Angle Bisectors in a Triangle

ID: 8889

Time required
40 minutes

Activity Overview

In this activity, students will explore the relationships between an angle bisector and segments in a triangle. They will determine the distances from an angle bisector to the sides of the bisected angle. In a triangle, proportional relationships occur when an angle bisector divides the opposite side into two parts.

Topic: Triangles and Their Centers

- *Use inductive reasoning to postulate a relationship between an angle bisector and the arms of the angle.*
- *Apply the Angle Bisector Theorem and its converse.*

Teacher Preparation and Notes

- *The Angle Bisector Theorem states:*
  If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.
- *In a triangle, when an angle bisector divides the opposite side into two parts, the segments created are proportional to the adjacent sides. For the diagram used in Problem 2, the following proportions are both true:*

$$\frac{BD}{AB} = \frac{CD}{AC} \quad \text{and} \quad \frac{BD}{CD} = \frac{AB}{AC}$$

- *Problem 3 is an optional extension involving the incenter of the triangle, which is the point of concurrency of all three angle bisectors.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student TI-Nspire document (.tns file) and the student worksheet, go toeducation.ti.com/exchange and enter "8889" in the quick search box.***

Associated Materials

- *AngleBisectorsInATriangle_Student.doc*
- *AngleBisectorsInATriangle.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

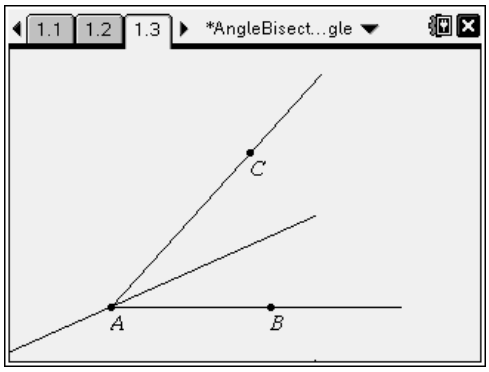
- *Altitude, Median, and Angle Bisector of a Triangle (TI-84 Plus family) — 4055*
- *Special Segments in Triangles (TI-Nspire technology) — 9421*
- *Angle and Perpendicular Bisectors in a Triangle (TI-Nspire technology) — 11392*
- *The Sprinkler and the Lawn (TI-Nspire technology) — 11674*

Problem 1 – The Angle Bisector Theorem

Read the directions on page 1.2.

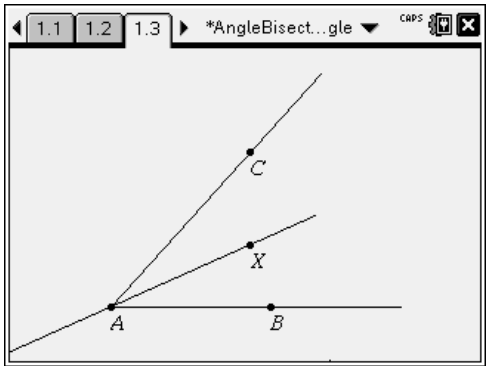
On page 1.3, $\angle BAC$ has been constructed.

Students should first construct the angle bisector of $\angle BAC$ using the **Angle Bisector** tool (**MENU > Construction > Angle Bisector**).



Direct students to place a new point on the angle bisector, using the **Point On** tool from the Points & Lines menu.

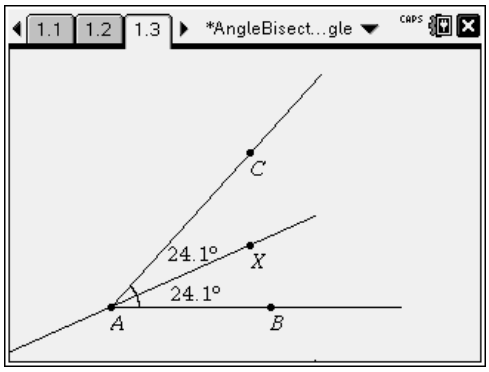
Label this point *X*. (A point may be labeled by typing its label directly after placing the point, or afterwards by using the **Text** tool (**MENU > Actions > Text**).)



Have students measure angles $\angle BAX$ and $\angle CAX$ using the **Angle** tool from the Measurement menu.

