The Timing Probability

Activity 9

NCTM Standards

- Problem Solving Standard – Solve problems that arise in mathematics and other contexts.
- Connections Standard – Recognize and apply mathematics in contexts outside of mathematics.
- Representation Standard – Use representations to model and interpret physical, social and mathematical phenomena.
- Algebra Standard – Understand patterns, relations, and functions. Approximate and interpret rates of change from graphical and numerical data. Understand and compare the properties of classes of functions.

Materials

- TI-89

Topics in Calculus:

Integrals, Applications of Integrals

Overview:

In this activity, the students will find the probability of a certain thing happening at a distinct time or within a distinct time. The students will find the probability of things in their life happening, which allows the students personalize the activity.

Name:_____ Date: _____

Probability is most commonly associated with winning the lottery, playing a card game, or even being struck by lightning. However, using some basic calculus techniques, you can find the probability of some other interesting facts that occur everyday. In this activity, you will find the probability of certain things happening at a specific or within a specific time.

Probability Density Functions (pdf)

A probability density function *f*(*x*) is a function that represents the probability of a given event that can occur at an infinite number of points in time. The domain of all real numbers (from $-\infty$ to ∞) such that:

$$f(x) \geq 0 \text{ for all } x \text{ and } \int_{-\infty}^{\infty} f(x)dx = 1.$$

Finally, the probability that is related to the function with an interval [*a*, *b*] is:

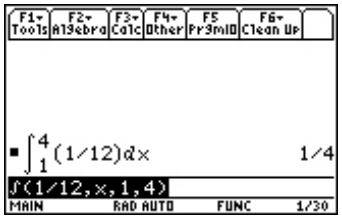
$$\int_a^b f(x)dx$$

This example will show how to solve these problems using the TI-89 Graphing Calculator. Suppose you own a battery-operated clock and you put batteries that are almost dead in the clock. What is the probability that the clock stops between 1:00 and 4:00?

STEP ONE: Create a probability density function *f*(*x*) such that $\int_{-\infty}^{\infty} f(x)dx = 1$. In this case, since there are 12 hours in a day, and since we are looking to find times between certain hours, we will use the function: *f*(*x*) = 1/12. Since the probability must be positive and equal to or between one and zero, we will write the function in the following format:

$$f(x) = \begin{cases} 1/12 & 0 \leq t \leq 12 \\ 0 & otherwise \end{cases}$$

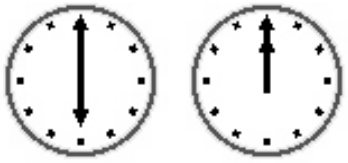
STEP TWO: Now, evaluate the integral at the given interval. Press [2nd] [∫] *function* [,] [X] [,] *lower bound* [,] *upper bound* [)] [ENTER]. The example is shown in the screen to the right. The solution is the probability of the event occurring. Remember that the solution should always be between 0 and 1. If your solution isn't, retrace your steps to find the error.



Why must $\int_{-\infty}^{\infty} f(x)dx = 1$ for *f*(*x*) to be a probability density function?

Find the following probabilities for the following events using the method from the first page.

1. Find the probability that clock stops between the two times indicated.



PDF Function: _____

The probability is: _____

What other method could be used to determine this probability? Explain.

_____

_____

2. Find the probability that a fire drill occurs during this class. (HINT: Let the pdf function equal the 1/number of classes.)

PDF Function: _____

The probability that a fire drill occurs: _____

3. Find the probability that you finish this activity at least ten minutes before class ends.

PDF Function: _____

The probability that you finish early: _____

4. Find the probability that you are **at least** five minutes late to class.

PDF Function: _____

The probability that you are late: _____

5. Find the probability that you are five minutes late to class.

PDF Function: _____

The probability that you are late: _____

What is the difference between the probability you found in questions four and five? _____

_____

6. Find the probability that you arrive at the bus stop at the exact same time as your bus. Before you begin, list the assumptions you will use to find each pdf. You will need to evaluate each pdf and add the probabilities together.

Your PDF Function: _____

Bus's PDF Function: _____

The probability that you arrive at same time: _____

What do you notice about the probability? Find the "odds" of this happening by turning the probability into a ratio. Write the ratio below. Does the ratio make the probability easier to understand? Why or why not?

_____

_____