EXPLORATIONS

Activity 2

# Integer Subtraction—What's the Difference?

## Objective

- To explore subtraction of integers using a number line.
- To develop strategies for subtracting integers.

## Materials

- TI-73 Calculator

# Teacher Notes

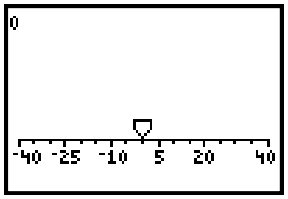
## Introduction

This activity builds on Activity 1 by having students use the Number Line to investigate integer subtraction.

## Procedure

1. Distribute copies of the Student Activity pages. Have students work in pairs.
2. Have students access the Number Line application by pressing [APPS] and pressing [▼] to select the NUMLINE application. Press [ENTER] twice to get to the NUM/FRAC LINE menu and access the NUMBER LINE option by pressing [ENTER].


   To display more of the Number Line, have students access and change the window settings. Press [WINDOW] and change the **Min** to **-40**, **Max** to **40**, **Scale** to **5**, and **Start** to **0**. Highlight **LabelOn** and press [ENTER]. Return to the number line window by pressing [GRAPH]. Press [CLEAR] **1** (or [CLEAR] [ENTER]) to clear any previous work.




3. Present a quick review of adding negative integers, explaining the concept of addition and subtraction being opposite or *inverse* operations.
4. Have students compare number lines for adding -18 and subtracting 18. Ask students why the number lines look the same even though subtraction and addition are opposites. The purpose of the question is to have students see the idea of "opposite" being conveyed in the sign of the number added. If one adds and subtracts the same number, then the number line views will look different. But when adding a negative number or subtracting the opposite of that negative number, the results will be the same (equivalent).
5. Students will begin to see a pattern, but should be cautioned to consider all cases.
6. The first variation to consider is when the largest number in absolute value is negative so the sum is no longer positive. If students merely subtract the largest number in absolute value from the smaller, their answer will have the wrong sign.

Students are asked to find a subtraction expression equivalent to **39 + ⁻45** and to verify their entry using the Number Line application. When students are asked to verify, no screen shot of the window will be provided. The calculator allows for student self-assessment. All too often, students get in the habit of asking "Is this right?" without first thinking for themselves. You can help minimize this habit by encouraging discussion and resolution in the student pairs. The activity encourages students to think ahead and then determine for themselves if their thinking is headed in an appropriate direction.

**7.** Here is a similar example. Depending upon the experiences of your students, you may not need to linger over several examples, but for those thinking about integer subtraction for the first time, the repetition of the thinking process is appropriate.

**8.** Here students should say they do subtract but the sign of the difference depends upon the sign of the number with the greater absolute value. You may want to use the language of absolute value or talk about the distance the number is from zero. So, for example, 39 + ⁻45 will have a negative answer since ⁻45 is farther away from zero than 39, not because ⁻45 is a bigger number than 39.

**9.** With this example, you can present the idea that subtracting a number is equivalent to adding the opposite of the number, an idea that arises from their viewing several examples and searching for a pattern.

| | | Equivalent Addition Problem | Prediction |
|---|---|---|---|
| a. | 23 - 5 | 23 + (⁻5) | |
| b. | 6 - 33 | 6 + (⁻33) | |
| c. | 18 - 28 | 18 + (⁻28) | |

**10.** The students begin to apply the rule. Notice that the calculator does not have a key similar to the (−) key to indicate positive numbers. Numbers without signs are taken as positive values.
Students are encouraged to stick with their mathematical reasoning even when it doesn't match their intuition. Recall the early mathematicians who thought negative numbers were "absurd"!

| | | Equivalent Addition Problem | Prediction |
|---|---|---|---|
| a. | 18 - (⁻13) | 18 + (+13) | |
| b. | 32 - (⁻5) | 32 + (+15) | |
| c. | 20 - (⁻19) | 20 + (+19) | |

**11.** A "different point of view" is presented.

**12.** Again, students return to adding a negative integer, something they should be comfortable with by now.

**13.** Finally, students consider subtracting a negative integer using the "opposite direction" thinking. Encourage students to share how they make sense of subtraction with integers.

**14.** These exercises are to help students develop and practice reasonable estimation skills in addition to developing some number line sense. For example, an estimate of the difference for **29 - 35** might be **⁻5**. The **⁻5** helps determine a **Min** for the window setting, but which number should the student focus on in determining a **Max**? 35 is larger than 29, but is that important for this subtraction? At what point on the number line will the vector for 35 start and stop? Which way will that vector be directed? These questions should help students understand the entire problem rather than just the final response.

| | | Min | Max |
|---|---|---|---|
| a. | 29 - 35 | ⁻7 | 30 |

| | | | |
|---|---|---|---|
| b. | 45 - 18 | 0, 25 | 46 |
| c. | 16 - (-26) | 0, 15 | 43 |
| d. | -23 - 17 | -41 | 0 |
| e. | -35 - (-29) | -36 | -5, 0 |

**15.** This is just another approach to helping students make sense of the "subtracting a negative integer" problem. Ask students if, based on the pattern they observe in the equations, the response they find for 18 - (-1) matches the response they would have found using previously developed rules.