Drag point *B* or *C* and observe the results. Ask: *Does this confirm the definition of an angle bisector?*

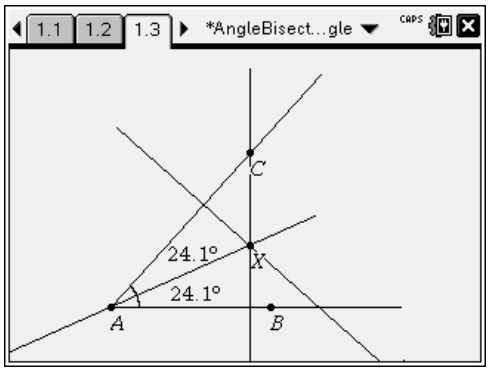
If desired, hide the angle measures with the **Hide/Show** tool (**MENU > Actions > Hide/Show**).



The distance from point *X* to the sides of the angle must be measured perpendicularly.

Students will construct a perpendicular line through *X* perpendicular to $\overleftrightarrow{AB}$ using the **Perpendicular** tool (**MENU > Construction > Perpendicular**).

Repeat to construct a line through *X* perpendicular to $\overleftrightarrow{AC}$.

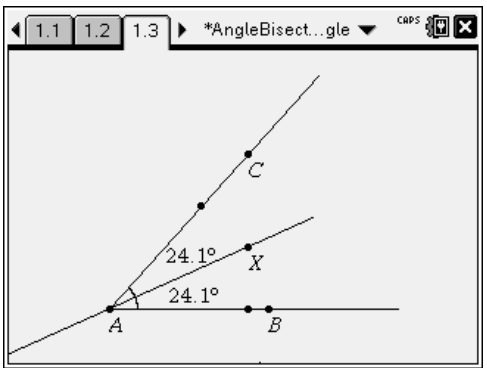


TI-Nspire Navigator Opportunity: *Screen Capture*

See Note 1 at the end of this lesson.

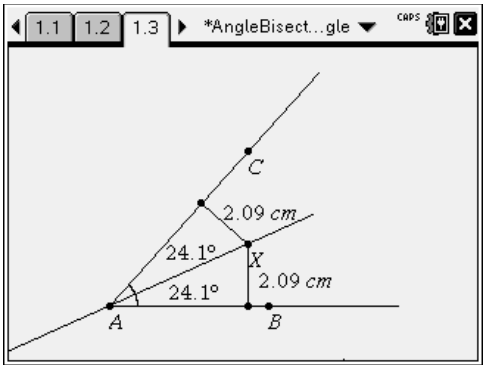
Students should use the **Intersection Point(s)** tool (**MENU > Points & Lines > Intersection Point(s)**) to place points at the intersection of $\overrightarrow{AB}$ and its perpendicular line and the intersection of $\overrightarrow{AC}$ and its perpendicular line.

Hide the perpendicular lines (**MENU > Actions > Hide/Show**).

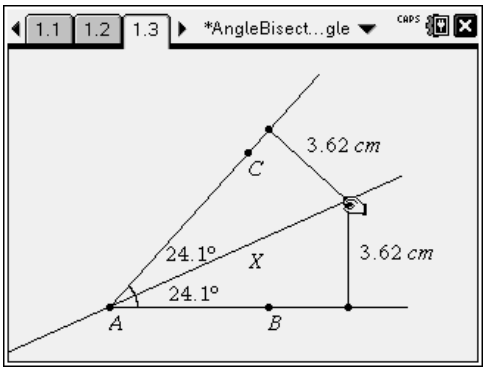


Have students use the **Segment** tool (**MENU > Points & Lines > Segment**) to connect *X* to each intersection point.

Measure the lengths of each segment using the **Length** tool (**MENU > Measurement > Length**).

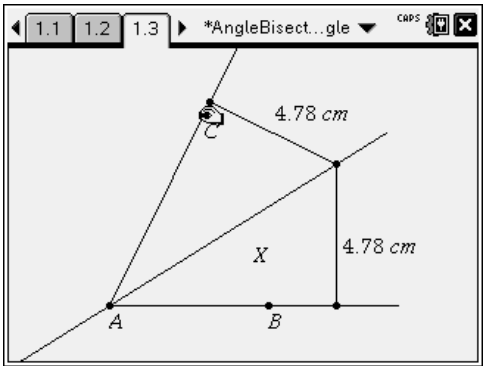


Students should drag point *X* and observe the changes in the measurements.



Then drag point *B* or *C* to change the size of the angle and observe the results.

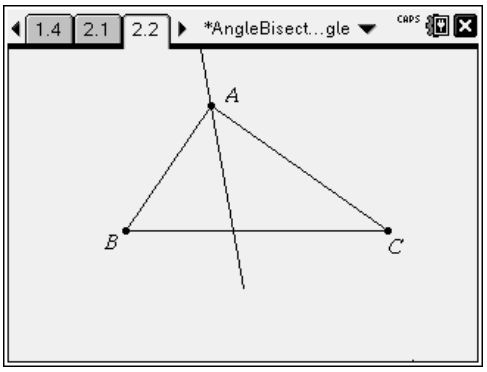
Record observations on the worksheet.



