Definite Integrals and Area Under a Curve

Activity 10

NCTM Standards

- Representation Standard – Use representations to model and interpret physical, social and mathematical phenomena.
- Connections Standard – Recognize and apply mathematics in contexts outside of mathematics.

Materials

- TI-89

Topics in Calculus:

Definite Integrals, Applications of Definite Integrals

Overview:

In this activity the area under a curve will be calculated using definite integrals.

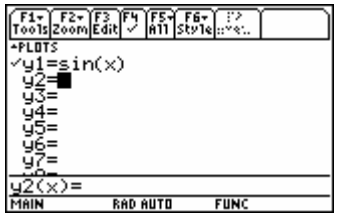
Definite Integrals and Area Under a Curve

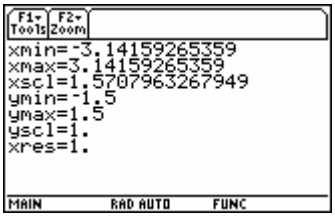
Exercise 1.

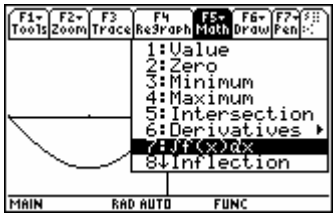
Evaluate the area bounded by the curve $y = \sin(x)$ and the x-axis between
a) $x = -\pi/2$ and $x = 0$.
b) $x = 0$ and $x = \pi/2$.
c) $x = -\pi/2$ and $x = \pi/2$.

Solution:

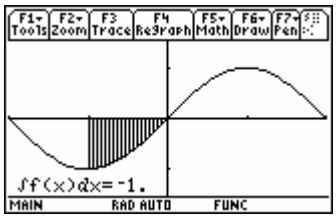
Graphical Interpretation:
Enter the equation in the Y= editor. In the **Window** editor set the window size to $[-\pi, \pi] \times [-1.5, 1.5]$.
GRAPH (♦, **F3**). Choose **F5**, **7:(∫*f*(*x*) *dx*)**.





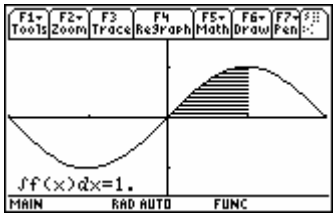


Set the limits of integration as: Lower Limit = $-\pi/2$ and Upper Limit = 0.



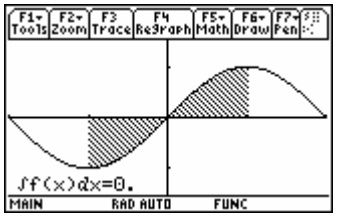
Answer:
The area bounded by the curve $y = \sin(x)$ and the x-axis between $x = -\pi/2$ and $x = 0$ is –1.

b) Set the limits of integration as: Lower Limit = 0 and Upper Limit = $\pi/2$.



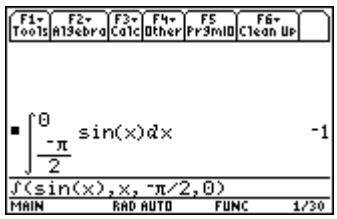
Answer:
The area bounded by the curve $y = \sin(x)$ and the x-axis between $x = 0$ and $x = \pi/2$ is 1.

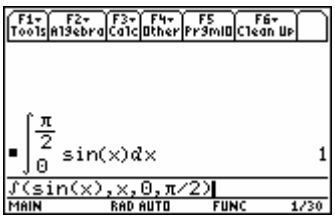
c) Set the limits of integration as: Lower Limit = - π/2 and Upper Limit = π/2.

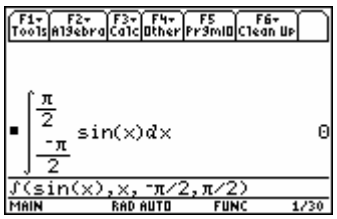


Answer:
The area bounded by the curve $y = \sin(x)$ and the x-axis between $x = -\pi/2$ and $x = \pi/2$ is 0.

Analytical Method:







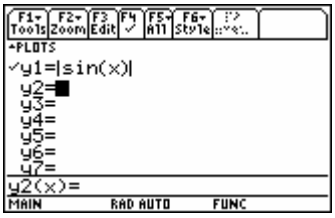
Answer:
a) The area bounded by the curve $y = \sin(x)$ and the x-axis between $x = -\pi/2$ and $x = 0$ is –1.
b) The area bounded by the curve $y = \sin(x)$ and the x-axis between $x = 0$ and $x = \pi/2$ is 1.
c) The area bounded by the curve $y = \sin(x)$ and the x-axis between $x = -\pi/2$ and $x = \pi/2$ is 0.

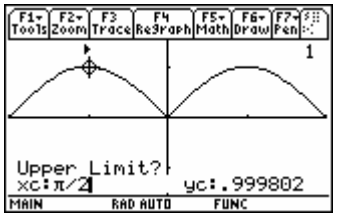
Exercise 2:

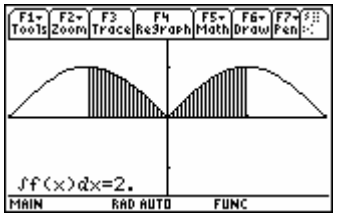
Compute the **total area** bounded by the curve $y = \sin(x)$ and the x-axis between $x = -\pi/2$ and $x = \pi/2$.

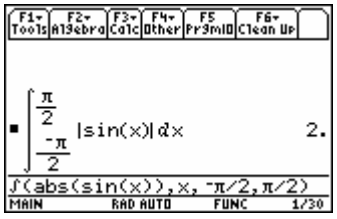
Solution:

To compute the total area, use the absolute value function as illustrated below.









Answer:
The total area bounded by the curve $y = \sin(x)$ and the x-axis between $x = -\pi/2$ and $x = \pi/2$ is 2.

Additional Exercises:

1. To evaluate the area bounded by the curve $y = x\sin(\pi x)$ and the x-axis between $x = -1$ and $x = 1$
   a) graph the function on the relevant interval and interpret the value of the integral as an area or as the negative of an area, and
   b) compute the definite integral.

2. Calculate the total area bounded by the curve $y = x\sin(\pi x)$ and the x-axis between $x = -1$ and $x = 1$.

3. To evaluate the area bounded by the curve $y = 15x^3(x-1)$ and the x-axis between $x = 0$ and $x = 1$
   c) graph the function on the relevant interval and interpret the value of the integral as an area or as the negative of an area, and
   d) compute the definite integral.

4. Calculate the total area bounded by the curve $y = 15x^3(x-1)$ and the x-axis between $x = 0$ and $x = 1$.