TEXAS INSTRUMENTS

Tracks of a Killer: Using footprints to estimate height ● ● ●

Analyze the relationships between shoe size, stride length, and height, and then use that information to identify the likely killer.

The body of famous pop music producer Jonathan Wallace was found in his bathtub. It is our hypothesis that an intruder surprised the victim and drowned him. The only clue at the crime scene was a set of muddy footprints leading from a nearby window to the bathroom and back again. The footprints were smeared, so their exact size could not be determined. The soles of the shoes had no pattern. It will be difficult to match the footprints to any particular pair of shoes.

Three suspects were questioned immediately following the murder:
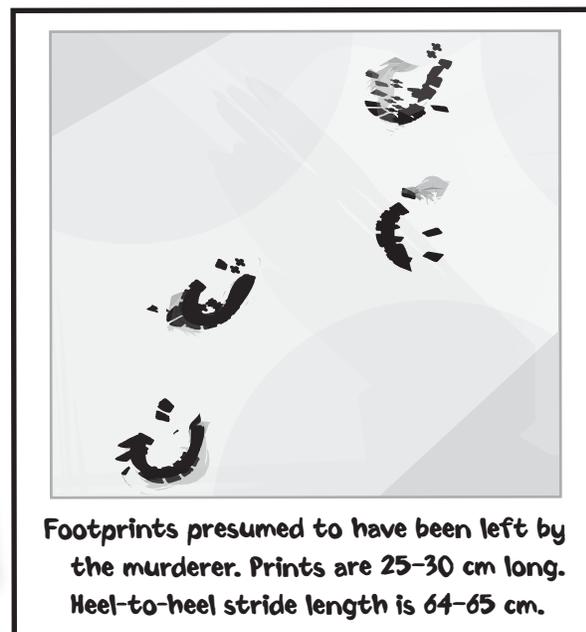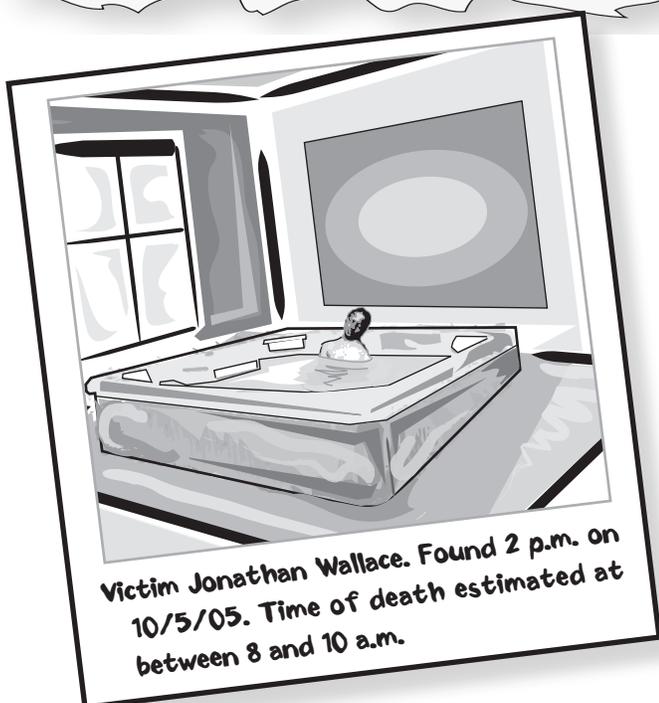**Penelope Paige, pop star: 5'4"/green eyes/blond hair**
Possible motive: She is suing Wallace over the failure of her last album.
Rex Chapman, rock guitarist: 5'8"/brown eyes/brown hair
Possible motive: He accused Wallace of stealing profits from his hit single "Walk It Off."
Dirty Dawg, rapper: 6'0"/brown eyes/black hair
Possible motive: He wants out of a record contract with Wallace.

Victim Jonathan Wallace. Found 2 p.m. on 10/5/05. Time of death estimated at between 8 and 10 a.m.

Footprints presumed to have been left by the murderer. Prints are 25–30 cm long. Heel-to-heel stride length is 64–65 cm.

