# Using Handheld Graphing Technology in Secondary Mathematics:

## What Scientifically Based Research Has to Say

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# INTRODUCTION

Handheld graphing technology—graphing calculators—have been widely heralded as a tool that can be used to enhance mathematics instruction and improve student learning. As early as the 1989 NCTM *Curriculum and Evaluation Standards for School Mathematics*, recommendations for grades 9-12 included an assumption that "Scientific calculators with graphing capabilities will be available to all students at all times" (p. 124). Yet despite extensive use of graphing handhelds in many classrooms, information on research about the effectiveness of their use remains unavailable to many educators.

The main purpose of this paper is to present the results of the most credible, scientifically based research available related to the use of handheld graphing devices in secondary mathematics instruction.

## Texas Instruments as a Leader in Educational Use of Graphing Handhelds

As a leader in developing graphing handhelds, Texas Instruments has a keen interest in scientifically based research on the effects of using such graphing devices in classrooms.

Texas Instruments provides a variety of products and services designed to meet the specific needs of students and teachers. The TI-8X graphing handheld series, and in particular the TI-83 Plus model, provides a full range of the graphing features that are widely considered to be appropriate for use in secondary mathematics courses. Texas Instruments also provides professional development for teachers who want to incorporate graphing handhelds into their mathematics and science curricula through the Teachers Teaching with Technology™ ($T^3$) program of courses, workshops, and related support resources. One of the purposes of this paper is to describe how these products and services match with instructional practices whose effectiveness has been demonstrated in scientifically based research.

## How the Studies Were Selected

In 2002, Texas Instruments commissioned a survey of research on handheld graphing technology in secondary mathematics (Burrill, Allison, Breaux, Kastberg, Leatham, & Sanchez, 2002). From a field of over 180 research reports, the research team for this project identified 43 studies that met criteria related to publication, relevance, inclusion of evidence, rigor, and scientific design.

Working from this set of 43 studies, Texas Instruments commissioned a further review based on the criteria for scientifically based research as described in the No Child Left Behind Act. Researchers found six studies that focused on the effectiveness of an instructional program or method that included graphing handheld devices, used an experimental or quasi-experimental design, and were of sufficient quality for inclusion in this report. One of the studies focused on college-level calculus.[1] The other five studies are described in this paper.[2]

## Overview of the Selected Studies

All five of the studies reviewed in this paper focus on use of handheld graphing devices in advanced algebra instruction. Summary information about each study is provided on the following page.

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[1] Burrill et al. (2002) included it in their report because college-level calculus is taught in high schools as well. It is not summarized in this paper, which focuses on use of graphing heldhelds in advanced algebra courses.

[2] The five studies featured in this paper are described in six research reports. One study is described in two reports: Harskamp, Suhre, and Van Streun (2000) and Van Streun, Harskamp, and Suhre (2000). In some cases (e.g., Thompson and Senk, 2001) only part of the results are summarized in this report because of the focus of the report on the effectiveness of using graphing handhelds.

| ***Study*** | ***Study Subjects*** | ***Curriculum Focus*** | ***Use of Graphing Handhelds in Experimental Treatments*** |
|---|---|---|---|
| Thompson and Senk (2001) | Classes with half 10th grade students and half 11th grade students in a rural high school near Chicago | Algebra 2 | Reform curriculum included "activities in which students generate graphs...[then] observe patterns and form conjectures or...make connections among algebraic, numeric, and graphical approaches" (p. 65). |
| Harskamp, Suhre, and Van Streun (2000); Van Streun, Harskamp, and Suhre (2000) | Classes from 7 Dutch upper secondary schools (ages 15-17) | Precalculus-level course | Graphing handhelds were used "(1) to explore graphs, (2) to find graphical solutions for different kinds of functions, and (3) to check answers found algebraically" (p. 41). |
| Ruthven (1990) | Classes of upper secondary students in England | Precalculus-level course (with some beginning calculus) | No prescription of how teachers and students were to use graphing handhelds; no previous experience or training with such graphing devices. |
| Schwarz and Hershkowitz (1999) | Classes of upper half grade 9 students in Israel | Approximately equivalent to an Algebra 2-level course on functions in the U.S. | Apparently open-ended student use of graphing handhelds to explore investigations through transformations on the algebraic models (e.g., stretches, shifts, sums) and direct manipulations of graphs and numerical data (zooming, scaling, scrolling). Unclear how much guidance was provided to students. |
| Hollar and Norwood (1999) | Classes of college students in a large state university | Intermediate college algebra (equivalent to high school Algebra 2 level) | Use of graphing handhelds to "explore, estimate, and discover graphically and to approach problems from a multirepresentational perspective" in class and for homework (p. 222). |

Research results from these studies fall into two main categories: general algebra performance, and understanding of functions. In both of these areas, use of handheld graphing devices was demonstrated to have positive impacts on student learning.

