students will drag point q towards p, while tracking the value of the slope of the secant. Eventually, students will identify the slope of $\overleftrightarrow{pq}$ when q coincides with p. Students will then look at this result numerically to gain a better understanding of how to estimate the slope of a tangent using the formula for slope.*

## Topic: The Concept of the Derivative

- *Slope*
- *Instantaneous rate of change*

## Teacher Preparation and Notes

*This investigation offers an opportunity to introduce the concept of instantaneous rate of change prior to having developed the concept of the derivative. Students should have some familiarity with limits, as well as continuity.*

- *This activity is designed to be **student-centered** with the teacher acting as a facilitator while students work cooperatively. The student worksheet is intended to guide students through the main ideas of the activity and provide a place to record their observations.*
- *For the most part, students will manipulate pre-made sketches, rather than constructing the diagrams themselves. Therefore, a basic working knowledge of the TI-Nspire handheld is sufficient.*
- *Students will write a formula in the spreadsheet application that uses cell references. They will also use the fill down command to copy their formula for the entire data set. You may want to review how this is done prior to beginning the activity.*
- *The ideas contained in the following pages are intended to provide a framework as to how the activity will progress. Suggestions are also provided to help ensure that the objectives for this activity are met.*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "13324" in the quick search box.***

## Associated Materials

- *EstimatingSlopeofTangentLine_Student.doc*
- *EstimatingSlopeofTangentLine.tns*

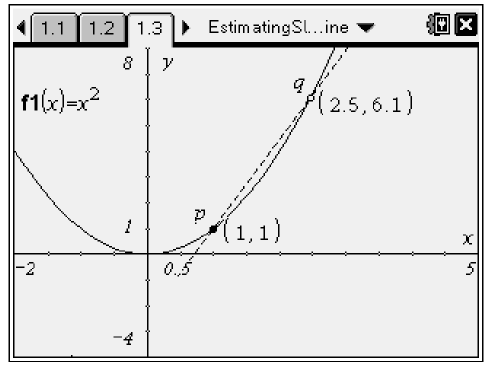
## Suggested Related Activities

*To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.*

- *The Tangent Line Problem (TI-Nspire technology) — 8315*
- *Limits (TI-Nspire CAS technology) — 8997*
- *Secant and Tangent Lines (TI-Nspire technology) — 11141*

**Slope of a tangent to a curve**

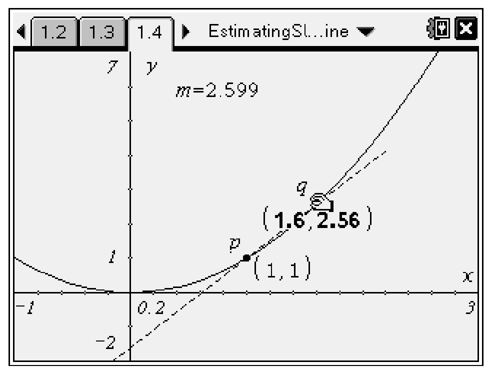
One focus question defines this activity: *How can you find the slope of a tangent to a curve?*




Discuss with students what they think will happen as point *q* is moved towards the fixed point *p*. Make sure that they understand that the slope of the secant can be calculated at regular intervals using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. In fact, have students calculate the slope of $\overleftrightarrow{pq}$ with the values given on page 1.3. Another key point relates to the fact that, when *p* coincides with *q*, the denominator of the slope formula equals zero and is therefore undefined. This issue will be explored in greater detail as the activity progresses.

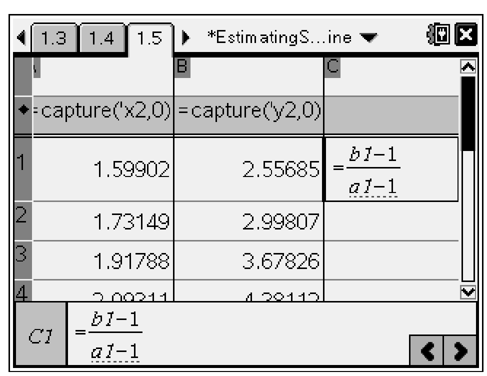
**Investigating the slope of a tangent line graphically**

**Step 1:** Remind students to periodically press ctrl + . as they drag point *q* towards *p*. This will capture the coordinates of *q* and place them in a spreadsheet. Students will also observe that the slope approaches 2 as *q* gets closer to *p*. You may also want to encourage them to move *q* slightly past *p* to give support to the notion that the left and right limits are equal.

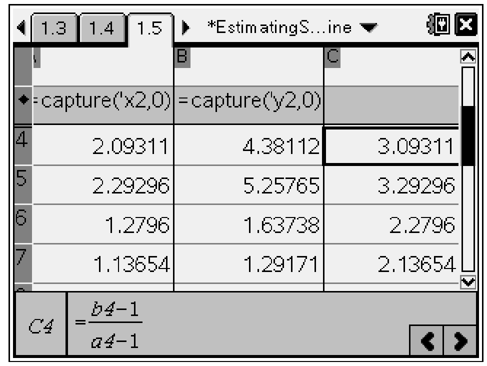



Students are asked to explain that the slope formula is undefined when both points coincide. They must also explain why the slope of the tangent is referred to as the instantaneous rate of change. Students should reason that the slope of a tangent represents the rate of change in the graph at that specific point. Another way to get at this concept is to ask students to imagine that the *x*-values represent time and, when they are brought closer together, you are looking at shorter and shorter intervals. Eventually these time intervals become zero or instantaneous.

**Step 2:** Students will see the coordinates of point *q* that were recorded in a spreadsheet each time ctrl + . was pressed with the *x*-values stored in Column A and the *y*-values stored in Column B. The formula shown in cell C1 will calculate the slope between the first captured point and *p*(1, 1). It is helpful to press ctrl + ÷ to access the stacked fraction symbol; and remind students to precede their formulas with an equals sign.



**Step 3:** Next, move the cursor to cell C1 and press ctrl + (alternatively, select **MENU > Data > Fill Down**) followed by ▼ on the NavPad to highlight each cell in Column C that has corresponding data in Columns A and B. By pressing enter, the formula will be copied to each highlighted cell. Notice in the screen at right that there is strong numerical support indicating that the slope of the tangent is 2.




**Step 4:** The screen at right shows an example of how students are expected to estimate the slope of the tangent to **f1**($x$) = $x^2$ at $x$ = 1 by using a point very close to (1, 1) as well as the formula for slope. Encourage students to use the stacked fraction symbol and function notation to set up this quotient.