

What's the Fastest Way to Get Rich?

Overview – Activity ID: 8933

Students investigate exponential growth and discover patterns by exploring different scenarios involving money. They will learn how repeatedly doubling, or multiplying by two, can also be expressed exponentially.

Math Concepts

- patterns
- nonlinear functions
- lists
- exponents
- number sense

Materials

- TI-34 MultiView™
- graph paper
- pencil

Activity

Give students the following scenario:

Your family wins the lottery and decides to present you with a reward for doing your daily chores for a month. They give you a choice of two payment plans:

Plan A: You can receive two pennies the first day, four pennies the second day, eight pennies the third day, 16 pennies the fourth day, and so on, each day doubling the amount you receive until the last day of the month (30 days).

Plan B: You can receive a one-time payment of \$1,000,000 at the end of the month.

Which plan should you pick?

Begin by having the students construct a table by hand. Help them get started by discussing what they'll need to include in each column of the table.

We know you'll get \$1,000,000 with Plan B. Therefore, it's Plan A we need to investigate. Let's begin by looking at how much money you'll receive in the first several days with Plan A. We'll need columns for the day number, the amount earned that particular day, and the total amount earned.

Work through the first several days with the students. (A visual is included at the end of this activity for classroom projection and discussion. The table below is for teacher reference only.)

Now that you've calculated how much you'd earn during the first week if you chose Plan A, what's your hunch? Should you chose Plan A or Plan B? Do you think we should continue to calculate?

Day number	Amount paid in cents	Total earned in cents
1	2	2
2	4	$2 + 4 = 6$
3	8	$6 + 8 = 14$
4	16	$14 + 16 = 30$
5	32	$30 + 32 = 62$
6	64	$62 + 64 = 126$
7	128	$126 + 128 = 254$
8	256	$254 + 256 = 510$



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Now try this: How much would your family pay you on day number 15? Do the work at your seat, and be prepared to discuss how you arrived at your answer.

Discuss the pattern created by doubling the previous day's earnings. While the first few days are easily calculated using mental math, as the month goes on, the numbers get much larger. Demonstrate how to use the TI-34 MultiView to simplify the process.

Now let's use the functionality of the TI-34 MultiView to make this process simpler. First, let's repeatedly multiply by 2 by using the Stored Operations feature of the calculator. You will quickly be able to see the pattern in progress. This is one way to calculate how much money your family would pay you on a given day with Plan A.


Have the students continue to complete the table individually at their seats. They should go all the way through day 30 so they see how much money they would earn on the last day of the month.

Move on to exponents, demonstrating how repeatedly multiplying by 2 can also be written using exponents.

Is there an easier way to calculate any given day's earnings than continually multiplying the previous day by 2?

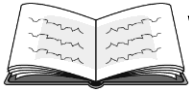
When repeatedly multiplying by the same factor, we can also use exponents. For example, $2^2 = 4$, $2^3 = 8$, and $2^4 = 16$. Notice that these are all payment amounts for certain days. Using the table we already created, identify how the amount paid on any day depends upon the day number. Use exponents.

Add a column to the initial table, asking students to express the amount paid on a given day as a power of 2.

 Follow these steps:

1. Press **2nd** [**set op1**].
2. Press **clear** if anything appears in this screen.
3. Press **2** **enter** to store the operation, then **2nd** [**quit**] to return to the home screen.
4. Press **1**, **op1**, then **op1** repeatedly to multiply each answer by 2.
5. The screen should show this:

	DEG	↕
1×2	n=1	2
2×2	n=2	4
4×2	n=3	8
8×2	n=4	16



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Day number	Amount paid in cents	Total earned in cents	Power of 2
1	2	2	2^1
2	4	$2 + 4 = 6$	2^2
3	8	$6 + 8 = 14$	2^3
4	16	$14 + 16 = 30$	2^4
5	32	$30 + 32 = 62$	2^5
6	64	$62 + 64 = 126$	2^6
7	128	$126 + 128 = 254$	2^7
8	256	$254 + 256 = 510$	2^8

The students may need assistance seeing the pattern of 2^n where n is the day number. It's not crucial they be able to write the expression algebraically, but they should be able to see that on any day, the amount earned is $2^{(\text{day number})}$.

Again ask the following question:

*How much would your family pay you on day number 15?
This time, use an exponential expression to arrive at your answer.*

Students should use the formula $2^{15} = 32,768$.

How much money is this? In cents and in dollars?

Review the conversion: $32,768 \text{ ¢} = \$327.68$

*Now, determine how much money you would earn over the entire 30-day month with Plan A. Can you see a pattern?
The total in cents after any given day is twice the amount earned that day, minus 2 cents.*

To conclude, ask "What plan should you pick?" Students will need to find the sum of every day's payment in Plan A, THEN convert to dollars. Plan A would pay a total of \$21,474,836.46. Plan B would pay the flat rate of \$1,000,000. Plan A is a better choice.


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Date _____



Directions: Read the problem below, then record your results in the tables. After each table has been completed, answer the questions that follow it.

Problems with  should be calculated using mental math rather than the TI-34 MultiView™.

You are offered an executive position by a millionaire businessman who asks you to choose which one of two ways you prefer to be paid. With Proposal 1, you would receive 2 cents the first day, and then your pay would be doubled each day for 30 straight days. With Proposal 2, you would receive \$1,000 the first day and an additional \$1,000 each day until the end of the month. That is, you would receive \$2,000 the second day, \$3,000 the third day, and so on. Which method of payment would you prefer?