## About This Paper

The remainder of this paper includes the following sections:

- Using Graphing Handhelds as an Integrated Part of an Algebra 2 Reform Curriculum—Research results from Thompson and Senk (2001)
- Incorporating Graphing Handhelds into an Existing Curriculum in a Precalculus-Level Course—Research results from Harskamp, Suhre, and Van Streun (2000) and Van Streun, Harskamp, and Suhre (2000)
- Undirected Addition of Graphing Handhelds in Precalculus-Level Classes—Research results from Ruthven (1990)
- Using Graphing Handhelds in an Investigation-Based Algebra 2-Level Course on Functions—Research results from Schwarz and Hershkowitz (1999)
- Integrating Graphing Handhelds into Intermediate College Algebra (Equivalent to High School Algebra 2 Level)—Research results from Hollar and Norwood (1999)

- Conclusion—A summary of key research results from the studies described in this paper; discussion of how uses of the graphing handhelds in these studies relate to capabilities of Texas Instruments handheld graphing technology for students; and specific strategies for using graphing handhelds that are modeled in the studies presented in this article, with an explanation of how the Texas Instruments T$^3$ professional development offerings support use of these strategies

## USING GRAPHING HANDHELDS AS AN INTEGRATED PART OF AN ALGEBRA 2 REFORM CURRICULUM

Use of computerized technology, including handheld graphing technology, is a key component of algebra curricula developed in response to the reform movement in mathematics education. Typically, use of handheld graphing devices in such curricula is seen as linked to an exploratory, problem-solving-based approach to student learning, as opposed to the more traditional emphasis on guided practice of algorithms and development of paper-and-pencil calculation and symbol manipulation skills. In actuality, many different curricula involve uses of the graphing technology but often do so in ways that align with their philosophical approach to teaching and learning.

Thompson and Senk (2001) conducted a study comparing the performance of matched pairs of Algebra 2 classes within the same school.[3] Within each pair of classes, one class used the second edition of the *Advanced Algebra* reform curriculum (Senk, Thompson, Viktora, Usiskin, Ahbel, Weinhold, Rubenstein, Jaskowiak, Flanders, Jakucyn, & Pillsbury, 1993) developed by the University of Chicago School Mathematics Project (UCSMP); the other class used a more traditional Algebra 2 curriculum.

In three of the four schools studied, both the UCSMP classes and the comparison classes had access to graphing handhelds, which were used by the teachers with their classes. However, in one school—a rural, virtually all-white school near the outskirts of Chicago—the comparison classes did not have access to graphing handhelds, and only 4% of students in those classes reported having access to graphing handhelds (presumably personally owned) on the posttest. For the purposes of this report, this school provides a contrast between a reform curriculum incorporating handheld graphic devices and a traditional curriculum without access to graphing handhelds.

### Treatments

Classes at the school (two matched pairs) consisted of half 10th grade students and half 11th grade students. Students in the comparison classes used *Algebra and Trigonometry: Structure and Method: Book 2* (Dolciani, Sorgenfrey, Brown, & Kane, 1986), one of the most widely used Algebra 2 texts at the time of the study. Specific contrasts between the UCSMP *Advanced Algebra* curriculum and Dolciani et al. curriculum included the following:

| ***UCSMP Advanced Algebra (Senk et al., 1993)*** | ***Comparison Curriculum (Dolciani et al., 1986)*** |
|---|---|
| Combination of “continual review...with a modified mastery-learning strategy” (p. 59) | No mixed review problems incorporating review of earlier material |
| General emphasis on realistic applications and use of “either a realistic context or some graphical representation” to introduce most lessons (p. 64) | Little use of real contexts or applications; application problems often separate |

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[3] Classes were matched on the basis of “a pretest measuring entering algebra and geometry knowledge” (p. 61). Pairs of classes were omitted if either the means or the variances of scores were significantly different ($p < .025$ for the means) or “additional information indicate[d] that the classes [were] substantially different in background or educational opportunity” (pp. 61-62).

| | |
|---|---|
| “Roughly equal emphasis” on skills, properties, uses, and representations, as four dimensions of understanding (p. 64) | Some coverage of all four areas, but with an “emphasi[s on] skills far more than the other three dimensions” (p. 64) |
| “Explicit connections between algebra and geometry” (p. 64) | “Few explicit connections” between algebra and geometry (p. 64) |
| Inclusion of multiple problem solution strategies | “Virtually no mention of multiple ways to solve particular problems” (p. 64) |
| “Small-group explorations and hands-on experiences to introduce or extend mathematical topics” (p. 65) | No use of cooperative problem solving or small-group work |
| “Extended projects...at the end of each chapter to encourage student investigation and written reports” (p. 65) | No use of projects |
| Assumption of the availability of graphing handhelds at all times. (Note, however, that in actual practice, students at this particular school used the graphing handhelds 2-3 times a week.) | No mention of graphing handhelds (text predates availability) |
| Activities featuring graphing handhelds “in which students generate graphs...followed by questions asking students to observe patterns and form conjectures or to make connections among algebraic, numeric, and graphical approaches” (p. 65) | Graphs “generally not integrated with other topics and...approached as ends in themselves rather than as means to solve problems” (p. 64) |

In short, use of graphing handhelds was an integral part of the UCSMP curriculum, with student generation of graphs used to promote active learning on a variety of algebra topics. In contrast, handheld graphing technology was unavailable in the comparison classes, which used a much more traditional approach relegating graphs to treatment as a separate topic.

