Coefficient of Friction (Skid Pad)

STEM Lesson for TI-Nspire™ Technology

Objective: Calculate the coefficient of friction from a sample set of data. Explain how maximum speed relates to the coefficient of friction. Explain how maximum speed relates to the radius of a curve.

About the Lesson: Because racing can happen on various surfaces, teams must be prepared to choose the best tires for the surface on which they are racing. Knowing the coefficient of friction between different tires or tire coatings and surfaces will allow teams to choose the best tire for a particular surface and race. A higher coefficient of friction will allow for a higher speed without sliding. This keeps you from plowing into walls as you turn corners. In a professional skid pad test, a test-driver drives at a constant speed around a circular track. The driver gradually increases speed until the car reaches its limits of control, the point just before it flies off the track (under-steer) or spins out (over-steer). You'll do the same thing on your 1:10 scale track.

Materials: RC Car (optional – data provided if you are not collecting your own)
Skid Pad track with a lane width of 5 feet
Student Worksheets
TI-Nspire handheld

Timing: 90 minutes for full exploration

Prerequisite skills: Students should have an understanding of graphing and basic knowledge of forces and kinematics. Students should also be proficient in manipulating equations algebraically.

Procedure:

1. Layout a skid pad track: Circles with an inner diameter of 20 ft, a diameter of 30 ft, a diameter of 40 ft and a diameter of 50 ft.
2. Do some practice laps to determine if the surface is so slippery that you need to add weight so the car drives without slipping. If necessary for traction, add weight. Make sure you measure and record the exact weight and use that for every trial.
3. Drive the car at its limits of control around the inner circular track at least ten times without stopping. You should be ALMOST flying off the track on each lap. You want to be at the limits of control.
4. Record the total elapsed time after every lap.

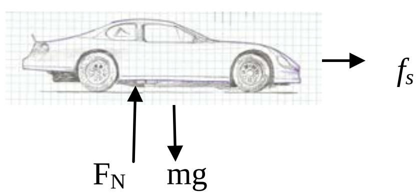
5. Repeat the trial without changing anything at least two more times.
6. Now move to the next circle and repeat steps 1-5. Continue this process for all circles.

Teacher Tip: *Make sure each person in the group has a job. If there is a dispute on who will drive, repeat the procedure for all drivers. Just make sure the students record the data for each driver separately and have them put the name of the driver at the top of the page. If there is a big enough area, feel free to setup more than one track.*

Analysis:

F_N - The Normal Force : The force exerted on the car by the ground
m - mass of the object
g - gravity
f_s - The static frictional force exerted on the tires
a_c - the direction of the centripetal acceleration which results in a centripetal force pointed toward the center of the circle.
μ_s - coefficient of static friction

Below is a simple force-diagram of a car rounding a flat curve (flat curve as opposed to a banked turn).



Newton's 2nd Law states that the net force on an object is equal to its mass times acceleration.

First, sum the forces in the x direction, in other words find the net force. Since there is acceleration, all the forces going in the direction of acceleration minus all the forces in the opposite direction equals the centripetal force. In this case there is only one force in the direction of acceleration and it is the force due to static friction.

$$ma_c = f_s$$

Second, sum the forces in the y direction. Since there is no acceleration, the net force is zero. Therefore, all the forces going up equal all the forces pointing down.

$$F_N = mg$$

The relationship between the friction and normal force is given by the equation below.

$$f_{s\,max} = \mu_s F_N$$

1. Substitute the first two equations into the third equation. Show your work below.

   Answer: $a_c = \mu_s g$

Teacher Tip: Make sure the students cancel the masses.

Centripetal acceleration is related to the speed and radius of the circle in which the car is traveling as stated below.

$$a_c = \frac{v^2}{r}$$

2. Substitute this definition in for centripetal acceleration. Show your work below.

   Answer: $\frac{v^2}{r} = \mu_s g$

Since velocity is distance divided by time, the velocity of a circle is the circumference divided by time.

$$v = \frac{2\pi r}{T}$$

3. Now substitute the definition for velocity into the latest equation. Show your work below.

   Answer: $\frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \mu_s g$

Navigator Tip: *Do a screen capture to make sure students have the right equation.*

4. What is your independent variable?

   Answer : Radius

5. What is your dependent variable?

   Answer : **Period or time**

6. Based on your answers above, arrange this equation in slope-intercept form (hint: one of your variables will be squared). Show your work below.

