EXPLORATIONS

Activity 9

Used Cars

You will most likely need to buy a car in the future. Due to the high prices of new cars and the large number of cars coming back from new car leases, used cars have become a very popular alternative for cost-conscious consumers and people looking for their first car. New cars *depreciate*, or lose value, as soon as they are purchased, but used cars have already had that depreciation built into the price. New cars have sticker prices posted on their windows and there are many consumer magazines and buying guides for new cars, but how are used car prices determined? Various companies publish guides to used-car values for dealers and consumers to use.

Introduction

In this activity, you will use the TI InterActive!™ browser to gather data from used car web sites and determine appropriate mathematical models for the price of used cars with respect to the age of the car. In the activity, you will discover what happens to the value of a car over time.

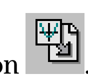
Equipment Required

- TI InterActive! software
- A working Internet connection

Collecting the Data

1. Start TI InterActive! The software opens to a new, blank document.
2. Title your new document *Used Cars* and add your name and the date. Click the Save button to save and name your document.
3. Click the Spreadsheet button to open the spreadsheet.
4. Click the Web Browser button to open the TI InterActive! browser. Click the Data Sites button. Under the **Activity Book Links** category, click on *TI InterActive! Data Collection and Analysis*. Choose **Activity 9: Used Cars**.

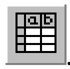
5. Once the page has been loaded in the browser, scroll down to the **And/Or** section and check the box for the Ford Mustang. Scroll down to the bottom of the page and click on the **CONTINUE** button. You will use the Mustang convertible for your first investigation, so check the box for the convertible. (Just the plain convertible, nothing fancy.) Scroll down and click on the **GO SEARCH** button. You should find lots of data for the values of Mustang convertibles.
6. When the page of Mustang data appears, scroll down to the data and drag the cursor over it to select. Click 

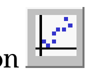
to import the data into your TI InterActive!™ spreadsheet.
7. Before you proceed, clean up the data in the spreadsheet.
   - **a.** Click on Edit, Replace.
   - **b.** In the **Find What** text box, type **Convertible** and in the **Replace With** box, type a space.
   - **c.** Click Replace All. This removes the word convertible from all the cells in your spreadsheet.
   - **d.** With the Replace dialog box still open, type **$** in the **Find What** text box. (Leave the space in the **Replace With** box.)
   - **e.** Click Replace All.
   - **f.** Click Close.
   - **g.** Click on the **B** at the top of the second column. Press and hold the shift key, then click on columns **C** and **D**.
   - **h.** Click on Edit, Delete. (In the Delete dialog box, *Entire Column* will be selected.) Click **OK**.
   - **i.** Columns **F** and **G** become the new columns **C** and **D**. Using the directions in steps **g** and **h**, delete these new columns **C** and **D**.
8. Click on the Save to Document button .

Investigating the Data

1. Look through the data. You may need to drag the bottom handle of the spreadsheet frame to display all of the data. In your TI InterActive! document, record any observations you may have about the values of Mustang convertibles with respect to the year they were produced.
2. In looking at the data, is it clear that the value of the Mustang convertible is a function of its age? In other words, its current value is dependent upon how old the car is. Many people consider old Mustangs to be classic cars. What do you think it means to be a "classic car?" Look at the data. At what time does the car change from being a classic car to being just another used car? Explain your answer in your TI InterActive! document.

3. Your first investigation will focus on the years 1983 through 1998 when the Mustang convertible is losing value, or depreciating in value. The car's value depends on its condition, or in other words, it depends on how well the previous owners took care of it. For this exercise, assume that the car is in excellent condition.

a. Set up a new spreadsheet in TI InterActive!™ by clicking on the Spreadsheet button . The age of a 1983 Mustang is determined by subtracting 1983 from the current year. Column **A** will represent the number of years since the current year, and column **B** will represent the value of the Mustang. The car's value depends on its condition. Assume the car is in excellent condition. Using the year 2000 as an example, in cell **A2**, type the age in years, **17** (2000 - 1983 = 17). Press the down arrow key to move to the next cell. Continue until you have entered all of the years since 2000 into column **A**. In column **B**, put the value according to Hemmings of the Mustang in excellent condition for each of the years.

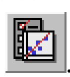
b. Highlight the data in columns **A** and **B**, and click the Scatter Plot button . A graph of the data, a StatPlot, appears. The viewing boundaries are adjusted automatically to show all the plotted data.