PROPOSAL 1: (Use class notes.)



Day number (<i>n</i>)	Amount paid in cents	Total earned in cents	Total earned in dollars
1			
2			
3			
4			
5			
6			
7			
10			
14			
21			
30			

1. In Proposal 1, how much would you earn on day number 7?
2. What is the total amount earned in dollars after 7 days in Proposal 1?
3. If you replaced “day number” with "*n*” in Proposal 1, what would be the formula to generate the amount paid in cents?
4. How much would you earn on day number 21 in Proposal 1?
5. In Proposal 1, compare the “Amount paid in cents” and “Total earned in cents” columns. Can you see a correlation between the numbers? Describe what you observe.

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- Write a formula to calculate the total earned in cents. Use n for the day number.
- Using the formula you just wrote, calculate the total earned in Proposal 1 after 30 days.

PROPOSAL 2:



Day number (n)	Amount paid in dollars	Total earned in dollars
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
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22		
23		
24		
25		
26		
27		
28		
29		
30		

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8. In Proposal 2, how much would you earn on day number 7?

9. Which proposal pays more after one week? How much more?

10. Which proposal pays more after 21 days? How much more?

11. Which proposal would you prefer? How much more money would you earn with your selected proposal?

12. On which day does the total earned in Proposal 1 surpass the total earned in Proposal 2?



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Answer Key

PROPOSAL 1:

Day number (n)	Amount paid in cents	Total earned in cents	Total earned in dollars
1	2	2	0.02
2	4	6	0.06
3	8	14	0.14
4	16	30	0.30
5	32	62	0.62
6	64	126	1.26
7	$2^7 = 128$	254	2.54
10	$2^{10} = 1024$	2046	20.46
14	$2^{14} = 16,384$	32,766	327.66
21	$2^{21} = 2,097,152$	4,194,302	41,943.02
23	$2^{23} = 8,388,608$	16,777,214	167,772.14
24	$2^{24} = 16,777,216$	33,554,430	335,544.30
30	$2^{30} = 1,073,741,824$	2,147,483,646	21,474,836.46

- In Proposal 1, how much would you earn on day number 7?
 $\$1.28$
- What is the total amount earned in dollars after 7 days in Proposal 1?
 $\$2.54$
- If you replaced “day number” with n in Proposal 1, what would be the formula to generate the amount paid in cents?
 2^n
- How much would you earn on day number 21 in Proposal 1?
 $2^{21} = 2,097,152 \text{ ¢}$ or $\$20,971.52$
- In Proposal 1, compare the “Amount paid in cents” and “Total earned in cents” columns. Can you see a correlation between the numbers? Describe what you observe.
Answers will vary. One possibility: The total after any number of days is double the amount earned that day, minus 2 cents.
- Write a formula to calculate the total earned in cents. Use n for the day number.
 $2(2^n) - 2$
- Using the formula you just wrote, calculate the total earned in Proposal 1 after 30 days.
 $2(2^{30}) - 2 = 2(1,073,741,824) - 2 = 2,147,483,646 \text{ ¢} = \$21,474,836.46$



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PROPOSAL 2:

Day number (<i>n</i>)	Amount paid in dollars	Total earned in dollars
1	1,000	1,000
2	2,000	3,000
3	3,000	6,000
4	4,000	10,000
5	5,000	15,000
6	6,000	21,000
7	7,000	28,000
8	8,000	36,000
9	9,000	45,000
10	10,000	55,000
11	11,000	66,000
12	12,000	78,000
13	13,000	91,000
14	14,000	105,000
15	15,000	120,000
16	16,000	136,000
17	17,000	153,000
18	18,000	171,000
19	19,000	190,000
20	20,000	210,000
21	21,000	231,000
22	22,000	253,000
23	23,000	276,000
24	24,000	300,000
25	25,000	325,000
26	26,000	351,000
27	27,000	378,000
28	28,000	406,000
29	29,000	435,000
30	30,000	465,000

8. In Proposal 2, how much would you earn on day number 7?
\$7,000
9. Which proposal pays more after one week? How much more?
Proposal 2 pays more by \$6,997.46 (i.e., \$7,000 – \$2.54).
10. Which proposal pays more after 21 days? How much more?
Proposal 2 pays more by \$189,056.98 (i.e., \$231,000 – \$41,943.02).



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11. Which proposal would you prefer? How much more money would you earn with your selected proposal?

Proposal 1. You would earn \$21,009,836.46 more!

12. On which day does the total earned in Proposal 1 surpass the total earned in Proposal 2?

On day number 24

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Transparency

Day number	Amount paid in cents	Total earned in cents
1	2	
2	4	
3	8	
4	16	
5		

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Extension Activity

Directions: Use the tables completed in the previous activity to create a graph comparing the growth rates of proposals 1 and 2. Use your own graph paper, and attach your completed graph to this worksheet. Label the x -axis "Day Number," and label the y -axis "Total amount earned in dollars."

After the graph is completed, answer the following questions:

1. After which day will Proposal 1 pay you more?
2. After which day will Proposal 2 pay you more?
3. On which day will the total amount earned with Proposal 1 surpass the total amount earned with Proposal 2?
4. Describe the graphs of Proposal 1 and Proposal 2. Do they appear to be linear? Exponential? Explain.