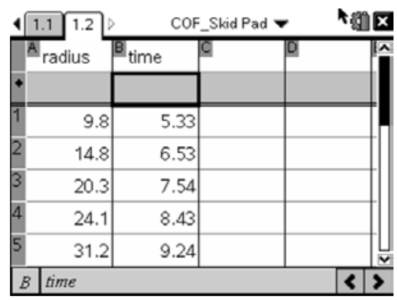
   Answer : $T^2 = \frac{2\pi}{\mu_s g} r$

In this instance T^2 is plotted on the *y*-axis and *r* is plotted on the *x*-axis. This implies that $2\pi/\mu_s g$ is equivalent to your slope. Keep in mind we are trying to determine the coefficient of static friction between the tires and the road based on the data collected. If we can find the slope through graphing, then we can set it equal to the slope in this equation and solve for the coefficient of static friction.

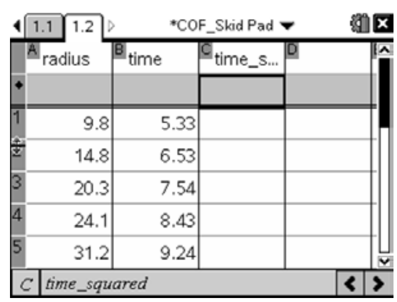
Teacher Tip: If you prefer, you can have the students do the next part of the examples using the following sample set of data:

Radius (ft)	Time (s)
9.8	5.33
14.8	6.53
20.3	7.54
24.1	8.43
31.2	9.24

7. On your handheld, go to My Documents and open the file named *COF_Skid Pad.tns*.

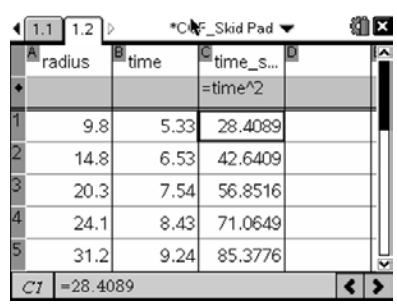


8. Use (ctrl) ▶ to move to page **1.2**. Enter your radius and period data into the appropriate columns. Make sure to start in the white box beside the number 1.

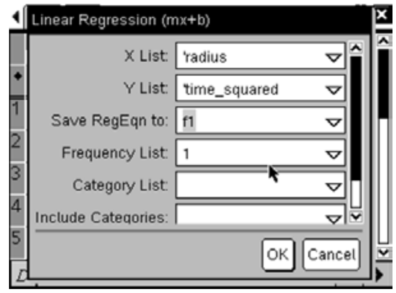


9. Since our equation has time squared we need to square the period data. Move to the top of column C and type **time_squared**. Press (enter).

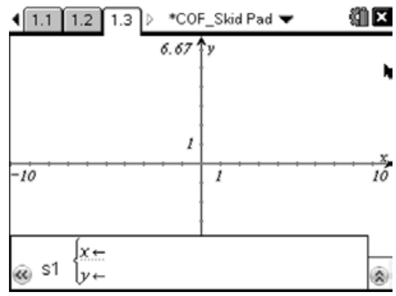
10. In the shaded cell type **time** then press (x^2). This should automatically fill the column with each time squared.



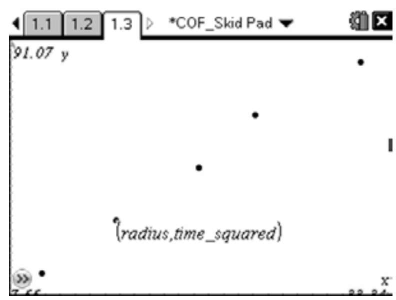
11. Do a linear regression analysis on your data. Press (menu) then choose **Statistics > Stat Calculations > Linear Regression (mx+b)**. The regression template will pop up. Remember to press (tab) to switch boxes. Click the arrow at the right of each box to access the drop down menus. Choose *radius* from the drop down menu for the X List and *time_squared* for the Y list. Tab through the other options until you get to 1st Result Column. Click beside the letter and change it to 'e' if it isn't already. Click OK.