Forensics Objectives

- determine if there is a relationship between the length of a person's stride and his or her height
- determine if there is a relationship between the size of a person's shoes and his or her height

Science and Mathematics Objectives

- efficiently gather data to test for correlations between height, shoe size, and stride length
- use a linear regression model of the data to predict height based on stride length

Materials

- TI-83/TI-84 Plus™ Family
- for station 1: metric tape measure or meterstick
- for station 2: metric ruler
- for station 3: metric tape measure or meterstick
       straight walkway at least 10 m long
       chalk or tape

Procedure

Part I: Collecting the Data ● ● ●

1. Set up three stations with two people at each, one person to collect data and one person to record data.
   a) At station 1, use the tape measure or meterstick to measure each person's height without shoes to the nearest half centimeter, and record it in the Evidence Record next to the person's name.
   b) At station 2, have each person remove his or her right shoe. Turn the shoe over and use a ruler to measure the distance from the tip of the toe to the end of the heel. Record the length of the person's shoe in the Evidence Record.
   c) At station 3, mark a starting line with chalk or tape. Have each person stand with the backs of his or her heels at the edge of the starting line. Starting at this point, each person should take 10 normal-length walking steps in a straight line (see the diagram below). After the 10th step, the person should stop and bring his or her heels together. Mark the final position of the back of the person's heels, and measure the distance in centimeters between that mark and the edge of the starting line. Calculate the average stride length by dividing this distance by 10. Record each person's average stride length in centimeters in the Evidence Record .
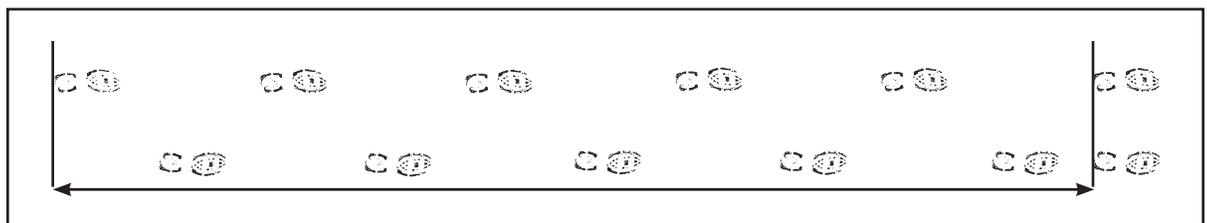
2. When all of the data are collected, compile a complete record for all individuals on a master Evidence Record.

Part II: Analyzing the Data ○●●

3. Enter your data in the calculator.
    a) Turn the calculator on.
    b) Press ⊂STAT⊃ ⊂ENTER⊃. This will bring up three lists: L1, L2, and L3. If the lists contain old data, you can clear them by highlighting the list number (use the arrow keys to move around) and then pressing ⊂CLEAR⊃ ⊂ENTER⊃.
    c) Enter each student's height in L1, shoe length in L2, and average stride length in L3. Make sure that all of the data for a particular student are in the same row. Do *not* enter the students' names in the data lists in your calculator.

Use the arrow keys to move between L1, L2, and L3. If you make a mistake typing in the number, highlight the wrong number, press ⊂ENTER⊃, and then key in the correct number. To delete an extra number, press ⊂DEL⊃. This will delete the number and move the entire row up. To insert a number, press ⊂2nd⊃ ⊂DEL⊃ and then the number.

4. Begin analyzing the data by graphing height versus stride length.
    a) Press ⊂2nd⊃ ⊂Y=⊃. This will take you to the **STAT PLOT** screen.
    b) Choose **PLOT1** by pressing ⊂ENTER⊃.
    c) In the resulting screen, use the arrow keys to highlight **On** and press ⊂ENTER⊃. Use the arrow keys to move down to **Type**. Choose a dot (scatter) plot by highlighting the first of the pictured graphs and pressing ⊂ENTER⊃.
    d) To put the height data on the *x*-axis, move down to **Xlist**. Choose **L1** by pressing ⊂2nd⊃ ⊂1⊃.
    e) To put stride length on the *y*-axis, move down to **YList**. Choose **L3** by pressing ⊂2nd⊃ ⊂3⊃.
    f) To mark each data point with a box, move down to **Mark**, use the arrow keys to highlight the box, and press ⊂ENTER⊃.

    g) To set the graph scaling values, press ⊂ZOOM⊃.
    h) Choose option **9: ZoomStat** to scale the axes of the graph to show your data correctly. Your screen should now change to show a graph of your data.

