EXPLORATIONS

ACTIVITY 9

Graphing an Extra Dimension
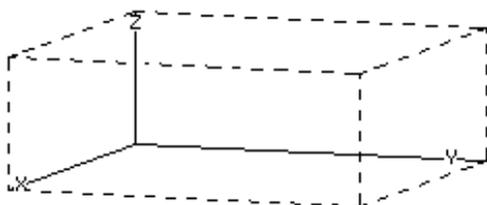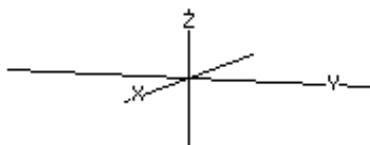
The real world is three-dimensional. Our motion is not confined to a plane. Rather, we are free to move up-and-down, back-and-forth, and side-to-side through space. Mathematically, we can describe the location of points in space in terms of three mutually perpendicular coordinate axes, as shown in the top illustration. The *x*-, *y*-, and *z*-axes, together with a box fashioned from dashed lines, are shown in the bottom illustration. Sides of the box represent portions of the three coordinate planes determined by intersecting pairs of axes.

The bottom of the box is a portion of the *xy* plane, the back of the box is a portion of the *yz* plane, and the left face of the box is a portion of the *xz* plane.

The graph of an equation in space is called a surface. In this activity, you will learn how to use the Voyage™ 200 PLT to graph in three dimensions. Particularly, you will focus on the characteristics of surface types called *paraboloids*.

Instructions

Part A—Reviewing the Basic Concept

How does the value of a affect the two-dimensional graph of function $y = ax^2$?

1. Press [ON] to turn on the Voyage 200 PLT. To reset to the default settings, press [2nd] [MEM] [F1], select 1:RAM, select 2:Default, and then press [ENTER] [ENTER].

2. Press [♦] [Y=] to display the Y= Editor. Press [F1], select 8:Clear Functions, and press [ENTER]. Press [ENTER] to move the cursor to the entry line. Press **A** [×] **X** [^] **2** [ENTER] to define the function y1 = ax².

3. Return to the Home screen by pressing [♦] [CALC HOME]. Store 1 to the variable a by pressing **1** [STO▶] **A** [ENTER]. Press [♦] [Y=] [F2], and select 4:ZoomDec to see the graph.

Graphing an Extra Dimension *(Continued)*

4. To change the value of a, press $\boxed{\blacklozenge}$ $\boxed{\text{CALC HOME}}$ to return to the Home screen, type a new value for a, and press $\boxed{\text{STO▶}}$ **A** $\boxed{\text{ENTER}}$. To see the new graph, press $\boxed{\blacklozenge}$ $\boxed{\text{GRAPH}}$.

    Repeat this procedure for several different values of a. Particularly, focus your attention on the categories in the table below. Summarize your findings by completing the right-hand column of the table.

a-value	effect on the graph compared with $y = x^2$
$0 < a < 1$	*moves away from the y-axis*
$a > 1$	
$-1 < a < 0$	
$a < -1$	*reflects over the x-axis; moves toward the y-axis*

Part B—Graphing Three-Dimensional Figures

To extend the exercise you just completed to three dimensions, consider the graph of $z = ax^2 + by^2$. How do the constants a and b affect the graph of this surface?

1. Press $\boxed{\text{MODE}}$ $\boxed{\text{⊙}}$, select 5:3D, and then press $\boxed{\text{ENTER}}$ to put your Voyage™ 200 PLT in three-dimensional graphing mode.

2. Press $\boxed{\blacklozenge}$ $\boxed{\text{Y=}}$ to display the Y= Editor. Press $\boxed{\text{F1}}$, select 8:Clear Functions, and press $\boxed{\text{ENTER}}$. Press $\boxed{\text{ENTER}}$ to move the cursor to the entry line, and then press **A** $\boxed{\times}$ **X** $\boxed{\wedge}$ **2** $\boxed{+}$ **B** $\boxed{\times}$ **Y** $\boxed{\wedge}$ **2** $\boxed{\text{ENTER}}$ to define the 3D equation z1 = ax^2 + by^2.

3. Change the graph format to display and label the axes by pressing $\boxed{\text{F1}}$ **9** $\boxed{\text{⊖}}$ $\boxed{\text{⊙}}$ and selecting 2:Axes. Then press $\boxed{\text{⊖}}$ $\boxed{\text{⊙}}$ and select 2:ON. Press $\boxed{\text{ENTER}}$.

4. Return to the Home screen by pressing $\boxed{\blacklozenge}$ $\boxed{\text{CALC HOME}}$. Store 1 to the variable a by pressing **1** $\boxed{\text{STO▶}}$ **A** $\boxed{\text{ENTER}}$. Store 1 to the variable b by pressing **1** $\boxed{\text{STO▶}}$ **B** $\boxed{\text{ENTER}}$.