**16.** Ask students to summarize their thoughts on subtracting integers.

Student Activity

Name

Date _____

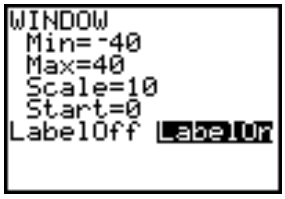
# Activity 2

## Integer Subtraction—What's the Difference?

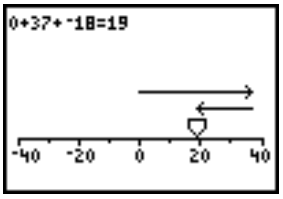
***Objective:*** *In this activity, you will use the number line to investigate subtraction of integers and to justify and support the rules you develop. You will be using the Number Line application on the TI-73 calculator.*

**1.** Access the Number Line application by pressing [APPS] and pressing [▼] to select the NUMLINE application. Press [ENTER] twice to get to the NUM/FRAC LINE menu and access the NUMBER LINE option by pressing [ENTER].

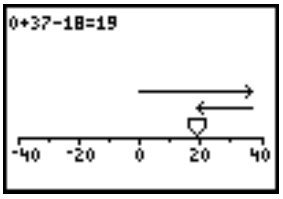

**2.** Press [WINDOW] and change the **Min** to **-40**, **Max** to **40**, **Scale** to **10**, and **Start** to **0**. Highlight **LabelOn** and press [ENTER]. Return to the Number Line window by pressing [GRAPH]. Press [CLEAR] 1(or [CLEAR] [ENTER]) to remove any previous work.




**3.** When adding a positive and negative integer, the vectors representing the two integers point in opposite directions. Vectors point in the *same* direction when you add two integers with the same sign.
Enter **37 + ⁻18** to verify this. Adding a negative 18 is represented by a vector pointing to the left with a length of 18.




**4.** Have another member of your group enter **37 - 18** on the calculator. Compare the number line showing the addition expression **37 + ⁻18** with the number line displaying the subtraction, **37 - 18**. Notice that subtracting a (positive) 18 is also represented by a vector pointing to the left with a length of 18.




Subtraction and addition are opposite operations, yet when you view the two number line windows showing the addition and subtraction problem, you see the *same* picture, not an opposite picture. Why is this so?

Press [CLEAR] **1** and use two calculators to compare number lines for these two problems:
**29 + ⁻16** and **29 - 16**.

Based on these two examples, it would appear that adding a negative integer to a positive integer looks just like subtracting a positive integer from a positive integer. Let's do more investigating to examine all possible variations of subtraction of integers.

**6.** Predict the sum of 39 and -45.

Verify your response using the Number Line application.
How does this example differ from **37 + -18** and **29 + -16**?

When working just with whole numbers (numbers greater than zero), you generally subtracted (took away) the smaller number from the larger number. If adding a positive and negative integer is just like

your "familiar" subtraction, you might think that **39 + -45** is the same as **45 - 39**. Is that the case? You can verify your thinking using the calculator.

How is **39 + -45** similar to **45 - 39**?

Based upon your earlier observation about subtraction and addition being opposite operations, what subtraction entry would give the same results as **39 + ⁻45**? Verify your entry on the calculator.

**7.** Try another example. Before using the calculator , predict whether the sum of 17 and ⁻28 will be positive or negative. Explain.

How are you mentally determining the distance of the sum from zero?

Press CLEAR **1** and enter **17 + -28**. What subtraction is equivalent to **17 + -28**? Enter your prediction to verify.

**8.** What "new" information did these last two examples, **39 + ⁻45** and **17 + ⁻28**, provide? Is the addition of a positive and negative integer just like subtraction? Explain.

