Name _____

Date _____

EXPLORATIONS

A C T I V I T Y   4

Using Linear Programming in Real-Life Problems

Is it possible to use mathematics to solve real-life problems? Linear programming techniques allow mathematicians to do just that!

A linear programming problem consists of a function to be maximized or minimized together with a system of linear inequalities. The function to be optimized is called the *objective function*; it might describe the profits a company earns or the costs associated with a certain manufacturing process. The linear inequalities are called *constraints*; they represent bounds on such things as labor and material requirements.

You can graphically represent the solution of the inequality constraint system as a region in the shape of a polygon. This is called the *feasible region* because it shows all the possible points for which the objective function makes sense. If the objective function is evaluated at all points in the feasible region, *the optimal value of the objective function, if there is one, will occur at a vertex of the feasible region*. This idea is the key to solving linear programming problems.

Instructions

Mr. Edwards is going to bake some cookies for his algebra class. He will make two different kinds, oatmeal-raisin and chocolate chip. He figures it will cost $1.70 to bake a dozen oatmeal-raisin cookies and $1.20 per dozen for chocolate chip cookies. The number of chocolate chip cookies must be at least twice the number of oatmeal-raisin cookies. He will bake at least six dozen cookies total, with no more than five dozen chocolate chip.

What is the minimum cost to bake the cookies? How many dozens of each kind will he be able to bake for that cost?

Using Linear Programming in Real-Life Problems *(Continued)*

1. Let x represent the number of dozens of oatmeal-raisin cookies and y represent the number of dozens of chocolate chip cookies. Complete the objective function below, representing the total cost, C, of baking the cookies:

    $C = 1.7\,x + \,?$

2. The fact that the number of chocolate chip cookies must be at least twice the number of oatmeal-raisin cookies can be represented mathematically as $y \geq 2x$. Use the other information given in the problem statement to complete the system of inequalities. Record this information in the space below.

    (a) $y \geq 2x$

    (b)

    (c)

3. For each inequality listed above, solve for y. Record your answers below.

    (a) $y \geq 2x$

    (b)

    (c)

4. Press ON to turn on the Voyage™ 200 PLT. To reset to the default settings, press 2nd [MEM] F1, select 1:RAM, select 2:Default, and then press ENTER ENTER.

    Press ♦ [Y=] to display the Y= Editor. Press F1 and select 8:Clear Functions. Press ENTER.

    Enter each of the three inequalities you identified in step 3 into the Y= Editor as linear equations as follows:

    Press ⊙ or ⊙ to move the cursor to an undefined function.

    Press **2** ⊠ **X** ENTER to enter the first function. Type the expressions to define the other two functions.

    When you have finished, the function list should show three functions, which are automatically selected for graphing.

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5. The graph of a linear inequality divides the coordinate plane into two regions. The solution to the inequality is represented by shading the region above or below the boundary line, depending on whether the relation is ≥ or ≤ when the inequality is solved for y.

   Move the cursor to highlight the linear equation $y = 2x$. To represent the corresponding inequality, press F6 and select 7:Above to shade the region above the boundary line because the relation is ≥ in the inequality $y \geq 2x$.

   Repeat this procedure, selecting appropriate graphing styles for the two remaining boundary lines in the Y= Editor.

6. Press F2 and select 6:ZoomStd to see a graphical representation of the inequality system.

   Notice that the *feasible region*—the region satisfying the given constraints where all the shaded sections overlap—is triangular. For a better view of this region, press F2 and select 2:ZoomIn. Move the cursor to the approximate center of the triangular region, and press ENTER. Alternatively, you could press ◆ [WINDOW] and adjust the x and y boundaries manually.

   Sketch and shade the feasible region in the area to the right.
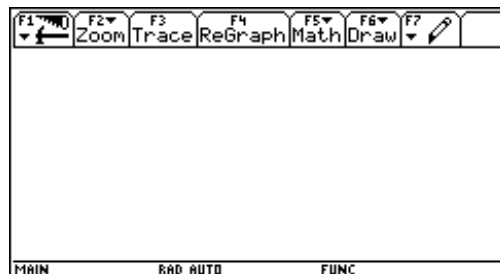
Using Linear Programming in Real-Life Problems *(Continued)*

7. Now you can identify the vertices of the feasible region. These vertices occur at boundary line intersection points.

    From the graph, press F5 and select 5:Intersection. Press ⊙ or ⊙ to select one of two lines that intersect at a vertex of the feasible region, and then press ENTER. Select the second line and press ENTER.

