



Gear Ratio Activity

STEM Lesson for TI-Nspire™ Technology

Objective: Students will collect data and analyze data using graphs. Students will also investigate the meaning of the x and y values of a quadratic curve derived from a quadratic regression on real-world data. Students will then use the graphs to decide which gear ratio is the best.

About the Lesson: Tracks on which Sprint Cup Series races occur vary in length, banking and surface among other things. On all tracks, proper set up to handle the turns is critical to team success but this is especially true for short tracks. For example, in Bristol the cars are almost never at top speed but are either accelerating or breaking around the short 0.533 mile track. During testing, NASCAR teams use data collection systems and technology to gather data on how the car performs with various setups. During a race however, the driver is the only way for a race team to collect data on how the car performs, so some drivers have learned to break a turn down into 16 points so he or she can communicate how the car feels along the turn. For both NASCAR and T80 SRC drivers, as a driver negotiates a turn, the car decelerates as it enters the corner then accelerates back up to top speed when it exits the turn. How fast your car can accelerate and how fast your car can go on the straight-aways are greatly dictated by the gear set. To NASCAR teams, it is almost a no-brainer that they'll install the right set because that decision rarely changes for a particular track. T80 SRC teams need also need to understand how gears impact performance and will model that in this investigation. For three different gear sets, gather data on how long it takes to drive (or use data provided) short, medium, and long straight-aways. Graph drive time as a function of gear ratio (ratio of spur gear teeth to pinion gear teeth) for each track (as a different series) to find the best gear ratio for each one. Given a table of available gears, choose the best set for the race. If you have Ten80 SRC cars, test your prediction.

Materials: RC car (optional – data provided)
3 drag strips: 50, 75, and 100 ft
At least 3 different sets of spur and pinion gears and tools to install
Stop watches (Number depends on the number of students in the group)
Student Worksheets

Prerequisite skills: The students need to understand how to interpret graphs and be familiar with scatter plots. Students should also be familiar with averages.

Procedure:

1. With a fully charged battery. Set the car at the start line and decide what the “GO” signal will be.
2. Record the pinion and spur gear sizes installed in the car.
3. Make sure all time keepers are ready and have another student give the “GO” signal. Make sure all time keepers can see the start and finish line clearly.
4. Drive from the start line to the finish line on the 50 ft track. Time keepers measure the time it takes to drive the drag strip while the driver drives.
5. Record lap number and each measured drive time. Drive the track repeatedly until times are consistently fast, at least three laps that are within 0.5 seconds of each other.
6. Change the pinion gear to a bigger one and repeat steps 2-5.
7. Change the pinion gear to a smaller one than the original one and repeat steps 2-5.
8. Repeat steps 1-7 for the 75- and 100-ft tracks.

Teacher Tip: *If time is an issue, this activity can be done with small groups and 3 different tracks. The groups can rotate tracks so everyone is working at the same time. Make sure each person in the group has a job. The procedure can be repeated for different drivers if time allows. Usually all students want to drive the car but don’t force them. One student can do both a stopwatch and record times for small classes.*

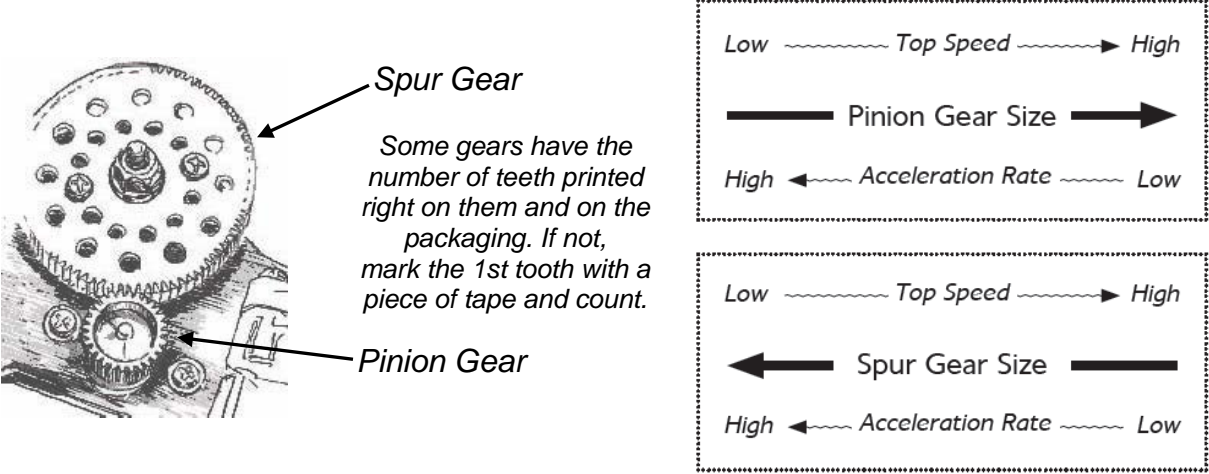
Analysis:

