EXPLORATIONS™

# Activity 2

## Objectives

- See the derivative as an indicator of increasing/ decreasing function behavior
- See the derivative as an indicator of local maxima/ minima function behavior
- Graphically associate a function with its derivative

## Materials

- TI-84 Plus / TI-83 Plus

# Graphs of Functions and Their Derivatives

## Introduction

When studying relationships between quantities, engineers, scientists, and economists often use information about a rate of change of one quantity relative to a second quantity. In this activity, you will see how a function's rate of change information (in the form of the graph of the derivative) describes many things about the behavior of the function itself.

## Exploration

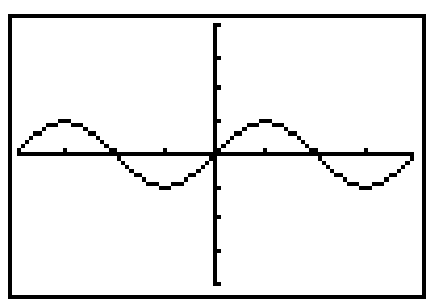
The derivative provides an easy indicator of whether a function is increasing or decreasing. Increasing means the function values are going up as $x$ goes up. Decreasing means the function values are going down as $x$ goes up.

Suppose that a function $f$ is defined on an interval $I$. Let $x_1$ and $x_2$ be any real numbers in $I$ where $x_1 < x_2$.

The function $f$ is increasing over the interval $I$ if $f(x_1) \le f(x_2)$.

The function $f$ is decreasing over the interval $I$ if $f(x_1) \ge f(x_2)$.

The graph of $y = \sin(x)$ is shown.



1. Draw vertical lines on the graph where the function changes from increasing to decreasing and from decreasing to increasing.
2. On your graphing handheld, generate the graphs of $y = \sin(x)$ and $y = \cos(x)$ by selecting **7:Ztrig** in the **ZOOM Menu**. Sketch the graph of $y = \cos(x)$ on the graph of $y = \sin(x)$ on the previous page.
3. Remember that $\cos(x)$ is the derivative of $\sin(x)$. What property of the derivative seems to predict whether the function is increasing or decreasing? Write down the rules that seem to connect the derivative with increasing and decreasing function behavior.

Now you will explore why the sign of $f'$ seems to predict where the function $f$ is increasing or decreasing.

Suppose that $f$ is increasing, $h$ is positive, and $f$ is differentiable for all real numbers.

4. Is $f(x+h)-f(x)$ positive or negative? Why?
5. Is $\dfrac{f(x+h)-f(x)}{h}$ positive or negative? Why?

Suppose that $f$ is increasing, $h$ is negative, and $f$ is differentiable for all real numbers.

6. Is $f(x+h)-f(x)$ positive or negative? Why?
7. Is $\dfrac{f(x+h)-f(x)}{h}$ positive or negative? Why?
8. If $f$ is increasing, is $\lim\limits_{h \to 0^+} \dfrac{f(x+h)-f(x)}{h}$ positive or negative?

   ***Note:*** $h \to 0^+$ *means that* h *approaches zero through positive numbers.*

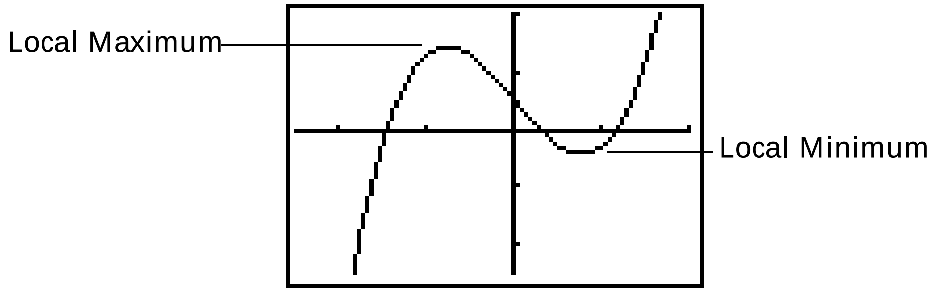
   Explain your reasoning.
9. If $f$ is increasing, is $\lim\limits_{h \to 0^-} \dfrac{f(x+h)-f(x)}{h}$ positive or negative?

   ***Note:*** $h \to 0^-$ *means that* h *approaches zero through negative numbers.*

   Explain your reasoning.

   Similar reasoning can be used to investigate the behavior of $f'$ when $f$ is decreasing.

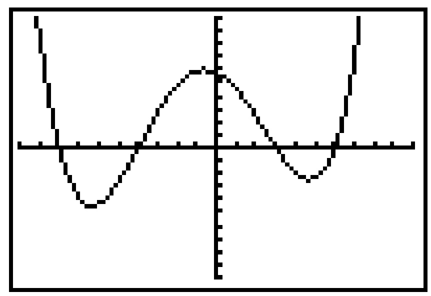
The derivative is also an indicator of whether a function has achieved a local maximum or a local minimum. Graphically, a local maximum is the function value *at the top of a hill* and a local minimum is the function value *at the bottom of a valley*, as the following figure illustrates.