## Research Results

At the end of the school year, Thompson and Senk tested students in all classes on a 36-item multiple-choice posttest developed by UCSMP focusing on “core content of second-year algebra” (p. 66). To correct for differences in content coverage between UCSMP and the comparison curriculum, they asked teachers to evaluate each of the items on the test to judge whether their students had adequate opportunities to learn the content covered in that item. This resulted in a “Fair Test” version of the posttest consisting of 26 items for this school.[4]

Analysis of the Fair Test version results showed that for both matched pairs of classes, students in the UCSMP class performed significantly better than those in the comparison class.[5]

Given the wide range of practices contributing to instructional differences between UCSMP and the comparison curriculum, it is impossible to say how much of this difference in scores is due to use of graphing handhelds. Unequal access to graphing technology during the posttest may also have contributed to the difference: students in the UCSMP classes had access to graphing handhelds during the posttest, while as noted above, only 4% of students in the comparison classes reported such access (although 60%

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[4] K-R 20 = 0.635

[5] For one matched pair, mean scores were 66.8% (SD = 12.8) for the UCSMP class and 53.5% (SD = 12.9) for the comparison class, a difference that was statistically significant on a matched pair t-test at the $p = .001$ level. For the other matched pair, mean scores were 68.3% (SD = 15.6) for the UCSMP class and 51.2% (SD = 14.1) for the comparison class, a difference that was also statistically significant on a matched pair t-test at the $p = .001$ level.

had access to scientific calculators). At the very least, however, these posttest results indicate that a reform curriculum incorporating graphing handhelds can lead to enhanced student performance with Algebra 2 topics considered standard content in both courses, compared to a more traditional curriculum that does not include handheld graphing technology.

# INCORPORATING GRAPHING HANDHELDS INTO EXISTING CURRICULUM IN A PRECALCULUS-LEVEL COURSE

A more focused study in Holland examined the effects of integrating graphing handhelds with an existing curriculum. Harskamp, Suhre, and Van Streun (2000) conducted research on students ages 15-17 from seven Dutch upper secondary schools in the next-to-last year of a university preparation course (precalculus-level). All of the classes used the same textbook.[6] Content of the course included four main topics: functions and graphs, rate of change and derivatives, exponential functions, and periodic functions.

## Treatments

Students from 12 classes were divided into three treatment groups: three classes in the first experimental group, five classes in the second experimental group, and four classes in the comparison group. Comparison group classes used the existing textbook without alteration and used only scientific calculators. Classes in the first experimental group used graphing handhelds with all four topics. Classes in the second experimental group used handheld graphing devices with the first topic only. The graphing handheld model used by students was the Texas Instruments TI-81.

For the two experimental groups, textbook sections were altered to incorporate use of graphing handhelds. Within the adapted topics, all types of functions were introduced through use of graphing technology:

> Firstly, an exemplary problem was introduced in which a certain function had a central place and in which students learned to discern the relevant aspects of the function that were needed in formulating a solution to the problem.... Secondly, students were given additional problems and were asked to tabulate some intermediary results, find a formula, and then plot the data. Finally, students learned how to apply their new knowledge to related problems that could be solved using the function. (p. 41)

Students used the graphing handhelds in three ways: “(1) to explore graphs, (2) to find graphical solutions for different kinds of functions, and (3) to check answers found algebraically” (p. 41). Other, more advanced features of the graphing technology (such as calculating the derivative using the function NDERIV) were not incorporated into the course. Between 25% and 50% of the textbook exercises in each topic involved use of the graphing handhelds.

## Research Results

### *Posttest Results*

At the beginning of the school year, students completed a six-item pretest on content taught the previous year.[7] At the end of the year, a 19-item posttest was administered covering the content of the course.

Results of the two tests were compared and analyzed for the three treatment groups. The researchers found that

> of those students who had few correct answers on the pretest (scores 0 and 1), those who were in either of the experimental groups achieved a significantly higher mean posttest score ($p < 0.05$) than students in the control group. Of those students with a score of 2 on the pretest, only those in the first

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[6] Van Streun (1995).

[7] The pretest obtained a Cronbach’s alpha of 0.52. According to the authors of the study, “This rather low reliability was presumably due to students’ forgetting what they had learnt the previous school year” (pp. 41-42). The relatively low reliability of the pretest may cast some doubts on results based on pretest-posttest comparison.

> experimental group [who used the graphing handhelds for the entire year] outperformed students in the control group ($p < 0.01$). (pp. 47-48)[8]

No significant differences among groups were found for students scoring 3 or above on the pretest.

Taken together, these results demonstrate a general advantage for low-achieving students from using handheld graphing technology as an integrated part of a precalculus-level course. This advantage was greater in classes where graphing handhelds were used for an entire year than for classes where graphing devices were used only part of the year.

***Solution Strategies***

For the 13 posttest problems where students were required to show their work, researchers identified student solutions for each question as falling into one of three categories:

- *algorithmic*—"solutions which included algebraic procedures such as solving an equation, calculating the coordinates of a point in a graph of a function, computing the derivative, and so on";
- *graphical*—"solutions which depended on plotting or drawing, either on the graphics calculator or by hand"; or
- *heuristic*—"informal methods of solving problems [that] often contain an element of trial and error" (p. 42).