Teacher Tip: *If you do not have time, materials, or skill to gather your own drive-time data, the following data has been collected using a Ten80 Student Racing Challenge RC car for you.*

Spur Gear: 88 tooth

Straightaway Length (ft)	Drive Time for 20 Tooth Pinion Gear (sec)	Drive Time for 25 Tooth Pinion Gear (sec)	Drive Time for 30 Tooth Pinion Gear (sec)
50	3.35	3.10	3.45
50	3.21	3.19	3.51
50	3.38	3.17	3.52
75	4.6	3.89	4.36
75	4.57	3.87	4.29
75	4.68	3.95	4.28
100	5.98	4.75	4.96
100	5.92	4.75	4.87
100	5.95	4.82	4.91

In a NASCAR car, the engine is attached to a transmission which has a set of four gears. The drivers use the first three gears to get up to speed from pit lane or a restart then they use only fourth gear while they are lapping at speed. Typically fourth gear has a 1:1 ratio which means that there is no torque multiplication. However, the transmission is connected to a gear at the back of the car, which depending on the track can be as low as 3.2:1 to as high 6.5:1. In a radio controlled car there are only two gears, one on the motor and one attached to the rest of the drive train. The picture shows the gear arrangement in the Ten80 SRC cars.



The gear ratio is calculated by dividing the number of teeth on the spur gear by the number of teeth on the pinion gear:

$$\text{Gear Ratio} = \frac{\text{Number of Teeth on Spur Gear}}{\text{Number of Teeth on Pinion Gear}}$$

If the gear ratio is 3:1, it means that the motor has to turn 3 times to get the spur gear to move one rotation. The bigger the gear ratio the slower the top speed of the car will be, but there will be more torque available to move the car. If the gear ratio is too low, the motor will not have enough power to get the car moving.

1. Determine the gear ratio for your data.

Answer: Answers will vary depending on the gears you have. For the data given for the 20 tooth pinion gear, the ratio is 88/20 or 4.4:1.

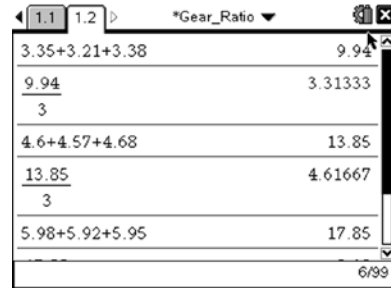
2. If the motor in an RC car is moving at 12,000 RPM (rotations per minute), how fast is the spur gear turning?

**Answer: (12000/x)=4.4
x = 2727 RPM**

Navigator Tip: Quick Poll the students for their answers to #1 and #2 and show the results to make sure the students understand the concept of gear ratios.

3. On your handheld, go to My Documents and open the file named Gear_Ratios.tns.

4. Use **(ctrl)▶** to move to page 1.2. Choose the 3 most consistent times for each track and calculate the average drive time for each gear set at each straightaway length.



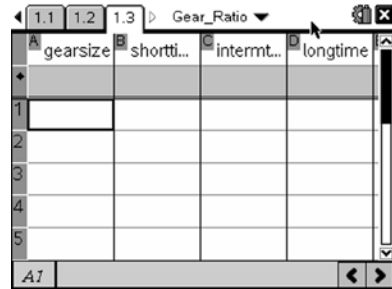
5. Fill in the table below with the average drive times.

PINION GEAR SIZE (teeth)	SHORT TRACK (50 FT)	INTERMEDIATE TRACK (75 FT)	LONG TRACK (100 FT)

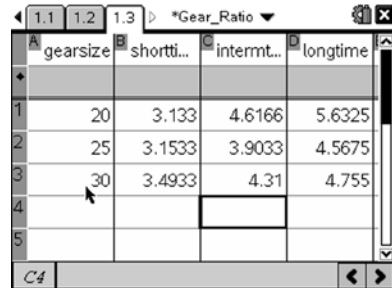
ANSWER: Answers should vary with driver and gear sets. Here are the answers for the data set provided.

