EXPLORATIONS

# Activity 9

## The Golf Ball Challenge

### Math Concepts

- Algebra
- Number Sense

### Science Concepts

- Data Collection
- Physical Science

### Materials

- 6 different golf balls
- TI-73 calculator
- CBR™
- Data link cable
- Masking or other type of tape
- Meter and/or yard stick

### *In this activity you will:*

- Form a pattern called a geometric sequence by dropping a golf ball from successive heights
- Explore the constant factor of this sequence, called the *constant* or *common ratio*
- Develop a plan for data collection and analysis to demonstrate that the stated hypothesis (the manufacture's golf ball will travel the greatest distance) is valid
- Demonstrate the decay of the bounce is in the form of $10 * 0.8^n$ (or $305 * 0.8^n$) where **n** is the bounce number

### Introduction

A certain golf ball manufacturer claims to sell a ball that travels the greatest distance. To support their claim, they did a comparison of elasticity using their ball and its major competitor. When dropped from any height onto a hard surface, their ball will rebound 0.8 (80%) of the original height, whereas the competitors ball will have a rebound ratio of only 0.7 (70%). Suppose their ball is dropped from a height of 10 feet (305 centimeters). The chart on the next page shows the rebound heights after the first 4 bounces of the manufacturer's golf ball and how each succeeding height is about 0.8 * the previous one.

| Bounce | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Height(ft) | 10 | 8.00 | 6.4 | 5.12 | 4.096 |
| Height(cm) | 305 | 244<br>(305 * .08) | 195<br>(244 * .08) | 156<br>(195 * .08) | 125<br>(156 * .08) |

### The Problem

In this activity, you will compare different golf balls based on their ability to bounce when dropped from a set height of about 5 feet (152 centimeters). You will look at the ratio of bounce heights for consecutive bounces, examine the pattern to see if it appears to be geometric, and determine the rate of decay of the bounce.

### The Set Up

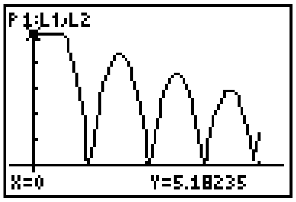
1. To begin with, you will need to measure heights of about 5 feet (152 cm) and 6.5 feet (198 cm).
   a. Mark these heights on the wall with masking tape.
   b. The ball-bouncing area needs to be a cleared area with a hard smooth surface floor. With two of the team's members facing each other, one student will hold the CBR™ at a height of 6.5 feet (200 cm) while another student will hold the ball at a height of 5 feet (150 cm). The range from the CBR to the object that will be monitored must be at least 1.5 feet (46 cm) away. A sturdy stool or chair to stand on may be needed for the team member that is holding the CBR.
2. Using the ball bounce program on the CBR, you will collect data on 6 different golf balls, collecting at least 4 bounce heights on each ball. (See *Appendix C: CBL / CBR APPS* for instructions on running the Ranger program.) You will then examine the change in heights of consecutive bounces. This can be done by dividing the second peak height by the first peak height, and so on.

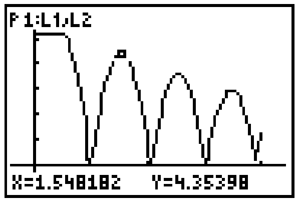
### Activity

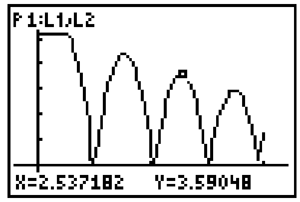
#### Collecting Data

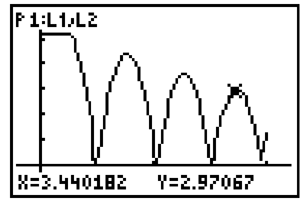
1. Press [APPS], select **2:CBL/CBR** from the **APPLICATIONS** menu. Press [ENTER] and then select **3:CBR** from the **CBL/CBR APP** menu. Run the **Ball Bounce** program following the directions on the screen.
2. After collecting "good data" on ball #1 (use option **5:REPEAT SAMPLE** from the **PLOT MENU** until you have the data you need), press [TRACE] and use [▶] to move to the top of each curve to determine the peak height of each bounce. For example: As you trace on the graph, as shown below, collect the value for the height (Y=) from the peaks representing each bounce.

✎ Record the values in question 4, Table 2, on the student data sheet.










**Start Height** **Y=5.182 ft.**

**Bounce 1** **Y=4.354 ft.**

**Bounce 2** **Y=3.59 ft.**

**Bounce 3** **Y=2.971 ft.**

✎ Record ball #1 data on Table 1 on the student data sheet as shown below.

| **Bounce** | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| **Height(ft)** | 5.182 | 4.354 | 3.59 | 2.971 |

**3.** To find the values of **A**, **B**, and **C** and store those to variables respectively, take the second height and divide it by the preceding height.

