

# Simple Harmonic Motion: $a \propto -s$

## Apparatus

Spring, 100g masses, stand, cbr, calculator, lead, Physics application.

## Method

A spring will be loaded, set in motion and its motion detected. The acceleration will then be plotted against displacement from the balanced position. Simple harmonic motion is characterised by:

$$a = -k s$$

where  $k$  is a constant connected to the mass on the spring,  $m$  and the spring constant,  $k$

First estimate the value of the spring constant for the spring. Measure the length of the spring. Hang a 100g mass from the spring and measure the length of the spring. Repeat for different masses. Enter the measurements for length in L1 and for mass in L2. These must be converted to extension / m and weight force / N.

Subtract the length of the unextended spring from the lengths: at the home screen:

L1 – length\_of\_spring STO L1

If the measurements are in cm then convert to m:

L1 / 100 STO L1

Convert mass from g to kg:

L2 / 1000 STO L2

Convert mass in kg to weight force in N:

L2 \* 9.81 STO L2

Where 9.81 is the value for  $g$ , the acceleration due to gravity.

## App Physics Enter

### Analyse Curvefit LinearL1L2

This fits a line to Force against extension. The value of the slope is the value of  $k$

Now hang the spring from a stand and hang a 200 g mass from it. Place a cbr ultrasonic motion detector underneath the masses so that when the spring is set moving, the masses come no closer than 30 cm from the detector. On the calculator:

## App Physics Enter

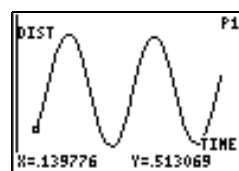
### Set up probes One Motion

Collect data Time graph .03 50

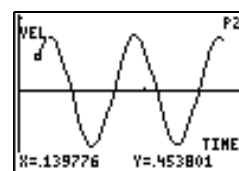
Enter Use time setup Enter

Enter Distance

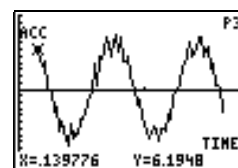
Examine the distance, velocity and acceleration against time graphs. If they show regular wave shape even for acceleration then proceed. If not, repeat until good data are obtained. The acceleration graph need not be perfectly smooth but should show a clear wave shape. Sketch these graphs with the appropriate labels and units.



screen 1



screen 2



screen 3

The smoothest acceleration section may be selected.

## Enter Next No

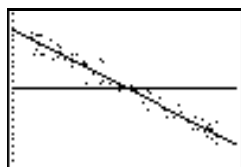
Analyse Select (acc) > Enter > Enter Enter

Exit app (an app can be exited at any stage by pressing ON or Enter and follow exit menus). For calculator instructions see below but, once again,

the program can be used if the lists are first manipulated. Copy time List 1 to List 2 (unused) to preserve this data. Copy displacement List 4 to List 1. This gives displacement in L1 and acceleration in L6. We need acceleration in List 4 so copy L6 to L4. So a plot of L4 against L1 i.e. the displacement plot in the app physics will give the desired plot of acceleration against displacement e.g. at the home screen:

```
L1  STO  L2  Enter
L4  STO  L1  Enter
L6  STO  L4  Enter
```

```
App  Physics Enter
Analyse View graph Displacement
Enter
Analyse      Curve fit Linear L1, L4
Enter
```



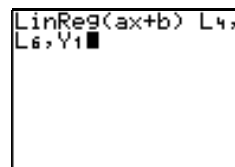
screen 4

While there may be a certain amount of scatter, the plot is clearly linear with a negative slope – just as predicted for simple harmonic motion. The slope should correspond to  $k / m$  but with negative sign. Check this for the measured  $k$  value and known mass ( 0.2 kg ).

For more adept calculator users simply exit the app, set Stat plot 1 to L6 (y axis ) against L4 (x axis ) with dotted data and ensure all Y= are deselected.

```
Zoom 9          Enter
Stat  > Calc Linreg (ax+b) L4, L6,
Vars  Yvars     Function Y1
```

Enter Enter Zoom 9



screen 5

It is also possible to show several graphs on the same screen – try a few. Sketch graphs (labels / units) and note summary data.

## Conclusion

A loaded spring oscillates with simple harmonic motion:

$$a \propto -s$$

The motion detector shows this more clearly and directly than other methods. Many other avenues can be explored in this and similar experiments. The values of  $k$  and  $m$  can be used to assess the accuracy of the fit:  $k / m = \text{slope}$ .

## Option

This is a very rich system. It is possible to hang the spring from a dual range force sensor and to attach an accelerometer to the masses. the accelerometer can be recalibrated using vertically up value as 9.81 and vertically down as  $-9.81 \text{ m s}^{-2}$ . The sensors should be zeroed before use. The motion detector can still be used at the same time as the other sensors. This setup allows the direct plotting of displacement against acceleration measured by accelerometer rather than calculated from displacement and time measurements. Much less scatter is observed in the plot and so in some ways this is a better demonstration of simple harmonic motion. However, there is an argument that the simplicity of the cbr only experiment makes for a compelling activity and it is possible that a whole class set of accelerometers may not be available. More details of setting up force and accelerometer sensors

are in experiment 2. This method also gives a very good demonstration of the variation of acceleration with force.

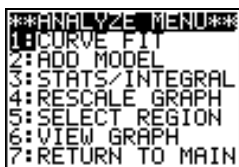
The details of the graphs of displacement, velocity and acceleration make for interesting comparisons and can lead to work on the modeling of these variations with time. Well known equations describe these variations. The various graphs can be modeled. Set up an equation in the calculator, after the experiment but before attempting to use a model:

$$Y1 = A \sin(2\pi BX + C) + D$$

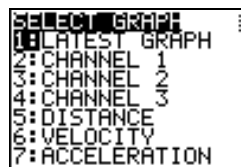
Then return to the application Physics and

### Analyse Add model Distance

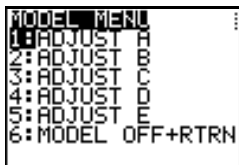
This sets up a model using the equation and values for the parameters A, B, C and D must be given. the graph of the equation is then plotted along with the data. Adjustment of the parameters should match the equation and data.



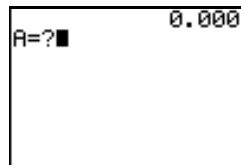
screen 6



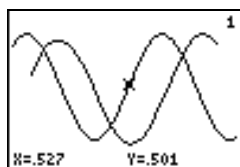
screen 7



screen 8



screen 9



screen 10

Screen 10 shows the two plots beginning to match up. But the process is not an easy one.