**9.** If **39 - 45 = 39 + ⁻45** and **17 - 28 = 17 + ⁻28**, then **19 - 35 =** ?

These examples suggest that subtraction is the same as adding the *opposite* of a number.
Write the equivalent addition problems and your predictions for differences (or sums) for the following problems.

| | | Equivalent Addition Problem | Prediction |
|---|---|---|---|
| a. | 23 - 5 | | |
| b. | 6 - 33 | | |
| c. | 18 - 28 | | |

**10.** Apply this idea to this problem: **15 - (⁻6)**. The previous reasoning would suggest that subtracting -6 from 15 is the same as adding the opposite of -6 (or +6) to 15. Rewritten, it looks like

**15 - (⁻6) = 15 + (+6)**

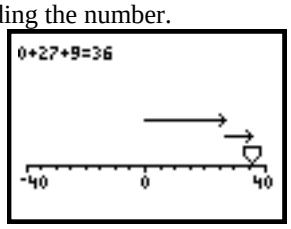
Verify this using the Number Line application. One person in your group should enter **15 - (-6)** while the other enters **15 + 6**.
Try the following examples. Write your predictions for the differences prior to entering them on the calculator.

| | | Equivalent Addition Problem | Prediction |
|---|---|---|---|
| a. | 18 - (⁻13) | | |
| b. | 32 - (⁻5) | | |
| c. | 20 - (⁻19) | | |

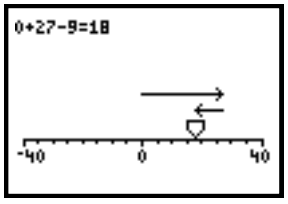
It is the subtraction of negative numbers that causes many students to look twice at their answer since the difference turns out to be greater than the numbers with which they started. For example, 18 - (⁻13). How can you start with 18 and subtract ⁻13 and end up with 31! Many only think of subtraction as "taking a number away", and so the difference of 31 just doesn't make sense. Yet, mathematics is about finding patterns, developing rules to describe those patterns, and then applying those rules to other examples. *You have done just that!* You have reasoned through many examples using a number line model, observed patterns between the direction of vectors and the operations, developed rules describing those patterns (for example: subtraction is adding the opposite), and then applied those rules in other cases. The rules appear to agree with your intuition except perhaps in the case of subtraction involving negative integers. It helps to rely more on reasoning and logical thinking than intuition. Using other models for integer subtraction will help support thinking and modify intuition.

**11.** Here's another view of subtraction with negative integers. Think about subtraction and addition as the direction of the vectors on the number line. When you subtract a number on the number line, the vector points (or directs a move) in the opposite direction than it would when adding the number.

For example, enter **27 + 9**. (Press CLEAR **1** if needed.) The 27 is a positive value. The vector shows a move, or points, in the positive direction and has a length of 27. To add a positive 9, the next vector continues to point in the *same* direction and "travels" for a distance of 9 units to arrive at the sum of 36.

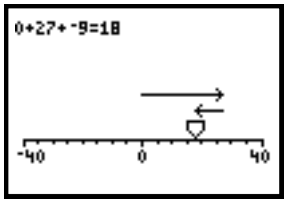



Clear the screen and enter **27 - 9**. Compare this to the previous expression. The first vector representing 27 is the same as before. The operation of subtraction, though, indicates a move in the *opposite* (or negative) direction for a distance of 9 units arriving at the difference of 18.




**12.** Press CLEAR **1**. Now consider **27 + (⁻9)**. Think through this first before entering the example.

The first vector will still have a length of 27 and point in the positive direction. The operation of addition does not change the direction of the vector. In this case, though, the vector changes direction because of the sign of the 9. We know a ⁻9 is a vector of length 9 that points in the negative (or left) direction. We need to add this vector to 27 so it will start at 27 and move left 9 units landing at 18. Verify using the Number Line application.




**13.** Finally, think about **27 - (⁻9)**.

The starting vector is the same as before. To subtract ⁻9, move in the *opposite* direction than you would if you were adding ⁻9. The addition of ⁻9 had the vector pointing towards the left, so you need to move in the opposite direction which would point the vector towards the right.

The number line window should look like which of the previous three windows? Verify your prediction.

**14.** Without using the calculator, find **Min** and **Max** values that would allow you to see all of the vectors for each of the following problems. You can be no more than 3 units away from the final difference and/or the endpoint of any vector.

| | | Min | Max |
|---|---|---|---|

| | | | |
|---|---|---|---|
| a. | 29 - 35 | | |
| b. | 45 - 18 | | |
| c. | 16 - (-26) | | |
| d. | -23 - 17 | | |
| e. | -35 - (-29) | | |

**15.** Study the patterns in the equations below and then respond to the following questions.

18 - 7 = 11

18 - 6 = 12

18 - 5 = 13

18 - 4 = 14

18 - 3 = 15

18 - 2 = 16

18 - 1 = 17

18 - 0 = 18

Describe any patterns you see as you work through the equations.

The integers being subtracted decrease by 1 each time. What should be the next equation in the list if you continue that pattern?

Does the solution match what you would have predicted using the rules from #14.

**16.** You've examined the subtraction of integers from several different views. What patterns have you seen that will help you in subtracting integers?