**a.** Using the sample data shown above, you would divide Bounce 1 by Bounce 0 (press **4.354** [÷] **5.182)**, and insert it as **A** (press [STO▶] [2nd] [TEXT], highlight the letter **A**, and press [ENTER]).

**b.** Select **Done** and then press [ENTER] from the Home screen to store this ratio.

**c.** Repeat this process when dividing Bounce 2 by Bounce 1 and storing to **B**, and dividing Bounce 3 by Bounce 2 and storing to **C**. Notice how these three values change.

**4.** To determine a constant or common ratio, find the mean average of **A**, **B**, and **C**.

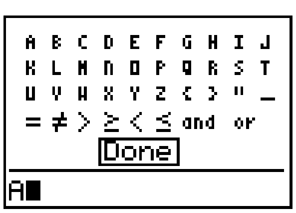
**a.** Press [2nd] [TEXT] and select the letters, with the [+] and [÷] keys to create the expression **(A+B+C)/3**.

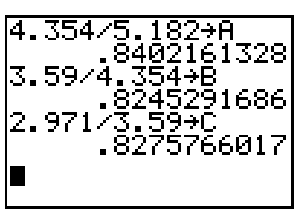
**b.** Select **Done** and press [ENTER] from the Home screen to do this calculation.

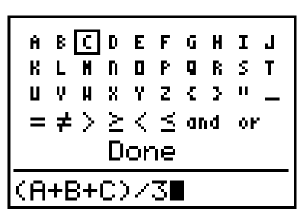
**c.** Verify this value with the mean function. Press [2nd] [STAT] [▶] [▶] (to **MATH** menu) and select **3:mean(** from the **MATH** menu.

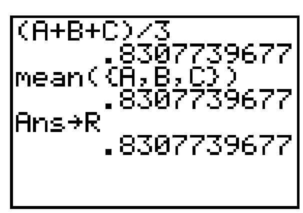
**d.** Now press [2nd] [TEXT] and move around and select the following symbols and the [,] [)] keys (pressing [ENTER] to pick each letter and the nixons) **{A,B,C}** and then pick **Done** [ENTER] from the Home screen.

**e.** Store this value to variable **R** by pressing [STO▶] [2nd] [TEXT], highlighting **R**, then selecting **Done** and pressing [ENTER] at the Home screen. This value **R** is the constant or common ratio.





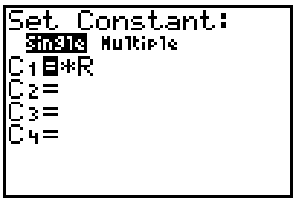


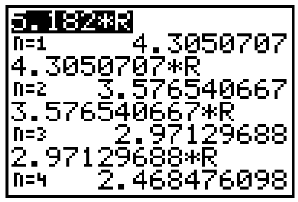


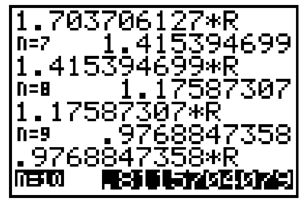
**5.** Use the [CONST] key to estimate how high the ball will bounce after five bounces, six bounces, ten bounces, and so forth.

- **a.** To do this, press [2nd] [SET], highlight **Single,** press [▲], and press [ENTER].
- **b.** Let **C1**=∗R by moving to that line and pressing [×] [2nd] [TEXT], selecting the letter **R** and then **Done**.
- **c.** Go to the Home screen ([2nd] [QUIT]) and type in the start height from Bounce 0 and press [CONST] any number of times, revealing the projected bounce height of consecutive bounces.

✎ Answer questions 1 - 3 on the student data sheet.



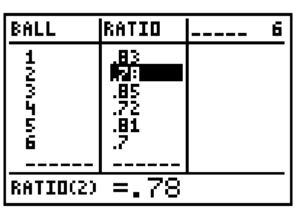



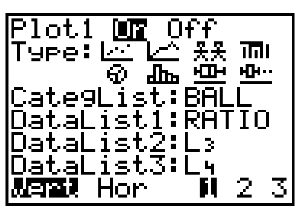

**6.** Collect data on the remaining five balls and record in Table 2 on the student data sheet.

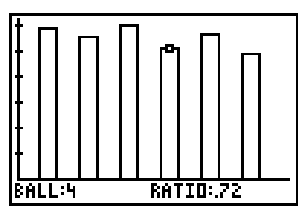
✎ Answer questions 4 - 7.