To select a menu item, either press the number corresponding to that item *or* use the arrow keys to highlight the item and press ⊂ENTER⊃.

The picture below shows a graph of some sample data. (Don't expect your data to look exactly like this!)
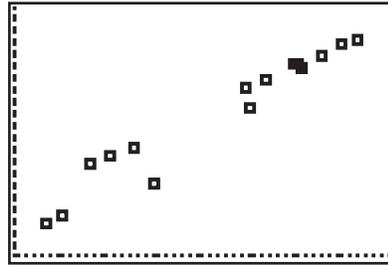
Next, you will use the calculator to determine the equation for the straight line that fits your data the best. It is important to have an equation that describes the relationship between height and stride length. If you have an equation, you can predict the height of *any* person based on the length of the person's stride.

There is a specific kind of mathematical formula that can be used to determine the equation for a straight line that best fits a group of data points. It is called a *linear regression*. In order to use this formula, we have to assume that the relationship between height and stride length is linear. In other words, we have to assume that height and stride length are related by an equation that is in the following form:

$$\text{stride length} = (a)(\text{height}) + b$$

where the a and b are constants. It is possible to calculate the equation by hand, but it takes a long time and is a little bit tedious. However, your calculator has a program called LinReg that will calculate the a and b for your data quickly.

5. Before using the calculator's LinReg program to calculate the linear regression, turn on the calculator's **Diagnostic** function. This will tell the calculator to calculate how well the line fits the data, in addition to calculating the a and b values.
   a) Go to the calculator's function catalog by pressing ⌑2nd⌑ ⌑0⌑.
   b) Use the arrow keys to scroll down until the **DiagnosticOn** option is highlighted, and then press ⌑ENTER⌑ ⌑ENTER⌑. Your screen should look like this:
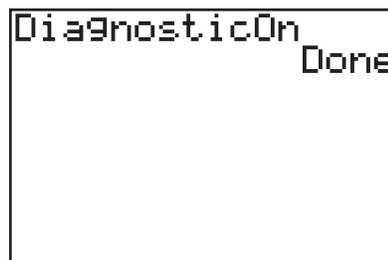
6. Use the LinReg function to perform the linear regression and store the resulting equation in variable Y1.
   a) Press ⌑STAT⌑, highlight **CALC**, and choose option **4: LinReg(ax + b)**. This will print **LinReg(ax + b)** on the Home screen.
   b) You need to tell the calculator where your data are and where to store the final equation. You want to find the line that describes the relationship between height (list L1) and stride length (list L3). Press ⌑2nd⌑ ⌑1⌑ ⌑,⌑ ⌑2nd⌑ ⌑3⌑ ⌑,⌑ to tell the calculator that lists L1 and L3 contain the data that you want to fit a line to.
   c) To indicate that you want to store the equation in variable Y1, press ⌑VARS⌑ ⌑)⌑ to select **Y-VARS**. Press ⌑ENTER⌑ ⌑ENTER⌑ to tell the calculator to store the equation in the Y1 variable. The Main screen should now show this: **LinReg(ax+b) L1,L3,Y1**.

d) Press ⟨ENTER⟩ to execute the LinReg function. This calculates the equation for the straight line that fits through your data best. The screen will display the a and b values that make the linear equation fit the data. The r^2 value tells you how well the line fits the data. An r^2 value that is close to 1 means that the line fits the data very well. The picture below shows the equation for the line that fits the sample data.

```
LinReg
 y=ax+b
 a=.7097997267
 b=-44.05196689
 r²=.9483776204
 r=.9738468157
```