Researchers found that students in both experimental groups used more than three times as many graphical solution strategies as students in the comparison group. Consequently, "students in both experimental groups more often made some attempt at a solution than students in the [comparison] group" (p. 45). Follow-up analysis suggested that this increased use of graphical solution strategies might be related to the improved scores for low-achieving students using graphing handhelds.[9]

Additional analysis of these results showed that for students who used the graphing devices all year long, increased use of graphical solution strategies was related to improved performance. For students who used the graphing handhelds for only one topic, however, there was no impact on performance, due to lower use of algorithmic solution strategies. Further analysis also showed that not all the improvement for students who used the graphing technology all year long was due to use of graphical solution strategies. This was interpreted to mean that students who used graphing handhelds for the entire year had a better understanding of functions.[10]

***Teaching Methods***

During the school year, each instructor was observed for two lessons to see what impact use of handheld graphing technology might have on instructional strategies. (For the second experimental group, observations were conducted during the portion of the class that used graphing handhelds.)

Notes were taken every 10 seconds on whether class activities involved problem solving (letting students explore, communicating solutions of students, or problem-solving dialogues between teacher and students) or use of graphs and tables (explaining an exercise with a graph on the blackboard or explaining an exercise with a graphing handheld).

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[8] These findings are based on LSD post hoc tests.

[9] This conclusion is based on comparison of several multilevel regression models relating posttest scores to various independent variables. A model including pretest scores, group, and frequency of different solution strategies showed a significant reduction in deviance compared to a model including only pretest and group ($p = 0.00$). A model including only pretest scores and frequency of different solution strategies showed no significant difference in deviance compared to the model including pretest scores, group, and frequency of different solution strategies.

[10] Results reported in this paragraph are from Van Streun, Harskamp, and Suhre (2000). This paper used a LISREL (linear structural equation analysis) technique and a chi-square test with a $p$ value of .09, which is acceptable for this form of analysis.

The researchers found that teachers in the experimental groups spent much more time using graphs and tables than teachers in the comparison group: 24% and 18% of class time respectively for the first and second experimental groups, compared to 1% of class time for the comparison group. Teachers in the experimental groups also spent more time in problem-solving activities (28% and 30% for the first and second experimental groups, versus 19% for the comparison group), though the difference was not as dramatic.

# UNDIRECTED ADDITION OF GRAPHING HANDHELDS IN PRECALCULUS-LEVEL CLASSES

Probably the most typical instructional use of handheld graphing technology at the secondary level is to reinforce student understanding of functions. Several studies have focused on this specific use, attempting to evaluate precisely how use of graphing handhelds changes the way students think about functions.

Ruthven (1990) reports the results of one such study, focusing on classrooms in England where graphing technology was introduced into an advanced-level, academic upper secondary (precalculus-level) course. Subsequently, students were tested on their comprehension of two types of problems related to algebraic functions: *symbolization* items, which require students to write the equation that matches a given graph; and *interpretation* items, which require students to extract information from and answer questions about a given graph.

## Treatments

Pairs of matched classes "similar in previous attainment and following the same mathematics course" (p. 433) were identified at four English secondary schools. Matched classes followed the same curriculum, with the sole exception that teachers in the experimental classes were provided with access to graphing handhelds for their students, while the comparison classes had no access to graphing handhelds or other graphing technology.[11] Content of the course included much of the material typically covered in precalculus and beginning calculus courses in the United States.

In contrast to the instructional situations reported in Thompson and Senk (2001) and Harskamp, Suhre, and Van Streun (2000), teachers in this study were given no guidance on how to use the graphing devices that had been made available to their classes:

> None of the participating teachers had any previous experience of graphic calculators.... Rather than following any prescribed programme of calculator activities, the teachers [were] free to plan the work of their own classes, meeting together periodically to exchange ideas and review progress. (p. 431)

No information was reported in the study about how teachers used the graphing handhelds, making it impossible to identify specific instructional practices that may have contributed to the research results.

## Research Results

After nearly a full year with the handheld graphing devices, students were administered a 40-minute test consisting of 12 items in which students answered questions based on presented graphs (six items each for symbolization and interpretation). Students from the experimental group were allowed to use their graphing handhelds on the test and were asked to record any use of the technology on their answer sheets. All students were asked to record reasoning that led to their answers whenever possible.[12]

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[11] Seven students in the comparison classes who owned personal graphing handhelds were excluded from the research results.

[12] According to the researcher, the test "covered material drawn from two topic areas central to any advanced-level course, and where the use of graphs is normal practice: standard function families, and variation of functions.... Equally, items were designed to test competences for which there is no automatic graphic calculator procedure, so as to confer no direct advantage to the graphic calculator users, and to focus on mathematical expertise of continuing importance" (pp. 433, 438).