PINION GEAR SIZE (teeth)	SHORT TRACK (50 FT)	INTERMEDIATE TRACK (75 FT)	LONG TRACK (100 FT)
20	3.3133	4.6166	5.6325
25	3.1533	3.9033	4.5675
30	3.4933	4.310	4.755

- Use **(ctrl)▶** to move to page **1.3**. In the top row type titles for your columns. Use the **▲▼▶** keys to move around the table. The first column should be the gear size and the next 3 columns will be the drive times for each straightaway length.



- Enter the data into the table starting in row 1 of column **A**. Type the number and then press **(enter)** or **▼** to move to the next cell.



Now we want to use scatter plots and regression lines for each gear size and time. This will allow us to analyze the data and decide which gear set works best for which straightaway length.

- Which variable is the independent variable x ?

Answer: Gear Size

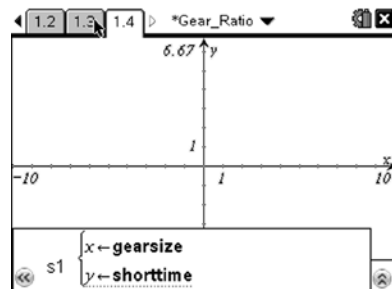
- Which variable is the dependent variable y ?

Answer: Drive Time

Teacher Tip: This is a place to discuss the fact that **time** is not necessarily the independent variable.

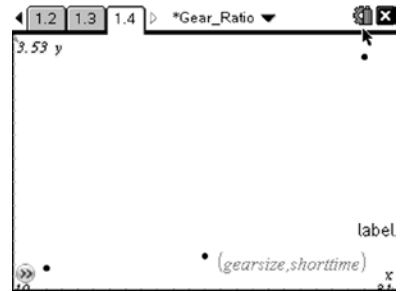
Navigator Tip: Quick Poll the students for their answers to #8 and #9 and show the results to start a good discussion and to make sure the students know which variable goes on which axis.

- Move to page **1.4**. Graph the scatter plot of average drive time vs. gear ratio. To do this first make sure the cursor is in the function bar. Press **(var)** and choose your title for gear size. Press **▼** then **(var)** and choose your variable for the short track time.

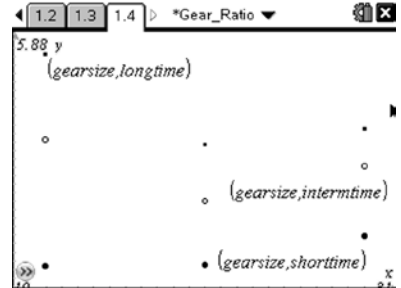


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- Press **(enter)**. Now adjust your window to view the data by pressing **(menu)** then choosing **Window/Zoom > Zoom – Data**. An example of a set of data is shown at the right.



- Press **(tab)** to bring the function bar back and repeat steps 10 and 11 for the intermediate and long tracks. You can move the labels by hovering over the label then pressing **(ctr)** then **(right arrow)** to grab then the touchpad to move if needed. An example set of data is shown at the right. Notice each data set has a different symbol for the points.



- Now we want to use regression analysis to derive curves for each set of data. Move back to page 1.3 and move to column E.

The figure shows a TI-Nspire spreadsheet view. Column E contains the following data points:

	longtime		
1	5.6325		
2	4.5675		
3	4.755		
4			
5			

- Press **(menu)** then choose **Statistics > Stat Calculations > Quadratic Regression**. The regression template will pop up. Remember to press **(tab)** to switch boxes. Click the arrow at the right of each box to access the drop down menus. Choose *gearsizes* from the drop down menu for the X List and *shorttime* for the Y list. Tab through the other options until you get to 1st Result Column. Click beside the letter and change it to 'f' if it isn't already. Click OK.

The figure shows the 'Quadratic Regression' dialog box. The settings are as follows:

- Y List: 'shorttime'
- Save RegEqn to: 'f1'
- Frequency List: '1'
- Category List: (empty)
- Include Categories: (empty)
- 1st Result Column: 'f1'

Buttons for 'OK' and 'Cancel' are visible at the bottom right.

- Repeat the process for the intermediate and long tracks.