Write the r^2 value that the calculator computed for your data into your Evidence Record. Also record the equation describing the data that the calculator computed.

e) The equation that describes the data is stored in the variable Y1. You can see this equation by pressing ⟨Y=⟩.

f) Press ⟨GRAPH⟩ to see how well a straight line fits your data points. The picture below shows the straight line that best fits the sample data.
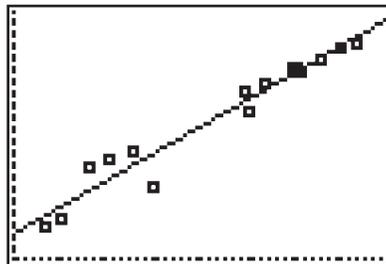
It might take a few seconds for the regression line to show up on your screen. If you pressed ⟨GRAPH⟩ and only your points are showing, look in the upper right corner of the screen. There probably will be a small moving line there. When that line is moving, it indicates that the calculator is "thinking." When the line goes away, the calculator has finished doing its computations.

7. Now determine whether or not there is a relationship between height and shoe size. Repeat steps 4–6, but make the following changes:
   a) In step 4c, use the arrow keys and ⟨ENTER⟩ to turn Plot 1 off.
   b) Use the arrow keys to select Plot 2 and turn it on.
   c) For Plot 2, make sure height is on the *x*-axis by setting **XList** to **L1** (press ⟨2nd⟩ ⟨1⟩).
   d) Put shoe size on the *y*-axis by setting the **YList** to **L2** (press ⟨2nd⟩ ⟨2⟩).
   e) In step 6b, complete LinReg(ax + b) by typing ⟨2nd⟩ ⟨1⟩ ⟨,⟩ ⟨2nd⟩ ⟨2⟩ ⟨,⟩ ⟨VARS⟩ ⟨▶⟩ ⟨ENTER⟩ ⟨▼⟩ ⟨ENTER⟩. The screen should read **LinReg(ax+b) L1,L2,Y2**. The equation for the line describing these data is stored in variable Y2. Write down the equation describing the data, as well as the r^2 value for this line, in your Evidence Record.

8. Answer the questions in the Case Analysis, using your results. Remember that the equation describing the relationship between height and stride length is stored in variable Y1, and the equation describing height and shoe size is in variable Y2. To see these equations, press ⟨Y=⟩. If you forgot to write down one of the r^2 values, repeat steps 7a–7c with the appropriate inputs to the equation. This will print the r^2 value on the screen again.

Remember that you can get back to the Home screen by pressing ⟨2nd⟩ ⟨MODE⟩.

NAME: _____

DATE: _____

Evidence Record

Student Name	Height (cm) (L1)	Shoe Length (cm) (L2)	Stride Length (cm) (L3)

r^2 value for height versus stride length: _____
Equation describing the relationship between height and stride length:

_____

r^2 value for height versus shoe size:_____
Equation describing the relationship between height and shoe size:

_____

Case Analysis

1. Based on your data, is there a linear relationship between height and stride length?
2. What is the value of r^2 for the straight line that best describes your data for height versus stride length? Do you think the straight line fits these data well?
3. Based on your data, is there a linear relationship between height and shoe size?
4. Do you think that it is possible to infer a person's height from his or her shoe size? Explain your answer.
5. Using the relationship between height and stride length that you calculated, determine the approximate heights of people with the following stride lengths: a) 75.5 cm, b) 45.5 cm, and c) 50.0 cm.
6. Using the relationship between height and stride length that you calculated, predict the stride length of a person who is not a student in your class (for example, your teacher, your principal, or a student in a different class) based on his or her height. Then measure the person's actual stride length. How close was your prediction to the actual stride length?
7. Suppose you measure the stride length of a set of footprints, you predict that the person who made the footprints is 175 cm tall, and you later find out that the person who made the footprints is actually only 152 cm tall. Give one possible reason that your prediction was incorrect.
8. Using the relationships that you calculated, determine which of the three suspects most likely left the footprints to and from Jonathan Wallace's bathroom. Show all your calculations. (Hint: In the equation that you wrote down, x is stride length and y is height.)