In order to establish that students in the matched experimental and comparison classes were academically comparable, test results for both the interpretation and symbolization items were analyzed separately against students' previous grades in standardized mathematics tests used throughout the United Kingdom.[13]

Analysis of the test scores showed no significant difference in performance on the interpretation items. However, experimental group members achieved significantly higher results on items involving symbolization—more than a standard deviation.[14]

Symbolization results were subjected to further analysis, using a scale that marked both answers and reasoning reported by students on the test in terms of *recognition* (correct identification of the type of function) and *refinement* (finding the precise equation for the graph). Results of this analysis "provide[d] strong evidence both of superior recognition and of superior refinement by the project group" (pp. 445-446).[15]

Analysis of the symbolization results also showed a significant interaction between treatment group and gender. In the comparison group, male students outperformed females. However, these results were reversed in the experimental group. This difference was found to be statistically significant.[16]

## Interpretation of Results

The researcher identified several possible reasons why use of graphing handhelds might lead to improvements in student symbolization but not interpretation. First,

> Regular use of a graphic calculator is likely to rehearse specific relationships between particular symbolic and graphic forms, as it is through such relationships that the calculator itself is operated, albeit in the reverse direction to that tested. Moreover, reliable access to graphic calculators is likely to encourage both students and teachers to make more use of graphic approaches in solving problems and developing new mathematical ideas, not only strengthening these specific relationships, but rehearsing more general relationships between graphic and symbolic forms. (p. 447)

The finding of Harskamp, Suhre, and Van Streun (2000) that teachers in classrooms with handheld graphing technology spend more time on problem solving and, especially, with graphs and tables seems to support the claim that use of graphing handhelds might improve student symbolization skills in this way. Ruthven argues further that

> availability of a graphic calculator when carrying out symbolisation tasks.... improves the quality of information available to students, facilitating checking within an analytic-construction approach, or enabling a graphic-trial approach. The effect of this is twofold: not only does it directly improve prospects of success; by reducing uncertainty, it is likely to diminish anxiety on the part of students carrying out such tasks, leading indirectly to improved performance. (pp. 447-448)

According to Ruthven, neither of these sets of reasons applies to interpretation. Translation between functions and graphs—changing a function to see corresponding changes in graphs, and vice versa—is the most obvious classic application of graphing handhelds within classes focusing on functions, including algebra, precalculus, and calculus. Such use of handheld graphing technology may strengthen students' abilities to symbolize the correct function for a graph (and vice versa), but does little or nothing to aid students in interpreting specific information from the graph. Ruthven notes that graphing handhelds could be used in more innovative ways to strengthen interpretation skills: e.g., by having students "[explore]

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[13] The exams used for the sake of comparison were the GCSE (or the O-level in some cases). The statistical procedure used was analysis of variance.

[14] *F*-value = 55.242, degrees of freedom = 1, $p = .000$.

[15] This result was based on a "directed Mann-Whitney rank-sum test, by normal approximation with correction for tied ranks" (p. 449). Students in the experimental group experienced a statistically significant advantage on 5 out of 6 items for recognition ($p < .03$) and 4 out of 6 items for refinement ($p < .03$).

[16] *F*-value = 4.938, degrees of freedom = 1, $p = .03$.

relationships between the graphs of functions, and those of their differential and integral functions." However, many of the experimental teachers "reported themselves as taking a cautious, fairly traditional approach to their teaching of calculus," so that such potentialities may have been largely unexplored in this case (p. 448).

## USING GRAPHING HANDHELDS IN AN INVESTIGATION-BASED ALGEBRA 2-LEVEL COURSE ON FUNCTIONS

Another study focusing on the effects of graphing handhelds on comprehension of functions was conducted with upper half grade 9 students in Israel in a one-year course on functions, roughly parallel to Algebra 2 in the United States.

Schwarz and Hershkowitz (1999) reported on the effects of two different but related curricula, one incorporating use of handheld graphing technology, the other not. These researchers examined student performance on items related to three separate dimensions of function understanding: *prototypicality*, *part-whole reasoning*, and *attribute understanding*.

- *Prototypicality* refers to student use of key examples and models to understand functions. "Prototypical examples are used as a frame of reference so that other examples are judged by reference to the prototype as a whole or reference to the self-attributes of the prototypes instead of by reference to the mathematical definition of the concept" (p. 364). Further: "prototypicality is assessed through the examples students invoke, the methods they use pertaining to specific examples (such as linear interpolation), and the links they make (justifications, transformations) between the problem under consideration and the examples invoked to solve problems on functions. In addition, prototypicality incorporates the degree to which examples evoked and justifications given depend on context." (p. 370). In this study, problems related to prototypicality are designed in part to test whether students recognize and use functions other than linear and quadratic functions in trying to understand a given set of data.
- *Part-whole reasoning*, as tested by these researchers, involves correctly matching parts of graphs with other parts of the same graph (generally with different scales).
- *Attribute understanding*, as tested by these researchers, involves identifying graphs that match a given equation (or vice versa) or that match a set of specific numerical values for the function.

Results related to these categories are reported below.

### Treatments

Researchers compared two versions of curriculum focused on the study of functions, one representing the second cycle of revision ($G_2$), the other representing the third cycle of revision ($G_3$). Three classes were selected: one using $G_3$, the other two using $G_2$. The two $G_2$ classes were selected "on the basis of their similarity to the $G_3$ group both in terms of the student populations and the teachers' teaching style" (p. 371).[17]

According to the researchers, "the second and third cycles differed in their use of computerized tools, in the nature and design of problem situations, in the process of introduction to the function concept and its definition, and in the social interactions in the classroom" (p. 369). Key points are summarized on the following page:

| ***Cycle 2 Curriculum*** | ***Cycle 3 Curriculum*** |
|---|---|

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[17] The researchers do not report any data for the two treatment groups on prior test performance, nor did the study apparently include any test of pretreatment ability differences. This represents a caveat for interpreting the research results.