Keep in mind that a function may have many local maxima and local minima.

Input the following expression into **Y1** in the **Y=** editor, and graph it in the standard viewing window: **0.01(x-6)(x-3)(x+4)(x+8)**

Your screen should match the screen shown:



**10.** Draw vertical lines on the graph where the function changes from increasing to decreasing and from decreasing to increasing.

**11.** Suppose that a function $f$ has a local maximum at $x_M$. How does the derivative change as input values change from a little less than $x_M$ to a little more than $x_M$?

**12.** Suppose that a function $f$ has a local minimum at $x_m$. How does the derivative change as input values change from a little less than $x_m$ to a little more than $x_m$?

**13.** To see an approximation of the derivative of this function, input the expression **nDeriv(Y1, X, X)** into **Y2** in the **Y=** editor, and graph it along with **Y1** (get nDeriv by selecting **8:nDeriv** in the **MATH Menu**). Draw this derivative approximation on the graph of **Y1** shown above.

Does the graph of the function and its derivative seem consistent with your answers to Questions **11** and **12**?

**14.** Suppose that a function $f$ is differentiable for all real numbers. What is the value of the derivative at a local maximum or local minimum?

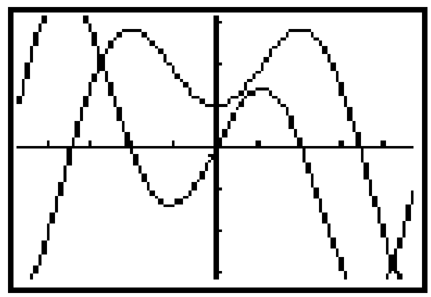
Generate a table with **Y1** and **Y2** selected using the following setup.


**15.** How does the derivative approximation behave from -7 to -6?

**16.** How does the derivative approximation behave from -1 to 0?

**17.** Explain how this behavior relates to the discussion so far.

**18.** The figure shows the graph of a function and its derivative. Which graph is the function, and which is the derivative? Trace over the graph of the derivative with your pencil.



**19.** Match the graphs of the functions in the first column below with the graphs of their derivatives in the second column below.

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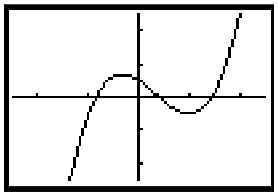
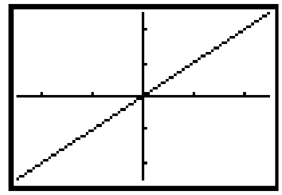


**A**



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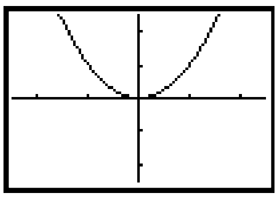
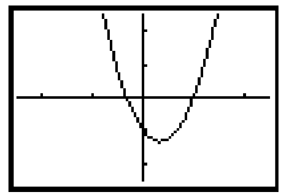


**B**



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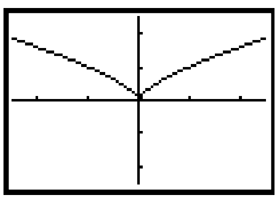
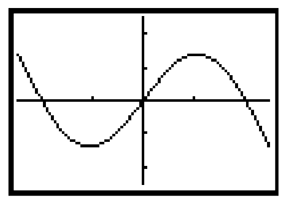


**C**



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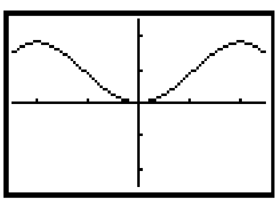
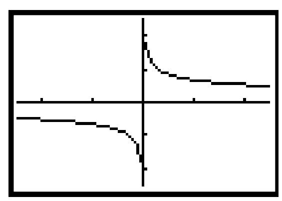
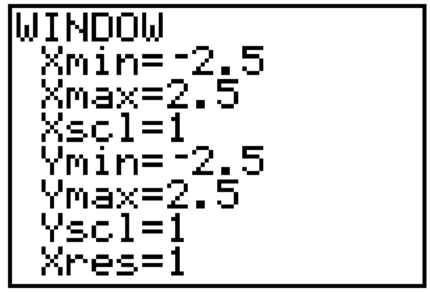


**D**



For the next three questions, work with a partner so that you can compare results on two different graphing handhelds. One person should input the expression $x^3 - 2x^2 - x$ into **Y1**, and the other person should input the expression $x^3 - 2x^2 - x + 2$ into **Y1**. Graph your functions using the viewing window given.




**20.** How are these functions alike?

**21.** How are these functions different?

Now graph an approximation of the derivative of your functions by entering the expression **nDeriv(Y1, X, X)** into **Y2** and graphing it together with your function in **Y1**.

**22.** Write a paragraph that compares the derivatives of these two functions and explains why you got the results that you did. Conclude with a rule that connects the derivative of a function $f(x)$ with the derivative of $f(x) + k$ (where $k$ is a real number).