**7.** Determine which ball is better and analyze graphically. Choose any type of a plot to display your findings and to support your analysis.

- **a.** To create a bar graph press [LIST] and type a name such as **Ball** for one list (see the TI-73 manual for instructions on naming lists and graphing statistical plots) and name a second list **Ratio**.
- **b.** Enter 1 through 6 to represent the six different balls in the list named **Ball**.
- **c.** Enter the respective ratios in the list named **Ratio**.
- **d.** Set up **Plot1** as shown below.
- **e.** Display the graph.



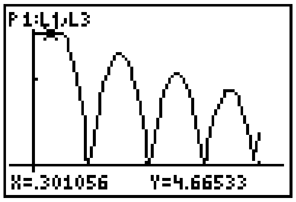



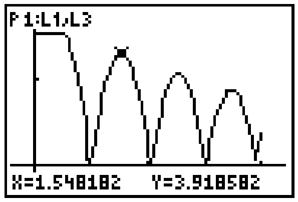

✎ Complete question 8 on the student data sheet.

## *Going Further*

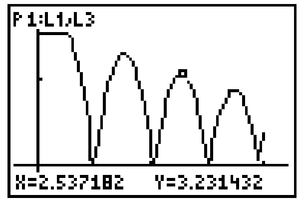
This data was collected in the same way you collected data. Use the graphs below to answer the following questions.



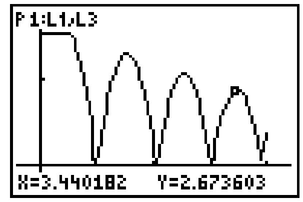

**Graph 1**




**Graph 2**




**Graph 3**




**Graph 4**

1. What was the starting height of the ball?________ Bounce 1 height?_________ Bounce 2 height?__________ Bounce 3 height? _________
2. Calculate the ratio of second peak $\boxed{\div}$ first peak, third peak $\boxed{\div}$ second peak, and fourth peak $\boxed{\div}$ third peak.
3. Find the average of these three ratios.

   This value is the *constant* or *common ratio*.
4. Estimate the heights of bounces 4, 5, 7, and 10.
5. Determine the constant ratio in each geometric sequence:

   **a.** 10, 7.5, 5.625, 4.21875

   **b.** 12, 7.2, 4.32, 2.592

   **c.** 4, 6, 9, 13.5
6. Find the next six terms in the sequences in **5**.
7. Can you think of another pattern in nature that could be geometric (exponential)?

Student Data Collection and Analysis Sheet

Name(s)______________________

Date ______________________

## Activity 9

# The Golf Ball Challenge

**Table 1**

| **Bounce** | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| **Height(ft or cm)** | | | | |

**1.** Complete.

**A** = _______

**B** = _______

**C** = _______

The average of **A**, **B**, **C** is **R**. **R** =_____

(**R** will be the estimated constant or common ratio.)

**2.** Record the first ten successive heights generated by using the constant key with **R** as the constant ratio. Round these values to a reasonable number of digits.

| n= | Height |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |

**3.** How do the first 4 terms in question **2** compare to the first four heights in Table 1?

______________________________________________

______________________________________________

**Table 2**

| Ball # | Start Height | Bounce 1 height | Bounce 2 height | Bounce 3 height | 2nd peak / 1st peak | 3rd peak / 2nd peak | 4th peak / 3rd peak | Average of 3 previous ratios |
|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | |
| 2 | | | | | | | | |
| 3 | | | | | | | | |
| 4 | | | | | | | | |
| 5 | | | | | | | | |
| 6 | | | | | | | | |

**4.** Are the ratios second peak $\boxed{\div}$ first peak, third peak $\boxed{\div}$ second peak, and so forth, for each ball constant? ______________________

**5.** Explain______________________________________

______________________________________________

Which golf ball had the highest rebound ratio average? ____________

How could you use this average to predict how high the ball will bounce on Bounce 5? ______________________________

Use the $\boxed{\text{CONST}}$ key to predict the heights on Bounce 6 = ____________; Bounce 10= ____________; and Bounce 100 = ________________.

**6.** Do you think that the numbers generated (of successive heights) for each ball appear to be geometric? Why? ______________________

______________________________________________

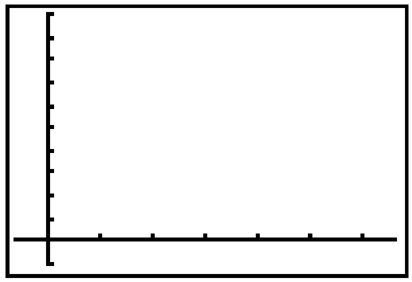
______________________________________________

**7.** Based on the data in Table 2, what would the equation be for ball 3 in the form of **a*****b**$^n$ where **a** is the starting height, **b** is the common ratio for the ball, and **n** is the bounce number. If you have a starting height of 10 feet and a ratio for the bounce of 0.8, you would have: $10*0.8^n$.