| | |
|---|---|
| Variety of functions presented, including “not only linear and quadratic functions but also polynomial functions, absolute value functions, greatest integer functions... Moreover, functions were treated in parallel in several representations” (pp. 367-368). | Same[18] |
| Formal definition of function presented early in the course | Short, informal definition of function provided later in the course |
| Students working “at times...in an explorative mode...on problem situations,” with exploration “followed by sessions of consolidation of the main concepts, procedures, and skills” (p. 368) | Students completed “a chain of problem situations for which the contextual frame was thought to facilitate the students’ development of the function concept. Students were given opportunities to construct functions while investigating problem situations” (p. 368). |
| Relatively traditional approach to classroom dynamics and social interaction, including teacher or textbook guidance of student explorations | More focus on student decision-making, collaborative work in small groups, written self- and peer-critique of problem solving processes, and flexible roles for the teacher (including “alternately a facilitator, a modeler, and a coordinator of debates concerning verbal reports and critiques in the whole-class synthesis sessions”) (p. 368) |
| No use of graphing technology | Availability of “either graph[ing] calculators or multirepresentational software tools,” which were generally used “during one of the two or three weekly periods” (p. 368) |

According to the researchers, graphing technology was used by $G_3$-students in two main ways: through transformations on the algebraic models (e.g., stretches [$af(x)$, $f(ax)$], shifts [$f(x + k)$], sums [$f(x) + g(x)$]) and direct manipulations of graphs and numerical data (zooming, scaling, scrolling). (All of these transformations and manipulations represent features of the Texas Instruments TI-8X series of graphing handhelds.) Presumably, the graphing technology was used as a tool in student problem solving, a process that was described as relatively student-directed. It is unclear from the study how much direction was provided to students on specific ways to use the graphing devices.

## Research Results

### *Performance*

At the beginning of 10th grade, the researchers administered an eight-item test (with multiple parts for many items) to students, including items related to prototypicality, part-whole reasoning, and attribute understanding. They found that the graphing technology-using $G_3$-students performed significantly better than $G_2$-students on items related to prototypicality, leading to significantly better overall performance.[19] There were no significant differences for items related to attribute understanding or part-whole reasoning.

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[18] This point of similarity is important for the research results because it relates directly to student performance in the area of prototypicality. Since students were exposed to multiple examples of functions beyond linear and quadratic functions in both cycles of the curriculum, differences in student performance on prototypicality can be assumed to be the result of differences in instructional methods, including the use of graphing technology.

[19] According to the researchers, “The MANOVA showed a significant advantage for $G_3$ over $G_2$ ($F(1, 101) = 7.66$, $p < .001$) with an effect size ($\eta$2) of .48.... The Bonferroni test showed that the difference between the two groups was significant for prototypicality ($F(1, 101) = 7.10$, $p = .009$) but not significant for attribute understanding ($F(1, 101) = 3.71$, $p = .057$) or for part-whole reasoning ($F(1, 101) = 1.51$, $p = .222$). These results show that the superiority of $G_3$ over $G_2$ was due mainly to prototypicality” (p. 377). The analysis was based on test item parts for which a quantitative comparison was appropriate.

***Idea Units***

Students were asked to provide justifications for their answers on many parts of items on the test. The researchers analyzed these student justifications, breaking them down into *idea units*, defined as "the primitive elements that constitute students' justifications" (p. 375).[20] Thus, a greater number of idea units represented greater complexity in student justifications.

Researchers found a significant correlation between correctness of answers and number of idea units in student justifications for the relevant test items for prototypicality and attribute understanding.[21] For seven out of eight parts of items related to these two areas, they found that $G_3$-students had provided significantly more idea parts than $G_2$-students[22]—suggesting that the experimental treatment may have resulted in more complex student thinking about these problems, which in turn correlated to more correct answers.

***Qualitative Analysis of Responses***

The researchers also performed an "analysis and interpretation...based to some extent on quantitative data" related to scores and idea units "but more so on qualitative considerations: particular examples and justifications given by students in the two groups" (p. 377). Such results, while not subject to quantitative data analysis, are nevertheless noteworthy as providing possible explanations for the quantitative research results.

In their analysis of student responses related to prototypicality, the researchers found that

> there were more $G_2$-students than $G_3$-students who [based] their justifications on the self-attributes of the linear function, even when such justification led to an absurdity.... In contrast, when using linear strategies or attributes, many $G_3$-students adapted their responses to the conditions under which the problem was posed. (381).

In other words, $G_3$-students were more sophisticated in their understanding and use of prototypical examples. The authors noted further that

> $G_3$-students' use of prototypes was often beneficial, either to give an example of a function with a given attribute...or to construct a function by using the prototype as a reference from which to begin.... $G_3$-students used linear, quadratic, or other functions...to exemplify attributes or to learn more about other examples. For these students, the prototypes were levers for concept learning. In contrast, $G_2$-students more often considered linear or quadratic functions as exclusive examples with a given attribute. They often chose a linear strategy, such as linear interpolation, not because it was useful or reasonable but because this strategy was the only way they knew to interpolate values of functions. (p. 385)

It is important to note once again that the two treatments that are being compared here are complete curricula, incorporating a variety of different instructional strategies. As with the UCSMP results reported by Thompson and Senk (2001), it is impossible to gauge how much of the difference between the two groups may be based on use of handheld graphing technology. What these results suggest, however, is that use of such graphing technology as part of a carefully designed curriculum can help secondary algebra students in particular ways by strengthening their understanding of the relationship between graphs and functions and broadening their sense of the range of possible functions that can be represented on graphs. In a general sense, this supports the finding by Ruthven (1990) that use of handheld graphing technology can be especially effective in improving students' symbolization of functions.

