## A Geometric Repr esentation of Trigonometric Functions Using TI-92

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Using similar triangles, a relationship is established between the 6 trigonometric functions of an angle and simple line segments. These relationships are then confirmed using line segment measurement capabilities of TI-92.


Figure 1:
Relationship (1) - (6) are developed using this diagram.


Figure 2:
TI-92's measurement of the 6 line segments.


Figure 3:
TI-92's calculation of the 6 trig functions. See Figure 2.


Figure 4:
As point A is "dragged", measurements are updated.


Figure 5:
New calculations based on measurements of Figure 4.

1. Construct a unit circle centered at the origin $(\mathbf{O A}=1)$.
2. At points $\mathbf{X}$ and $\mathbf{Y}$, draw tangents to the circle parallel to $\mathbf{x}$ and $\mathbf{y}$ axes. $\angle \mathbf{X O A}=\alpha$ is an angle in standard position with $\mathbf{A C} \perp \mathbf{O X}$ and $\mathbf{A S} \perp \mathbf{O Y}$, see Figure 1. Using similar triangles, we have:
$\frac{\mathrm{AC}}{\overline{\mathrm{O}} \mathrm{A}}=\frac{\mathrm{OS}}{\mathbf{1}^{-}}=\sin (\alpha)$
$\frac{O C}{\bar{O} \bar{A}}=O C=\cos (\alpha)$
$\underset{\overline{\mathrm{O}} \overline{\mathrm{X}}}{\mathrm{XT}}=\frac{\mathrm{AC}}{\mathbf{O} \overline{\mathrm{C}}} \Rightarrow \mathrm{XT}=\frac{\sin (\alpha)}{\overline{\cos } \overline{\mathrm{s}}(\alpha)}=\tan (\alpha)$

$\overline{\mathrm{Y}} \overline{\mathrm{O}}=\frac{\mathrm{AS}}{\overline{\mathrm{O}} \mathrm{S}} \Rightarrow \mathrm{YG}=\frac{\cos (\alpha)}{\overline{\sin }(\alpha)}=\cot (\alpha)$
$\frac{\mathrm{OG}}{\mathrm{OA}}=\frac{\mathrm{YG}}{\mathrm{SA}} \Rightarrow \mathbf{O G}=\frac{\cot (\alpha)}{\cos (\alpha)}=\frac{1}{\sin (\alpha)}=\csc (\alpha)$
3. In the geometry environment of TI-92 construct the diagram of Figure 1 and measure the length of the 6 line segments obtained in (1) through (6) as well as the measure of angle $\alpha$. These measurements can be to any desired degree of accuracy as shown in Figure 2.
4. Next, using a simple program we calculate the 6 trigonometric functions of angle $\alpha$ as it was measured in Figure 2. Figure 3 shows the result of these calculations.
5. Complete agreement between measurements of Figure 2 and calculations of Figure 3 as it was predicted by the relationships (1) through (6).
6. Now as we "drag" point A on the circumference of the circle, TI-92 continuously measures the new $\alpha$ as well as the 6 line segments. See Figure 4.
7. Figure 5 shows once again the result of the calculations of the trigonometric functions of new $\alpha$ and they are in complete agreement with the measurements of Figure 4.


Figure 6:
Split screen allows for simultaneous viewing.

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8. Using split screen feature of TI-92, we can combine Figures 2 and 3 into a single screen for simultaneous viewing, See Figure 6.
9. If model of Figure 1 is superimposed on a Cartesian coordinate system, it is easy to see that line segments OC,OS, XT, and YG will take on negative quantities as points $\mathbf{C}, \mathbf{S}, \mathbf{T}$, and $\mathbf{G}$ oscillate between the 4 quadrants.

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