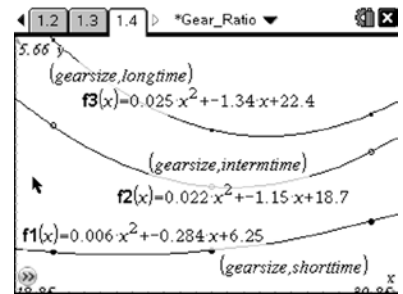
The figure shows a TI-Nspire spreadsheet view with two columns of quadratic regression results. The first column is for 'shorttime' and the second is for 'intermediate'.

	shorttime	intermediate
1	Title	Title
2	RegEqn	RegEqn
3	a	a
4	b	b
5	c	c

Values for the regression coefficients are:

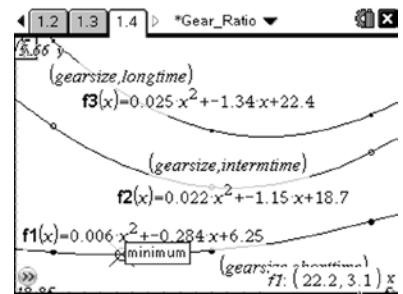
- shorttime: a = 0.0224, b = -1.15066, c = 18.6698
- intermediate: a = 0.02505, b = -1.34025, c = 22.4175

16. Move to page 1.4 and graph the regression curve. Press **(menu)** then choose **Graph Type > Function**. Press **▲** to see your function in f1 then press **(enter)**. You should see your line on top of your data points. Repeat for f2 and f3. Move the labels around the screen as needed to see everything clearly.



Since our goal is to achieve the lowest lap time possible, we look at the minimums of the curves. For the 50-foot data the minimum occurs at a pinion gear slightly below the 25-tooth run of the data provided. Note how the optimum pinion gear size gets bigger as the course length increases.

17. Find the exact x-value of the minimum for each curve by tracing. Press **Menu > Trace > Graph Trace**. The cursor will show up on one of the curves. Use the **◀** and **▶** keys to move along a curve and the **▼** and **▲** keys to switch curves. The word “minimum” will pop up when you are there. In the bottom right corner, the coordinates will appear as you trace.



Teacher Tip: This is a good time to discuss what minimums and maximums of curves are. This is also a good activity to use while introducing quadratics and ways they can be used in real-world applications.

18. Remember the x-value is the gear size, but real-world gears can only be bought to the whole tooth. What is the optimum gear size for each size track?

Answer: This depends on the original gear set. Remember we used an 88 tooth spur gear for our experiment and all data is based on this fact. For our car and the graph shown, you should use a 27 tooth gear for the 100-ft track, a 26 tooth gear for the 75-ft track, and a 22 tooth gear for the 50-ft track.

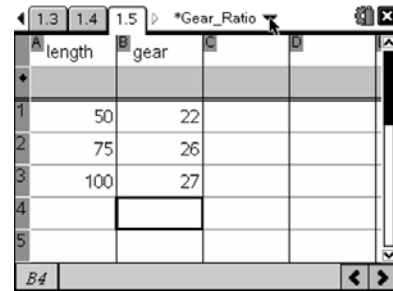
19. Now estimate the lap time a driver should get based on your gear choice by tracing the curve to the whole number gear sizes and record it below.

Answer: Based on our data 100 ft - 4.49 sec, 75 ft – 3.90 sec, and 50 ft – 3.1 sec.

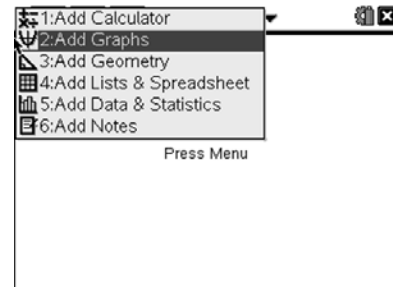
Navigator Tip: Quick Poll the students for their answers to #18 and #19 and show the results to start a good discussion about the best gear sizes and why.

Sometimes crew chiefs have to set up the car for a straightaway length that they haven't had the chance to test and gather data for yet. We can use the data we have collected to predict what gear set the track should need. Of course there are always adjustments the crews have to make after running a few laps, but at least they can get a good starting point.

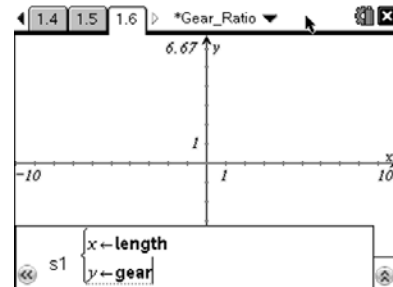
20. Move to page **1.5**. Create titles and input your straightaway length and best gear size data from questions 18 and 19. Remember you cannot use the same titles as before because the Nspire remembers all column titles as variables and the data in the column as the assigned values.



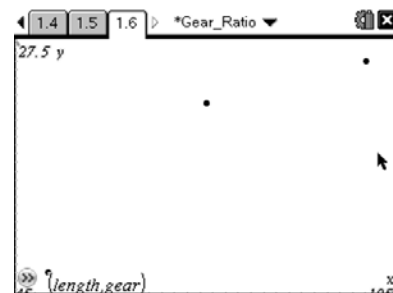
21. Graph the data as a scatter plot of gear size vs. straightaway length. Press **(ctrl)** then **(docv)** to add a page in the current document. Next click **2:Add Graphs** using the touchpad or choose it by pressing **▼** then **(s)**.