Graphing an Extra Dimension *(Continued)*

5. To see the graph, press ◆ [Y=] F2 and select
   A:ZoomFit. Your graph should look like the one
   shown to the right.
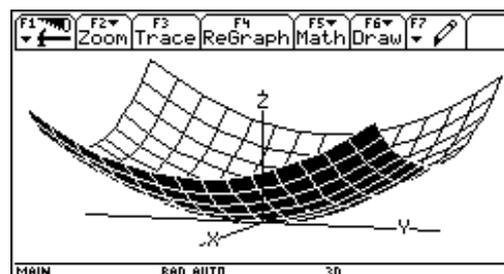
6. Notice that the xz plane intersects the surface
   $z = x^2 + y^2$ along a parabolic curve. This curve
   is called the *trace* of the surface in the xz plane.
   After a surface has been represented graphically
   in three dimensions, it is possible to visualize its
   traces in different planes. Usually, the traces of a
   surface in each of the three coordinate planes are
   considered first.

   Consider the intersection of the surface $z = x^2 + y^2$
   with the yz plane and the xy plane. What are the
   resulting traces? Summarize your results by
   completing the table below.

plane	trace
xz	*parabola, opening upward*
yz	
xy	

7. In the table in step 6, you described the trace
   of $z = x^2 + y^2$ in the xy plane. What is the trace of
   $z = x^2 + y^2$ in a plane *parallel* to the xy plane?
   In the space below, describe this trace completely.

   _____

   _____

8. How does changing the values of the constants a
   and b affect the graph of $z = ax^2 + by^2$? To change
   the value of a to 5, press ◆ [CALC HOME] to return to
   the Home screen and then **5** STO▸ **A** ENTER.

   To see the new graph, press ◆ [GRAPH]. Which
   coordinate-plane traces, if any, are affected by this
   change as compared with traces of $z = x^2 + y^2$? In
   the space below, give a complete description of any
   changes that occurred.

   _____

   _____

Graphing an Extra Dimension *(Continued)*

9. Change the value of a to 1 by pressing
    ◆ [CALC HOME] to return to the Home screen,
    and then pressing **1** STO▶ **A** ENTER. Change the
    value of b to 5 by pressing **5** STO▶ **B** ENTER.
    Before you view the new graph, predict how the
    coordinate-plane traces will appear. Record your
    predictions in the table below.

    To see the new graph, press ◆ [GRAPH]. Record your
    observations in the last column of the table. How do
    your predictions and observations compare?

plane	predicted effect	observed effect
xz		
yz		
xy		

10. Using the method described in step 8 and step 9,
    test several different *positive* values for a and b.
    After making each adjustment, press ◆ [GRAPH] to
    see the new surface. On the basis of your
    observations, when is a trace of $z = ax^2 + by^2$ in a
    plane parallel to the xy plane circular and when is it
    elliptical?

    _____

    _____

    _____

11. What if the values of a and b are not both positive?
    Press ◆ [CALC HOME] to return to the Home screen.
    Store the value of 1 to b and the value of –1 to a, as
    described above.

    To see the new graph, press ◆ [GRAPH]. How did this
    change affect the coordinate-plane traces compared
    with traces of $z = x^2 + y^2$? Give a complete
    description of any changes that occurred.

    _____

    _____

Graphing an Extra Dimension *(Continued)*

12. Return to the Home screen by pressing
    ◆ [CALC HOME]. Return the value of a to 1 and b
    to -1. Before you view the new graph, predict how
    the coordinate-plane traces will appear. Record
    your predictions in the table below.

    To see the new graph, press ◆ [GRAPH]. Record your
    observations in the last column of the table. How do
    your predictions and observations compare?

plane	predicted effect	observed effect
xz		
yz		
xy		

13. What if the values of a and b are *both* negative?
    Return to the Home screen by pressing
    ◆ [CALC HOME]. Store negative values to a and b.
    Then press ◆ [Y=] [F2], and select A:ZoomFit to see
    the new surface. Describe the resulting graph
    completely in the space below.

    _____

    _____

Graphing an Extra Dimension *(Continued)*

Questions

1. When a and b have the same sign, the surface
    $z = ax^2 + by^2$ is called an *elliptic paraboloid*.
    Explain why you think this name is appropriate.

    _____

    _____

    _____

    _____

2. Choose an appropriate name for the surface
    $z = ax^2 + by^2$ for the case when a and b have *opposite*
    signs. Explain the reasoning for your name choice.

    _____

    _____

    _____

    _____