Problem 2 – One Angle Bisector in a Triangle

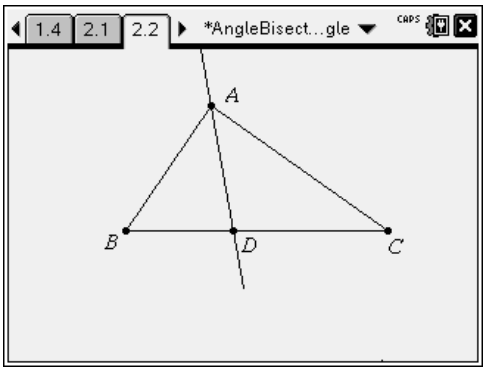
Students should advance to page 2.1 and read the directions.

On page 2.2, students should first construct the angle bisector of $\angle BAC$. Before continuing, have students drag a vertex of $\triangle ABC$ so that everyone gets different data.



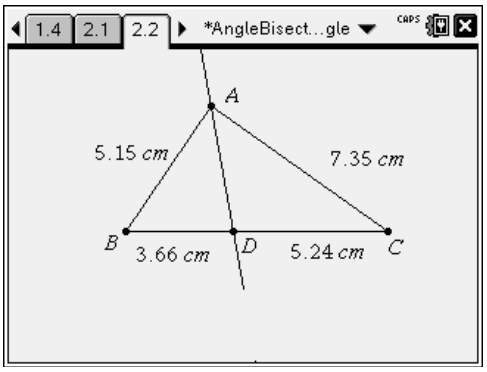
Have students plot the intersection point of the angle bisector and side $\overline{BC}$.

Label this point *D*.

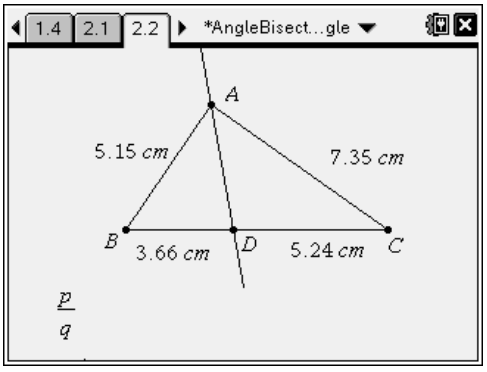


Direct students to measure the lengths of $\overline{AB}$, $\overline{AC}$, $\overline{BD}$, and $\overline{CD}$ using the **Length** tool.

Record these values on the worksheet. Drag a vertex of the triangle and record more data.

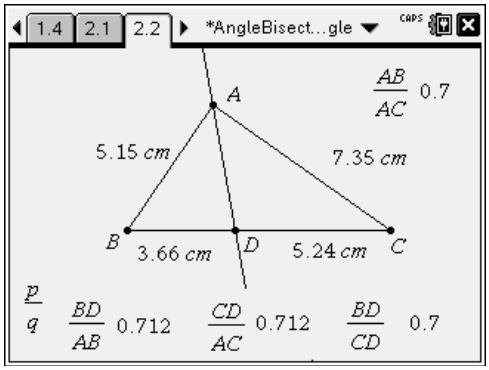


Have students use the **Text** tool to display the expression $\frac{p}{q}$ on the screen. They will use this expression to calculate ratios of the measurements.



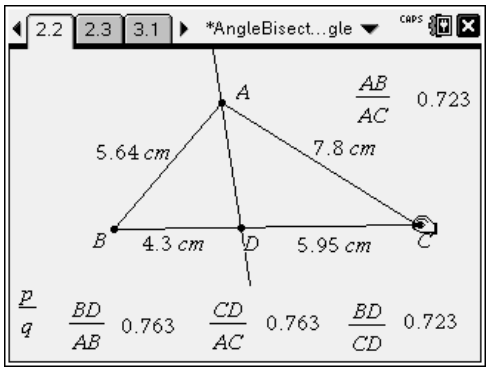
Divide pairs of measurements using the **Calculate** tool (**MENU > Actions > Calculate**). Examine the ratios that result. (The screenshot to the right provides labels for each calculated ratio, for your reference.)

Drag a vertex of △*ABC* and examine the ratios again. What do you notice?



Ask students to identify a pair of ratios that are equal. Then have them drag a vertex of the triangle to see if the equalities remain true.

Record observations on the worksheet.