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[20] Researchers independently identified the idea units with 90% agreement, then resolved disagreements.

[21] For prototypicality, $r = .22$, $p = .02$; for attribute understanding, $r = .54$, $p < .001$. No significant correlation was found for part-whole reasoning.

[22] Results were based on a MANOVA, with $p < .05$ for one item and $p < .001$ for six items.

# INTEGRATING GRAPHING HANDHELDS INTO INTERMEDIATE COLLEGE ALGEBRA (EQUIVALENT TO HIGH SCHOOL ALGEBRA 2 LEVEL)

Hollar and Norwood (1999) conducted another comparison of two curricula for the same course, one incorporating handheld graphing technology, the other not. The study involved college students enrolled in an intermediate algebra course, designed for "those students scoring lowest on the university's mathematics placement examination" (p. 221)—roughly equivalent to a high school Algebra 2 course.

At the end of the course, the researchers collected information on student performance on the standardized departmental final exam, but also conducted a more specialized test

> designed to assess...aspects of conceptual knowledge of functions without the use of the graphing calculator: (a) *modeling* a real-world situation using a function; (b) *interpreting* a function in terms of a realistic situation; (c) *translating* among different representations of functions, and (d) *reifying* functions. (p. 221; italics added)[23]

Reification is further defined as "the transition from the operational to the structural phase of concept development" (p. 221); i.e., the shift from viewing functions as simply rules that operate on numbers to seeing them as objects on which operations are performed (transforming functions to different functions).

## Treatments

Four classes were selected for the study, with "two instructors each teaching one experimental and one [comparison] class" (p. 222). Analysis of pretest scores on the O'Callaghan function test (used as both a pretest and posttest) revealed no significant differences among the four classes.

The experimental classes used the college text *Intermediate Algebra: A Graphing Approach* (Hubbard & Robinson, 1995), together with Texas Instruments TI-82 graphing handhelds. According to the authors,

> A balance of graphing calculator and traditional algebra work is found in the text, which includes exploration and discovery examples to help guide students to look for patterns and make discoveries. Use of the TI-82 enabled students to explore, estimate, and discover graphically and to approach problems from a multirepresentational perspective. The students had access to the calculators both in class and for homework exercises and tests but not for the O'Callaghan Function Test or the traditional final examination. (pp. 221-222)

The comparison classes used *Intermediate Algebra: Concepts and Applications*, fourth edition (Bittinger, Keedy, & Ellenbogen, 1994), which

> covered the same topics as the experimental text but emphasized memorizing isolated facts and procedures and becoming proficient with paper and pencil calculations. It focused on simplifying and transforming expressions and solving equations. The control group had no known graphing calculator access. (p. 222)

Instructors worked together within each treatment in their lesson planning.

## Research Results

Analysis of the O'Callaghan function test posttest results revealed that students in the experimental group using graphing technology performed significantly better on the test as a whole and on each of the four

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[23] This set of terms and the test used to measure student performance in these areas are taken from the work of O'Callaghan (1998).

components (modeling, interpreting, translating, and reifying).[24] The researchers accounted for these results by arguing that

> Because of the availability of graphing calculators, the graphing-approach curriculum can include examples and problems for modeling real-world situations with functions that would be either too time-consuming or impractical without a graphing calculator. The graphing calculator affords the user both the ability to create equations, tables, and graphs quickly and the facility to move among the representations rapidly. Thus, it can be concluded that the graphing-approach students who used the TI-82 were more comfortable than the traditional students when working with real-world data and situations. The experimental group had become accustomed over the semester to examining functions from different perspectives and accordingly performed significantly better than the traditional students on interpreting and translating questions. (p. 224)

It may be noteworthy that in the original O'Callaghan (1998) study, students using computer-intensive algebra (CIA) curriculum did not outperform students in traditional algebra courses in the area of reification. Hollar and Norwood speculated that the difference between their findings and O'Callaghan's may be accounted for by the fact that "In O'Callaghan's study...students had access to graphing technology only in a lab setting," whereas in the current study students were given access to the graphing-handhelds "during every class meeting as well as for homework and so had more opportunities to explore functions and to examine abstract applications" (p. 225).

Hollar and Norwood found no significant difference between the experimental and the comparison group on the departmental final exam, which "focused mainly on paper-and-pencil calculations and manipulations such as simplifying and transforming symbolic expressions and solving equations" (p. 225). This was interpreted as signifying that students' computational ability was not impaired, while their deep understanding of functions was enhanced through use of curriculum that incorporated handheld graphing technology.

# CONCLUSION

## Summary of Findings

Research cited in this paper shows that handheld graphing technology can have a positive impact on student learning in a range of settings and using a variety of instructional approaches. In particular, the research shows that use of graphing handhelds can have a positive impact both on general skill and understanding of algebra concepts and, more specifically, on student comprehension of functions.