22. Press **(menu)** then **Graph Type > Scatter Plot** to set up the graph. Press **(var)** and choose your title for the length as the independent variable **x**. Press **▼** then **(var)** and choose your title for the optimum gear size.

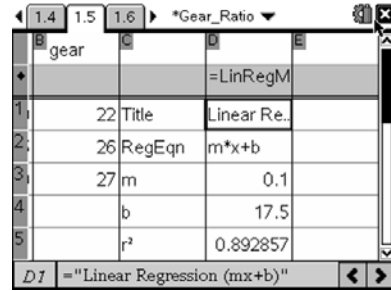


23. Press **(enter)** and zoom to the data by pressing **(menu)** then choosing **Window/Zoom > Zoom – Data**.

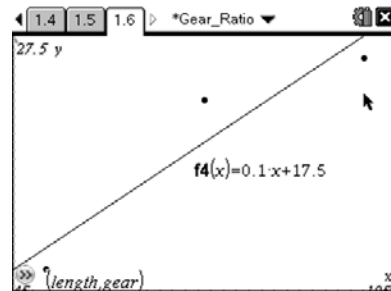


24. Move back to page **1.5** to do a linear regression on the data to derive a function that will predict a pinion gear size for any straightaway length.

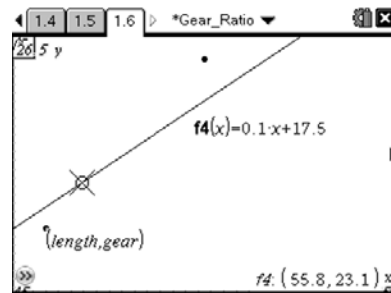
25. Press **menu** then choose **Statistics > Stat Calculations > Linear Regression (mx+b)**. The regression template will pop up. Remember to press **tab** to switch boxes. Click the arrow at the right of each box to access the drop down menus. Choose *length* from the drop down menu for the X List and *gear* for the Y list. Tab through the other options until you get to 1st Result Column. Click beside the letter and change it to 'c' if it isn't already. Click OK.



26. Move back to page 1.6 and graph the regression line. Press **menu** then choose **Graph Type > Function**. Press **▲** to see your function in f4 then press **enter**.



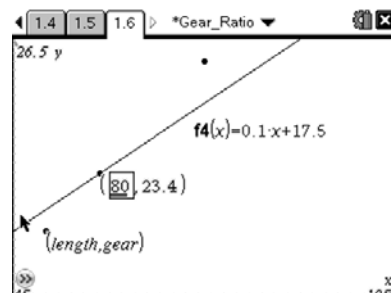
27. Press **Menu > Trace > Graph Trace** to trace the function and see an estimated gear size for each length in the bottom right corner.



28. Trace the function until you get to $x = 60$ (a straightaway length of 60 ft). What is the recommended pinion gear size (y value rounded to whole number)?

Answer: For our data a 24 tooth pinion gear

29. You can also go straight to a certain length by setting a point on the line. While in tracing, press **2nd** to create a moveable point on the line. This point should show coordinates when you set it. Next press **esc** and click on the x -coordinate twice. A cursor will pop up and a box will form around the number you are editing. Just press **del** and type the length you need.



30. Recommend pinion gear sizes for the following straightaway lengths.

70 ft –

80 ft –

95 ft –

Answer: For our data 25 tooth, 26 tooth, 27 tooth

Navigator Tip: Screen Capture student handhelds to show the students tracing and finding the sizes.

Teacher Tip: The students will probably have to adjust their window size to find the 95-ft data. This is a good place to discuss adjusting graphing windows and feasibility of some values such as negative numbers. You can't have negative teeth, but you could have negative money meaning you owe someone.

Navigator Tip: Quick Poll the students for their answers and show the results to start a good discussion on which way to round and make sure the students have feasible answers.

31. If you have a previous crew chief's data recommending a 90 tooth spur gear and a 26 pinion gear for an 80-ft straightway, what pinion gear would be the best if you only had a 79 tooth spur gear and wanted to achieve the same results?

Answer: You need to keep the same gear ratio so set up a proportion: $(90/26)=(79/x)$. $x = 22.8$, so 23 tooth would be the best.

Teacher Tip: This might be a good question to do as a class discussion to bring understanding of how the gear ratio ties in to the math the students just did. Sometimes the students get lost in the math and forget the overall reason for the math.