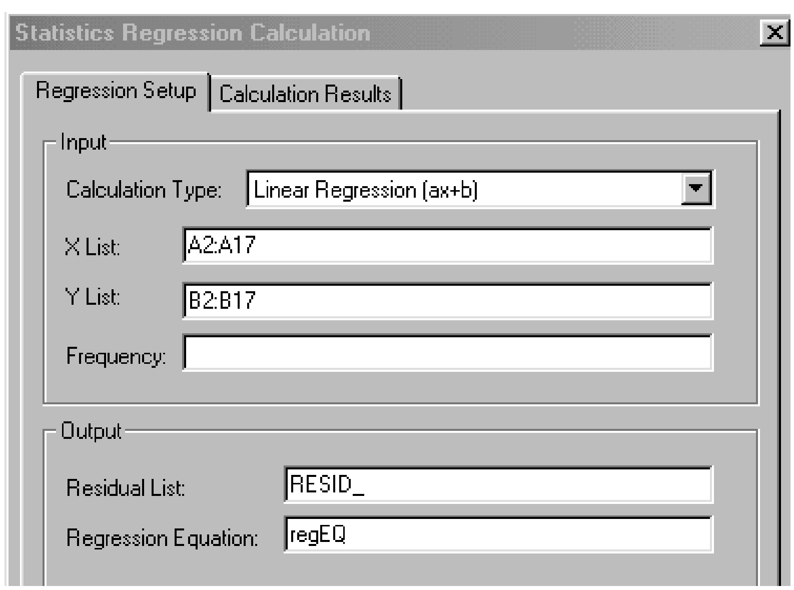
c. Click Save to Document to save the open graph in your TI InterActive! document.

d. Close the Data Editor window.

4. Look at the Scatter plot. Is the graph of the years since the current year versus value a linear function of the form $y = ax + b$?

5. To justify your answer, you need to try to fit a line to the data. TI InterActive! lets you determine values of a and b by calculating a regression line of best fit. Use a linear regression model on the data. To do the regression, double-click on the spreadsheet to open it again, then:

a. Highlight the same data in the spreadsheet, and on the toolbar, click Statistical Regressions . Note that Linear Regression ($ax + b$) is the default selection.



b. Click on the Calculate button at the bottom of the window. Click on Save Results. Close the Data Editor.

c. Double-click on the graph to open the Graph window. In the Functions dialog box, click on the f(x) tab. Type **regEQ(x)** in the first text box and click at the left to place a check mark in the checkbox. This adds the line to your graph.

> ***Note***: *The graph must be located below the Linear Regression results in your TI InterActive! document.*

d. Click on the Save to Document  button.

6. Does this line fit the data well? Explain your answer.

7. Click on the Save button  to save your TI InterActive!™ document.

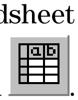
Continuing the Investigation

You can use the spreadsheet functionality of TI InterActive! to further the investigation. Linear functions increase or decrease at a constant rate. (See *Activity 2: Tight Rope*.) Look at the following spreadsheet of $y = 3x - 2$ on the domain $-5 < x < 5$. The spreadsheet on the right has all the rules to help you create your own linear function spreadsheet.

	A	B	C
1	x	y	differences
2	-5	-17	
3	-4	-14	3
4	-3	-11	3
5	-2	-8	3
6	-1	-5	3
7	0	-2	3
8	1	1	3
9	2	4	3
10	3	7	3
11	4	10	3
12	5	13	3

	A	B	C
1	x	y	differences
2	-5	=3*A2-2	
3	=A2+1	=3*A3-2	=B3-B2
4	=A3+1	=3*A4-2	=B4-B3
5	=A4+1	=3*A5-2	=B5-B4
6	=A5+1	=3*A6-2	=B6-B5
7	=A6+1	=3*A7-2	=B7-B6
8	=A7+1	=3*A8-2	=B8-B7
9	=A8+1	=3*A9-2	=B9-B8
10	=A9+1	=3*A10-2	=B10-B9
11	=A10+1	=3*A11-2	=B11-B10
12	=A11+1	=3*A12-2	=B12-B11

As the *x*-values increase by 1, what is the constant rate of increase in the *y*-values?

1. Set up a new spreadsheet in TI InterActive! by clicking on the Spreadsheet button . Create a linear function spreadsheet similar to the one above, but choose different values for the slope and *y*-intercept in the linear function model $y = ax + b$. Describe your observations. Share values of a and b with other members of your class. Make general conjectures about the differences of linear functions.

2. Close this spreadsheet and double-click on the Mustang data spreadsheet in the TI InterActive!™ document. Examine the differences in your Mustang values. Using the method shown above, create a column with the differences in values. Do they increase at something close to a constant rate? Explain.

3. If not, look at the ratio of successive values. For example: $\frac{B2}{B3}, \frac{B3}{B4}, \frac{B4}{B5}$, and so forth. Is the ratio relatively constant? When the *y* values of a function increase or decrease at a constant *ratio*, the function is *exponential*. Put the ratios in column C and use the average function on the spreadsheet, **=AVERAGE(C3:C17)** to calculate the average of these ratios. It will not be perfect, but is the ratio of successive values relatively constant?

Record this average ratio in your TI InterActive! document, and also record any conjectures that you are willing to make about the Mustang data. This ratio can be thought of as the *depreciation ratio* for the Mustang. What do you think that means in the analysis of the data?

Analysis and Questions

When the relationship between the variables *x* and *y* is exponential, it can be expressed in the form $f(x) = ab^x$, where *b* is the constant ratio between the *y* values.

