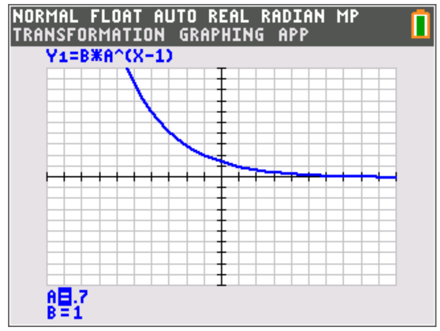
Exploring Geometric Sequences



About the Lesson

In this activity, students will explore geometric series. They will consider the effect of the value for the common ratio and first term using the **Transfrm** App for the TI-84 Plus family. As a result, students will:

- Consider the effect of the value for the common ratio and first term.
- Graphically analyze geometric series using graphs.

Vocabulary

- geometric series
- common ratio

Teacher Preparation and Notes

- This activity serves as a nice introduction to geometric series. Students will need the **Transfrm** App on their calculators.

- An extension of this activity could include solving infinite geometric series that converge using sigma notation and the limit of the partial sum formula.

Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  TI-84 Plus C Silver Edition
  TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*

Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

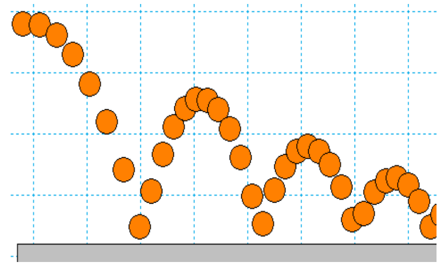
Lesson Files:

- Exploring_Geometric_Sequences_Student.pdf
- Exploring_Geometric_Sequences_Student.doc

Example of a Geometric Sequence

Students are shown the path of a ball that is bouncing. Show that the common ratio of the heights is approximately the same.

$\frac{2.8}{4.0} = 0.7$ $\quad \frac{2.0}{2.8} \approx 0.71$ $\quad \frac{1.4}{2.0} - 0.7$

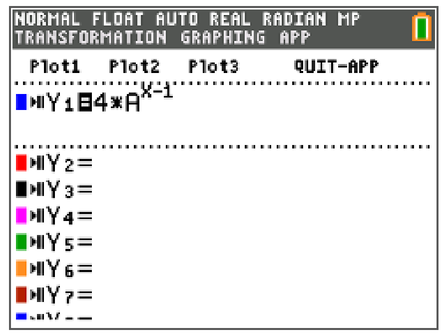


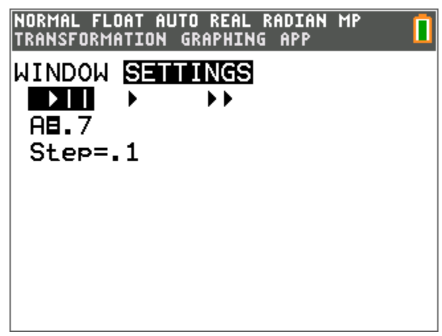
Problem 1 – Changing the Common Ratio

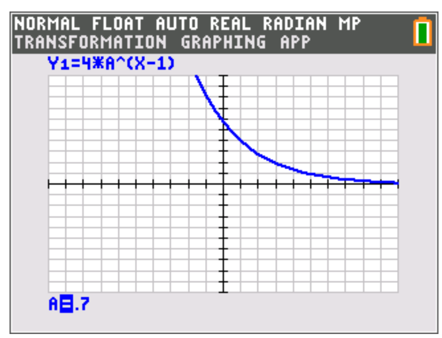
For the first part of this activity, students explore geometric sequences graphically by varying the value of *r*, the common ratio. Pressing the right or left arrows will change the value of *r* and updates the graph each time.

Note that students will not see a graph for **A** values less than zero.

They should see that *r*-values between 0 and 1 could model the heights of the ball in the example.







1. Why do you think the *r*-value is called the common ratio?

<u>Answer</u>: The ratios of consecutive terms are the same; they have that in common.

2. What did you observe happens when you change the common ratio from positive to negative? Explain why this happens.

<u>Answer</u>: The smooth continuous graph disappears. The *y* values of the graph oscillate between positive and negative values.

3. What would happen if you added all the terms of this sequence? For what common ratio conditions do you think the sum will diverge, (get larger, and not converge to some number)?

<u>Answer</u>: The sum would converge to some finite value. If *r* is greater than or equal to 1, the sum will diverge.

