

Exploring the Fundamental Theorem of Calculus

by – Jeff VanArnhem, Olmsted Falls High School

Activity overview

The purpose of this activity is to help students be able to visualize the fundamental theorem of calculus.

Students will look at the graph of $F(x) = \int_a^x f(t) dt$ and see how it compares to the graph of $f(x)$.

The activity will facilitate discussion on how to sketch the graph of F from the graph of f and visa versa.

Concepts

- *Fundamental Theorem of Calculus*
- *Integration*
- *Differentiation*
- *Graph sketching*
- *Increasing/Decreasing and concavity*

Teacher preparation

Teacher will upload the Integrals.tns file to all student calculators.

Classroom management tips

Allow students 20 minutes to explore the relationships between the function $F(x)$ and $f(x)$. Then have class discussion with an example displayed for the entire class.

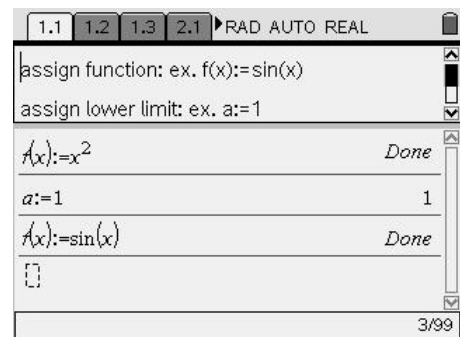
TI-Nspire Applications

Note pages, Graphs pages, Spreadsheet page, and Calculator pages

Step-by-step directions

1) Define the function f and the lower limit of integration a for

the function $\int_a^x f(t) dt$.



Fundamental Theorem of Calculus

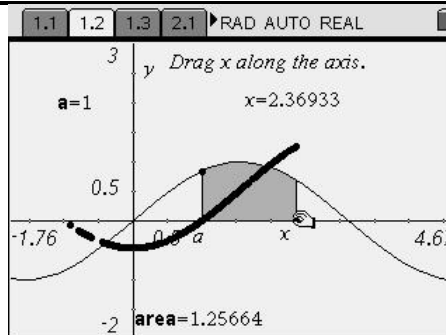
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Grade level: secondary

Subject: Calculus

Time required: 45 to 90 minutes

2) Students can drag the point x along the axis to visualize the integral function as an “area accumulation” function. This leads to discussion of how the integral is modeled by positive area (accumulation) and “negative” area (taking away).



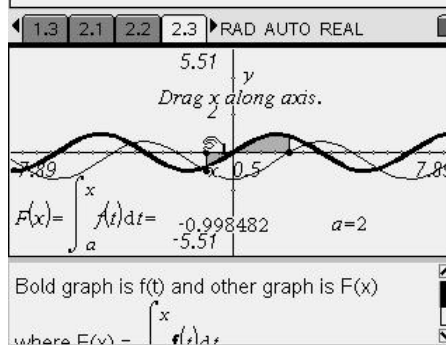
3) This page is where all the values are calculated. Students should not change any values or formulas on this page. However, in order to clear the plotted points on the graph on page 1.2, students should follow the directions on the notes page at the bottom of the screen. By double-clicking on the capture function and hitting enter, the column is “cleared” of its values and the capture function captures the first value.

	A l...	B y...	C	D xint	E x...	F	G
1	1	sin(1)		=captu	=captu		
2	1	0		-.78...	-.16...		
3				-.76	-.18		

4) Problem 2 is an offshoot of problem 1. This time however, a function is graphed as well as its antiderivative using the Fundamental Theorem of Calculus. Students can even build this problem on their own. It just transfers the integral value of the function from a fixed point a to a point x onto the y -axis. A locus of (x, y) points are created with parameter x . If you do not wish to have students create the activity, I have completed it here already.

Define f(x) and lower.	
$f(x):=\sin(x)$	Done
lower:=1	1
lower:=2	2
[]	

5) Students again can see the relationship between a function and its antiderivative. With the functions completely graphed, you can have discussion about why the antiderivative is negative, increasing, zero, etc. and how this relates to the original function.



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6) A few questions are built into the activity to help students self assess their understanding of the concepts covered in this activity. Students should be prepared to explain their answer. Students may need to move back and forth between the questions and the graph page.

2.1 2.2 2.3 2.4 ▸ RAD AUTO REAL

Question
When f is positive, what is true about the graph of F?
Answer ▾
The graph of F is increasing.

Assessment and evaluation

- *Self-assessment for the student is built into note pages in the activity*
- *Students will be orally assessed during the classroom discussion*
- *Students will work through AP Exam questions that use the Fundamental Theorem of Calculus*

Activity extensions

- *Students build the antiderivative grapher themselves to better see the relationships involved in the Fundamental Theorem of Calculus*
- *The Fundamental Theorem of Calculus Extension where the upper limit is a function and not simply x can be explored.*
- *The Mean Value Theorem for Integrals and the Average Value of a Function can also be tied into the graph discussions.*

Student TI-Nspire Document *Integral.tns*

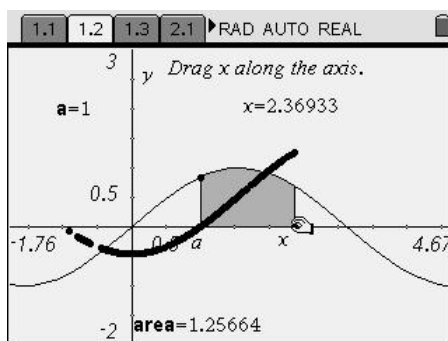
1.1 1.2 1.3 2.1 ▸ RAD AUTO REAL

assign function: ex. $f(x) := \sin(x)$

assign lower limit: ex. $a := 1$

$f(x) := x^2$	Done
$a := 1$	1
$f(x) := \sin(x)$	Done
[]	

3/99



1.1 1.2 1.3 2.1 ▸ RAD AUTO REAL

	A L...	B y...	C	D xint	E x...	F	G
				=captu	=captu		
1	1	$\sin(1)$		-91...	-07...		
2	1	0		-78...	-16...		
3				-76	-18		

A1 | = a

To clear the capture, double-click on the capture function and hit enter.

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1.1 1.2 1.3 2.1 RAD AUTO REAL

In this problem, you will again define a function and a lower limit of integration. The graph of the integral will automatically be graphed. Questions begin on page 2.4 which use the graph on page 2.3.

1.2 1.3 2.1 2.2 RAD AUTO REAL

Define f(x) and lower.

$f(x):=\sin(x)$	Done
lower:=1	1
lower:=2	2
[]	

3/99

1.3 2.1 2.2 2.3 RAD AUTO REAL

Bold graph is f(t) and other graph is F(x)
where $F(x) = \int_a^x f(t) dt$

2.1 2.2 2.3 2.4 RAD AUTO REAL

Question

When f is positive, what is true about the graph of F?

Answer

The graph of F is increasing.

2.2 2.3 2.4 2.5 RAD AUTO REAL

Question

When the graph of f is increasing, what does that tell you about the graph of F?

Answer

The graph of F is concave up.

2.3 2.4 2.5 2.6 RAD AUTO REAL

Question

When f(x)=0, what does that tell you about the graph of F?

Answer

The graph of F has a relative extrema.