In order to find an $f(x) = ab^x$ model for the Mustang data, you will only need to find the value of *a*, the initial value of the car, since you know *b* = the average of the ratios. At this point your model is $f(x) = a^* (R^x)$ and you can use the guess-and-check method. When *x* is zero, the current year, (R°) = 1 so *a* represents the value of the car in the current year.

1. Use TI InterActive! to find the value of *a*.

a. Double-click on the graph of the Mustang data.

b. Click the f(x) tab in the upper left corner of the Functions dialog box.

c. Start with an initial guess of *a* = $18,000. Type **f(x):= 18000*R^x**, where R is your constant ratio, in the uppermost text box of the f(x) tab.

d. Press Enter to superimpose the graph on the plotted data.

It is unlikely that your first guess for the value of *a* produced a model that matched the data closely. Click in the text box of the f(x) tab again and edit the exponential function, replacing the old value, *a* = 18,000, with your new guess for *a*. Press Enter to update the graph. Repeat the guess-and-check procedure until you find an *a*-value that models the data well, and record it in your TI InterActive! document.

2. With TI InterActive!, you can check the values of *a* and *b* you just found by calculating the exponential function, $y = ab^x$, of best fit.

a. Click the Graph window close box.

b. Double-click on the spreadsheet data.

c. Highlight the data and click Statistical Regressions.

d. Click the down arrow next to Calculation Type, scroll down the list, and click on Exponential Regression.

e. Click Calculate.

f. Click the Save Results button. TI InterActive!™ stores the results in variables, closes the Statistical Regressions tool, and displays the selected results in your document.

3. How does the value of *a* in the exponential regression equation compare with the *a*-value you found by guess-and-check? What does the value of *a* from the regression equation represent in this mathematical model?

4. What should the *b*-value from the regression equation represent in this mathematical model?

5. Double-click on the graph in the TI InterActive! document to refresh the Graph window. In the second text box of the f(x) tab, type **f(x):= regEQ** and press Enter. TI InterActive! graphs the equation that was created as the Statistical Regressions result. Which equation seems to fit the data better? Why?

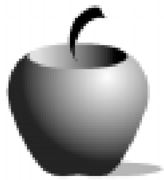
6. Save and print your TI InterActive! document.

Extensions

- Ford Mustangs built between 1965 and 1973 are considered classic cars. Instead of depreciating in value, the car's value is actually increasing or *appreciating* in value as the number of years since 2000 increases. Place the values for years since 2000 and value of the Mustang into your spreadsheet and use the differences and ratio technique to determine if this growth is linear or exponential. The years 1972 and 1973 seem to be *outliers* in the data set. You may want to disregard these two values. Use regression to determine an appropriate model and write up the investigation in a new document in TI InterActive! Carefully explain your model and justify your conjecture about linear growth versus exponential growth. Use appropriate graphs and/or tables of values to support your argument.
- Go back to the web site and repeat the investigation. This time use a Chevrolet Corvette instead of a Mustang. Corvettes built between 1956 and 1975 are considered classic cars, but thereafter are just used cars. Analyze the data from 1986 to 1998. Is the rate of depreciation and appreciation the same as for the Mustang? Why is it or is it not the same?
- Check out other Web sites to find values of used cars. Kelley's Blue Book at **http://www.kbb.com** is a good place to start. Do you find similar values for the Ford Mustangs that the Hemmings Motor News™ Web site gave? What factors determine the value of a used car?

Teacher Notes

Activity 9: Used Cars



Math Concepts

- Internet Data Collection
- Spreadsheets
- Ratios
- Linear Function
- Exponential Function

Activity Notes

- Most cars depreciate over time. This activity introduces an exponential decay model and compares the decay (growth) to the constant growth of linear functions.
- Using differences of the *y* values, students should determine that linear functions increase by the constant value of *m* as *x* increases by 1.
- Exponential models increase or decrease at a constant ratio (the growth or decay factor) as *x* increases by 1.
- The values of used cars are frequently changing. Check the Hemmings Motor News™ web site or use other appropriate web sites prior to starting this activity.
- The finite differences spreadsheet can be done by the teacher as a demo, or have all the student groups investigate and compare their results.

Investigating the Data - Key

1. Early Mustangs are appreciating in value, but Mustangs from 1983 forward are losing value.
2. Each year has only one value associated with it, so it is a function. The car changes from being a classic car to being just another used car after 1973, when it begins to depreciate in value.
3. No, it is not linear.
4. $a \approx -950$ $b \approx 16{,}500$ No, it does not fit the data well. For example, too many points are below the line, or look at the residuals.
5. The constant rate of increase in the *y*-values is 3.
6. The differences equal *m*, the slope.
7. No, because it is not a linear function.
8. Yes, the ratio is relatively constant. R ≈ .91

Analysis and Questions - Key

1. $a \approx 19{,}000$ $f(x) = 19{,}000 * .907^x$
2. $a = 19{,}135$ $b = .91^x$
3. *a* represents the resale value of the current year Mustang Convertible.
4. *b* represents the decay ratio.
5. The regression equation should be a better fit, but allow students to justify their answer.