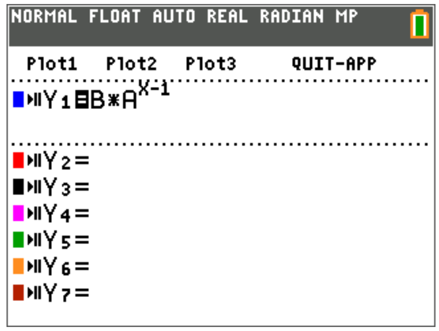
4. When the common ratio is larger than 1, explain what happens to the graph and values of *y*.

Answer: The graph increases from left to right making the values of *y* larger.

5. What *r*-values could model the heights of a ball bounce? Explain.

Answer: The *r*-value would be between 0 and 1 because the height of the ball becomes smaller with each bounce.

Problem 2 – Changing the Initial Value and the Common Ratio



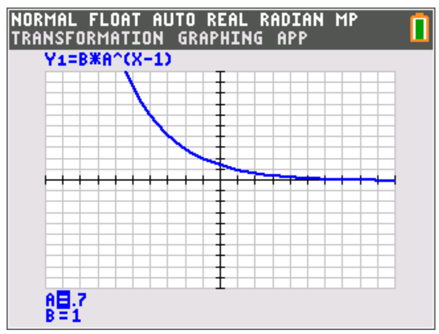
Students will now change the equation in the **Transfrm App** to explore the changes in the graph of the general series, $a_n = a_1 \cdot r^{n-1}$. Students are to use the up and down arrows to move from one variable to another and use the left and right arrows to change the value of the variable.

They should be sure to try negative and positive values for **B** for various values of **A**.

6. Explain your observations of what happens when **B** changes. What is **B** also known as?

Answers: The graph becomes more steep. B is the initial value.

Inquiry question



- Which variable seems to have a more profound effect on the sequence? Explain.

This question can spawn some constructive discussion. Students may have differing opinions.

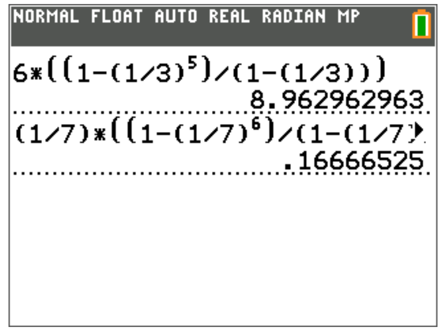
Extension – Deriving and Applying the Partial Sum Formula

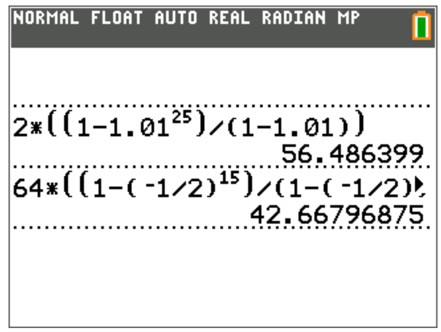
On the student worksheet, the derivation of the formula for the sum of a finite geometric series is shown. You may need to explain in detail the substitution in the third line.

For example, $a_2 = r \cdot a_1$, so $a_3 = r \cdot a_2 = r(r \cdot a_1) = r^2 \cdot a_1$.

In the fourth line, *r* is multiplied by both sides, changing r^{n-1} to r^n.

Students can apply the formula to find the sum of the series given on the worksheet.





8. $\frac{1}{7}+\frac{1}{7^2}+\frac{1}{7^3}+\frac{1}{7^4}+\frac{1}{7^5}+\frac{1}{7^6}=$

Answer: $\dfrac{\frac{1}{7}\left(1-\left(\frac{1}{7}\right)^6\right)}{1-\frac{1}{7}}=\dfrac{19608}{117649}\approx 0.167$

9. Find S_{25} for $a_n = 2(1.01)^{n-1}$.

Answer: $\dfrac{2\left(1-(1.01)^{25}\right)}{1-(1.01)}\approx 56.486$

10. $64-32+16-8+4-2+1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16}-\frac{1}{32}+\frac{1}{64}-\frac{1}{128}+\frac{1}{256}=$

Answer: $\dfrac{64\left(1-\left(-\frac{1}{2}\right)^{15}\right)}{1-\left(-\frac{1}{2}\right)}=\dfrac{10923}{256}\approx 42.668$