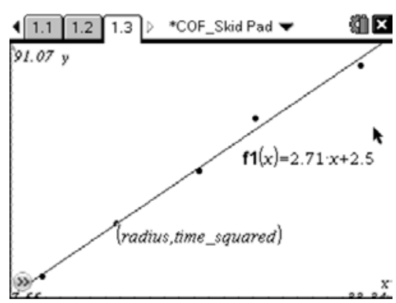
12. Move to page **1.3**. Press (menu) then choose **Graph Type > Scatter Plot**. Notice the function bar at the bottom of the screen changes from f1 to s1.



13. Press (var) and choose *radius*. Press ▼ then (var) and choose *time_squared*. Press (enter). Now adjust your window to view the data by pressing (menu) then choosing **Window/Zoom > Zoom – Data**.



14. Now graph the regression line. Press (menu) then choose **Graph Type > Function**. Press ▲ to see your function in f1 then press (enter). You should see your line on top of your data points.



Navigator Tip: *Do a screen capture to show student screens and the different equations and graphs.*

15. What is the equation of your function?

Answer: Answers may vary unless sample data is used. All equations should be in slope-intercept form.

16. What number in your equation represents the slope of the data?

Answer: Answers will vary. It will be the student's coefficient of *x* from their equation.

This slope is equal to the slope represented in the original equation:

$$m = \frac{4\pi^2}{\mu_s g}$$

17. Use your slope for m, 32 ft/s² for g, and solve for μ_s. Show all of your work.

Answer: Answers will vary. Calculations for the sample data is shown below.

$$2.71 = \frac{4\pi^2}{\mu_s g}$$

$$\mu_s = \frac{4\pi^2}{2.71g}$$

$$\mu_s = .455$$

Navigator Tip: Use screen capture to show the students' coefficients of friction. If the students are all on the same surface using the same car, they should have approximately the same coefficient.

When the coefficient of friction is 0, there is no friction. The larger the coefficient of friction, the more force you have to apply to get the object to move. When the coefficient of friction is equal to one, that implies that the horizontal force applied must equal to the weight of the object.

18. Based on this information, does your answer in question 17 make sense?

Answer: The students should evaluate their answer to find any errors until it does make sense.

19. What does your value of the coefficient of friction mean in terms of force?

    Answer: You would need a horizontal force equaling [student's coefficient] of the weight of the car to make it slide.

20. With a constant coefficient of friction, what happens to the maximum speed of the car as the radius increases? Explain your reasoning.

    Answer: It increases. If the coefficient of friction is the same, then the centripetal force needed to cause the car to slip remains the same. Therefore our centripetal acceleration should be the same. Since centripetal acceleration is velocity^2/radius, as the radius increases, so does the velocity.

21. On page **1.3**, you plotted T^2 vs. *radius*. Speed is determined from time and distance. Sketch what a plot of *speed* vs. *radius* would look like.

    Answer: The coefficient of friction tells you how "sticky" a surface is. For example walking on ice is difficult because you have no traction. That "traction" is friction.

22. When it rains outside, what happens to the coefficient of friction between tires and the road?

    Answer: It decreases.

23. What happens to how fast people drive:

    Answer: They slow down.

24. As you increase the coefficient of friction, what happens to the magnitude of your maximum speed? Explain your reasoning.

    Answer: It increases. The force necessary to cause you to slide is equal to your centripetal acceleration. Centripetal acceleration is based on velocity^2/radius. If you must decrease your centripetal force (to keep you from sliding), you can either increase your radius or decrease your velocity. If the radius is constant, then you must decrease your velocity to decrease your centripetal force. The maximum speed will therefore increase as the force needed to cause you to skid increases.

<u>Navigator Tip:</u> *Use quick pole to make sure all students agree and to start discussions.*