    Set the lower bound for x. Press ⊙ or ⊙ to move the cursor to the lower bound and press ENTER; or, just type its x-value. The lower bound is marked with a ▶ at the top of the screen.

    Set the upper bound, and press ENTER.

    The coordinates of the intersection point are displayed at the bottom of the screen. Record these values in the table below.

    Repeat this procedure for the two remaining vertices.

x-coordinate	*y*-coordinate

8. Finally, you can optimize the objective function you identified in step 1, $C = 1.7x + 1.2y$. Remember, if there is a solution, it will occur at a vertex of the feasible region. So, you need to evaluate the objective function at each vertex. To do this, use the Data/Matrix Editor.

    Press APPS and select Data/Matrix Editor. Then select 3:New. Press ⊙ ⊙ to move to Variable: and type **LINPROG1** as the name (for linear programming, problem 1).

    Press ENTER ENTER to display an empty data table. Type the vertex x-coordinates as recorded in the table from step 7 into column c1 and the corresponding y-coordinates into column c2.

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8. (continued)

Move the cursor to any cell in column c3 and press ☐F4. Enter the objective function at the prompt in terms of the x-values, c1, and the y-values, c2, as follows.

Press **1.7** ☒ **C1** ⊞ **1.2** ☒ **C2** ☐ENTER☐.

Record the vertex coordinates and the objective function values in the table below as they appear in the Data/Matrix Editor.

x-coordinates: c1	*y*-coordinates: c2	objective function: c3

9. Look at the objective function values in column c3 of the data table. Identify the minimum cost required to bake the cookies:

minimum cost = $ _____

To minimize the cost, the teacher should bake

_____dozen oatmeal-raisin cookies

and

_____dozen chocolate chip cookies.

Questions

1. In the problem you just solved, what was the maximum feasible number of oatmeal-raisin cookies? What was the minimum feasible number of chocolate chip cookies?

    maximum feasible number of oatmeal-raisin cookies = _____

    minimum feasible number of chocolate chip cookies = _____

2. Would it have been feasible not to bake any oatmeal-raisin cookies? Explain why or why not.

    _____

    _____

3. Describe how it is possible to tell what the feasible region might look like based on the number of inequality constraints given. What would be the likely shape of the feasible region if the objective function were to be optimized subject to five constraints?

    _____

    _____

    _____

4. Is it possible to have a linear programming problem for which the objective function has no maximum value? If so, describe what the graph of the feasible region would look like.

    _____

    _____

    _____

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Extra Practice

Follow the procedure outlined in this activity to solve each of the following linear programming problems. Complete your work on a separate sheet of paper. Whenever possible, represent your answers in table form. Include a sketch of all relevant graphs as they appear on your Voyage™ 200 PLT screen.

1. For (a) and (b) below, sketch the graph of the feasible region for the given constraint inequalities. Identify and label the vertices. Then find the minimum and maximum values of the given objective function.

   (a) Objective function:
   $$C = 2x + y$$

   Constraints:
   $$\begin{cases} y \geq 3x \\ x + y \geq 5 \\ y \leq 8 \end{cases}$$

   (b) Objective function:
   $$C = 5x - 3y$$

   Constraints:
   $$\begin{cases} y \geq 0 \\ y \geq 3 \\ 2x + y \leq 15 \\ x - 2y \geq -3 \end{cases}$$

2. A clothing company makes short-sleeve and long-sleeve shirts. Each long-sleeve shirt nets the company $4 in profit, and each short-sleeve shirt nets the company $3. Each shirt must be processed by two sewing machines—Machine I and Machine II. Long-sleeve shirts require 3 minutes on Machine I and 1 minute on Machine II. Short-sleeve shirts require 2 minutes on each machine. Every day, Machine I is available for 10 hours and Machine II is available for 7 hours.

   How many shirts of each type should the company produce each day so as to maximize profits?

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3. A shoe manufacturer produces high tops and running sneakers. The time required to cut and stitch each pair of shoes is given in the table below. The cutting machine is available for a maximum of 275 minutes each day, and the stitching machine is available for a maximum of 300 minutes each day.

    If the company nets a profit of $18 per high tops and $13 per running shoes, what is the maximum profit that can be made with the available resources?

	high tops	running sneakers
cutting time per pair (minutes)	3	1.5
stitching time per pair (minutes)	2.5	2