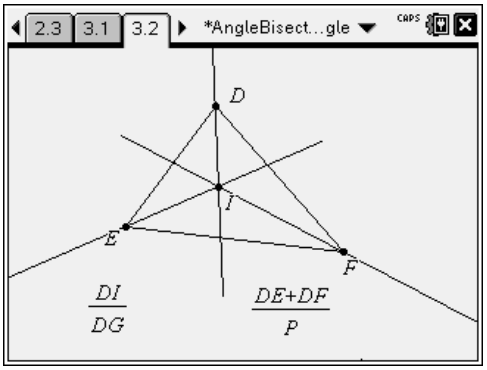
TI-Nspire Navigator Opportunity: *Quick Poll*

See Note 2 at the end of this lesson.

Extension – One Angle Bisector and the Incenter of a Triangle

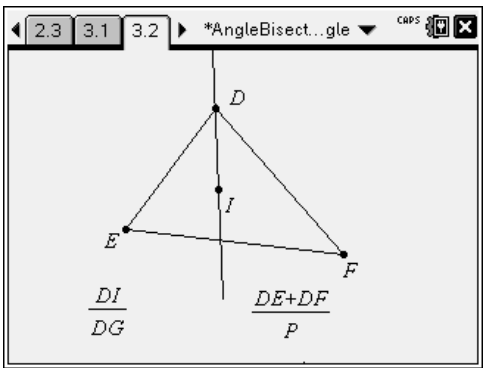
Students should advance to page 3.1 and read the directions.

On page 3.2, △*DEF* is constructed with all three angle bisectors created. The point at which the three angle bisectors intersect is called the *incenter* and is labeled *I*.

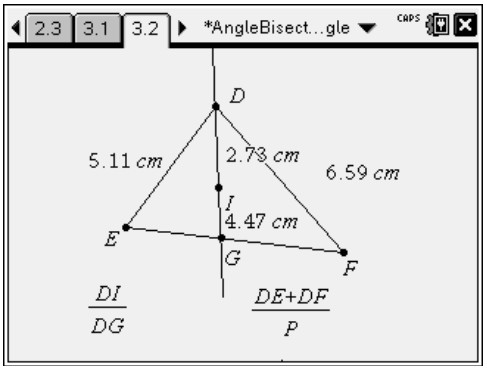


Students should hide angle bisectors $\overleftrightarrow{EI}$ and $\overleftrightarrow{FI}$ using the **Hide/Show** tool.

Use the **Intersection Point(s)** tool to find the point of intersection of the remaining angle bisector with the opposite side, and label the point *G*.

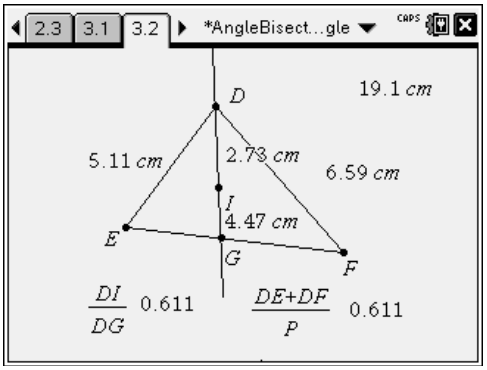


Direct students to measure the lengths of $\overline{DI}$ and $\overline{DG}$. Also measure sides $\overline{DE}$ and $\overline{DF}$ and the perimeter of the triangle.

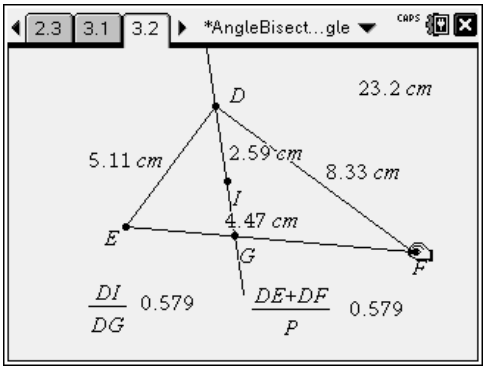


The expressions $\frac{DI}{DG}$ and $\frac{DE+DF}{P}$ are displayed on the screen.

Students should use the **Calculate** tool to evaluate these expressions and compare the resulting quotients.



Students should drag a vertex of $\triangle DEF$ and examine the quotients again. Ask: *What do you notice?*



TI-Nspire Navigator Opportunities

Note 1
Problem 1, *Screen Capture*
Throughout the activity, you may choose to use Screen Capture to verify students are able to construct the bisectors and use the **Length** function.

Note 2
Problem 2, *Quick Poll*
You may choose to use Quick Poll to assess student understanding throughout the activity. The worksheet questions can be used as a guide for possible questions to ask.