Case File 1
Tracks of a Killer: Using footprints to estimate height

Teacher Notes

Teaching time: one class period

This lab introduces the concepts of linear regression and r^2 values through an analysis of the relationship between stride length, shoe size, and height.

Tips

- Having three stations may not work well with some classes because the students will spend some time waiting for others to have measurements taken. You may want to break up the class into groups of three or four students; each group will make all three measurements.
- Make a transparency of the Evidence Record table to be used as a master Evidence Record. Students can record their individual data on the transparency, and then they can all copy the compiled data into their own Evidence Records.

Modifications

If time is short or students are less advanced, steps 1b and 7, as well as Case Analysis questions 3 and 4, can be eliminated.

Sample Data

Student Name	Height (cm) (L1)	Shoe Length (cm) (L2)	Stride Length (cm) (L3)
Student 1	146.5	23.0	58.0
Student 2	158.5	25.5	70.5
Student 3	186.5	28.0	88.0
Student 4	176.5	23.0	82.0
Student 5	180.0	30.5	85.0
Student 6	161.0	25.5	64.5
Student 7	174.0	28.0	77.5
Student 8	189.0	28.5	89.0
Student 9	181.5	23.5	84.5
Student 10	184.0	30.0	86.5
Student 11	149.0	23.0	59.5
Student 12	152.5	24.0	68.0
Student 13	155.5	26.0	69.5
Student 14	173.5	24.5	81.0
Student 15	181.0	30.0	85.0

r^2 value for height versus stride length: **0.948**

Equation describing the relationship between height and stride length:
    $y = 0.7098x - 44.05$

r^2 value for height versus shoe size: **0.409**

Equation describing the relationship between height and shoe size:
    $y = 0.1206x + 5.703$

Case Analysis Answers

1. Based on your data, is there a linear relationship between height and stride length?
   There should be a clear linear relationship between height and stride length. The data points should fall on a fairly straight line.
2. What is the value of r^2 for the straight line that best describes your data for height versus stride length? Do you think the straight line fits these data well?
   The r^2 values should be fairly close to 1 (0.95 or 0.90 is acceptable). If the values are significantly lower than this, it is possible that the students entered incorrect data or that their measurements were inaccurate.
3. Based on your data, is there a linear relationship between height and shoe size?
   There should not be a clearly linear relationship between height and shoe size.
4. Do you think that it is possible to infer a person's height from his or her shoe size? Explain your answer.
   No, it is generally not possible to predict a person's height from his or her shoe size.
5. Using the relationship between height and stride length that you calculated, determine the approximate heights of people with the following stride lengths: a) 75.5 cm, b) 45.5 cm, and c) 50.0 cm.
   Answers will vary depending on calculated height–stride-length equations.
6. Using the relationship between height and stride length that you calculated, predict the stride length of a person who is not a student in your class (for example, your teacher, your principal, or a student in a different class) based on his or her height. Then measure the person's actual stride length. How close was your prediction to the actual stride length?
   Answers will vary.
7. Suppose you measure the stride length of a set of footprints, you predict that the person who made the footprints is 175 cm tall, and you later find out that the person who made the footprints is actually only 152 cm tall. Give one possible reason that your prediction was incorrect.
   Possible reasons for incorrect predictions of height include the following: the person was running or was taking larger or smaller steps than usual, the person's normal stride does not follow the usual trend, and the stride length was measured incorrectly.
8. Using the relationships that you calculated, determine which of the three suspects most likely left the footprints to and from Jonathan Wallace's bathroom. Show all your calculations. (Hint: In the equation that you wrote down, x is stride length and y is height.)
   Answers will vary; based on the sample data here, Penelope Paige most likely left the footprints (her height is closest to the calculated height of 5 ft).

$$\text{stride length} = 0.7098\,(\text{height}) - 44.05$$

$$\text{height} = \frac{\text{stride length} + 44.05}{0.7098} = \frac{64.5 + 44.05}{0.7098}$$

$$= 152.9 \text{ cm} = 60 \text{ in}$$