Specific findings of the research cited here can be summarized as follows:

- Use of an Algebra 2 reform curriculum incorporating graphing handhelds can result in significant improvement in overall student performance. (Thompson & Senk, 2001)
- Incorporation of graphing handhelds as an integrated part of an existing precalculus-level curriculum can lead to the following results:
  - Higher achievement among low-performing students
  - Increased student use of graphical solution strategies, a trait linked to improved performance
  - Improved understanding of functions
  - Increased teacher time spent on presentation and explanation of graphs and tables, and on problem solving activities (Harskamp, Suhre, & Van Streun, 2000; Van Streun, Harskamp, & Suhre, 2000)

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[24] "The MANOVA, $F(4, 69) = 4.68$, revealed an overall significant treatment effect at the $p = .01$ level.... Univariate results of ANOVAs on the four components revealed significant differences at the $p = .05$ level in favor of the graphing-approach for modeling, interpreting, and translating, and at the $p = .01$ level for reifying" (pp. 223-224).

- Student use of handheld graphing technology can improve students' skill in creating algebra descriptions of Cartesian graphs (symbolization), even when teachers are inexperienced in the use of graphing handhelds and there is no specified structure for integrating use of graphing handhelds into the curriculum. (Ruthven, 1990)
- An investigation-based approach utilizing graphing handhelds can improve student knowledge of functions by promoting appropriate use of prototype examples. Such an approach can also lead students to include significantly more ideas in their justifications of answers, a trait that correlates in turn to correct student responses. (Schwarz & Hershkowitz, 1999)
- Integrated use of graphing handhelds as part of a course covering standard Algebra 2 content can improve student understanding of functions without diminishing performance on computations performed without use of the handhelds. (Hollar & Norwood, 1999)

It should be noted that each of these findings is based on a single study. No replication studies were found in the review of materials that met the standards for incorporation into this paper. These findings should therefore be taken as preliminary conclusions.

## Uses of Handheld Graphing Technology

Specific uses of handheld graphing technology modeled in the studies described here include the following:

- Creating graphs of functions from an algebraic representation of the function, including modifications of functions to create new graphs. It is likely that students used the capability to generate more than one graph at a time, on the same screen.
- Manipulating graphs of functions by zooming, changing scales, etc.
- Using graphs to find solutions to equations. (It is unclear from the descriptions whether this involved use of the graphing handheld solve feature, the tracing feature, or both.)
- Making tables from an algebraic representation of a function, and using the table setting and scrolling features to explore the tables.
- Checking answers found algebraically. (This may have included use of the evaluate feature.)

All of these represent features of graphing technology that are available with the Texas Instruments TI-8X handheld series, and in particular with the TI-83 Plus. The TI-83 Plus provides an interface that is especially well suited for use of these features in a classroom setting and by less experienced students.

There are additional features of the TI-8X series graphing handhelds (particularly the TI-83 Plus) that mathematics educators have found to be appropriate for use with high school algebra and precalculus-level courses, but that are not mentioned in these studies. Some of these include

- Finding maximums, minimums, and intersections on graphs
- Curve fitting
- Statistical calculations and graphs
- Programming (by students or teachers) of formulas, graphical demonstrations, etc.

## Strategies for Instructional Use of Graphing Handhelds

Studies described in this paper included both general implementation strategies and more specific instructional strategies for using handheld graphing technology. General implementation strategies modeled in these studies include the following:

- Use of written materials (e.g., textbooks) specifically prompting and/or modeling use of graphing technology (Harskamp, Suhre, & Van Streun, 2000; Hollar & Norwood, 1999; Schwarz & Hershkowitz, 1999; Thompson & Senk, 2001; Van Streun, Harskamp, & Suhre, 2000)

- Organization of students in small collaborative groups for problem solving (Schwarz & Hershkowitz, 1999; Thompson & Senk, 2001)
- Ongoing interaction among teachers to share strategies for using graphing handhelds (Ruthven, 1990)

Specific identifiable instructional strategies that are modeled in the studies described in this paper include the following:

- Estimation and checking of algebraic solutions using a graphing handheld (Harskamp, Suhre, & Van Streun, 2000; Hollar & Norwood, 1999; Van Streun, Harskamp, & Suhre, 2000)
- Investigations in which students are prompted to make connections and discover patterns related to functions and their algebraic, numeric, and graphical representations (Hollar & Norwood, 1999; Schwarz & Hershkowitz, 1999; Thompson & Senk, 2001)
- Introduction of new types of functions through modeling with graphing handhelds (Harskamp, Suhre, & Van Streun, 2000; Van Streun, Harskamp, & Suhre, 2000)
- Investigations in which students explore the effects of transforming algebraic representations of functions (Schwarz & Hershkowitz, 1999)
- Exploration of a representative variety of functions in problem contexts that highlight key aspects of functions (Schwarz & Hershkowitz, 1999)

These instructional strategies for using graphing handhelds are modeled in selected activities from the Texas Instruments $T^3$ professional development offerings for Algebra 1, Algebra 2, Connecting Math and Physics, and PreCalculus. As part of these professional development seminars, teachers typically develop lesson plans that may incorporate the instructional strategies listed here and general implementation strategies such as development of written materials for students prompting use of graphing handhelds and small-group collaboration on problem solving.

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