______________________________________________

**8.** As you trace a graph from the CBR™ data, describe the meaning of the **x** and **y** values.______________________________

**9.** Make a sketch (based on a type of graph from your TI-73) of the graph you created to support your findings (hypothesis) and tell what conclusions you made.



# Teacher Notes

## *Math Strands: Algebra and Number Sense*

The successive heights, 10, 8, 6.4, 5.12, 4.096... feet (305, 244, 195, 156, 125, … cm) form a sequence. This pattern is called a geometric sequence. In a geometric sequence, the ratio between any two successive terms is the same. The constant factor of 0.8 is called the *constant* or *common ratio*.

## *Science Strands: Data Collection and Physical Science*

Developing a plan for data collection and analysis to demonstrate that the stated hypothesis (the manufacture's golf ball will travel the greatest distance) is valid. In addition, the data demonstrates the decay of the bounce in the form of $10 * 0.8^{n}$ ($305 * 0.8^{n}$) where $n$ is the bounce number.

## *Classroom Management and Safety*

One set of materials for whole class approach, or one set per group if done in teams. Make sure the equipment and student are secure when dropping the golf balls.

## *The Set Up*

The best selection for golf balls would be a variation of manufacturers. A trip to a store that sells golf balls would give you data on prices as related to brands. Newer golf balls will work better. A good source for golf balls would be parents of kids (or grandparents) and driving ranges.

The TI-73/CBR™ will give data in meters or feet. As you work with this problem, centimeters and feet might be the best selection. Teacher choice is key here, but remember no one uses feet any more! The chance to change from meters to centimeters would be a good practice in the study of powers of 10 (multiplying by 100).

The CBR allows for several trials and the students should look for a graph with the pattern of smooth bounces as shown on page 2. Experimental technique and the limitation of variables are issues to be considered. The use of different kids to drop the balls will add some variations, but might be worth it if the problem is approached by the whole class rather than in teams in that more students get to participate.

A shorter height might help, as well as locating a "sweet" spot on the floor to drop on. The golf ball may have a point (the label) to aim at as you drop it.

The students might need to round values as they collect them or to set the mode to 2 or 3 digits, depending on grade level and units used (meters or feet).

***Note:*** *Sample data is in a program named* ***GOLF.73p****.*

## *Student Data Collection and Analysis Sheet – Key*

**1** and **2**:

**Table 1**

| **Bounce** | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| **Height(ft)** | 5.182 | 4.354 | 3.59 | 2.971 |

**3.** These values should be close but not exact due in part to the variation of the ratios (**A**, **B** and **C**). Look to **1-Var Stats** or a box-and-whiskers plot to make this clear to the students.

Table 2: Look for significant digits and a decrease in heights as the number of bounces increase.

**4.** They should be if rounded to the nearest tenth (hundredth with better kids, floor, golf balls). Students should note the significant digits issue in their responses.

**5.** The ball with the greatest average in the last column of Table 2. Use the constant key with the common ratio selected for **R** to test answers.

**6.** The pattern might be hard to see if done in feet, due to the lack of whole numbers to look at, but if looked at in measures to the nearest centimeter or inch, then the pattern should be obvious. Changing the values might be used to make the pattern clearer for students.

**7.** Discuss the nature of this equation showing exponential decay and test it with the equation solver or the constant key in the TI-73.

**8.** **x** is time in seconds, and **y** is height in meters or feet. The time is not significant, but the bounce number is.

**9.** Look for the use of various graphs (bar, multiple bar, pictograph, circle graph, scatter, xyLine), window settings, and types of data graphed (time versus height, ball number versus common ratio, ball bounce versus height, and so on) to make the point.

## *Going Further – Key*

**1.** 4.67 feet, 3.92 feet., 3.23 ft, 2.67 feet.

**2.** 0.84, 0.82, 0.83.

**3.** 0.83.

**4.** Look for use of constant key in solution: 2.22 feet, 1.84 feet, 1.27 feet, 0.72 feet.

**5.** a = 0.75, b = 0.60, c = 0.67.

**6.** Look for use of Constant key in solution: a = 3.16, 2.37, 1.78, 1.33, 1.00, 0.75; b = 1.56, 0.93, 0.56, 0.34, 0.20, 0.12; c = 9, 6, 4, 8/3, 16/9, 32/27.

**7.** Growth patterns, chemical compounds, or genetics.