Newton's Law of Cooling, Data Analysis and Differential Equations

Activity 3

NCTM Standards

- Problem Solving Standard – Solve problems that arise in mathematics and other contexts.
- Connections Standard – Recognize and apply mathematics in contexts outside of mathematics.
- Representation Standard – Use representations to model and interpret physical, social, and mathematical phenomena

Materials

- TI-89
- CBL2 and temperature probe

Topics in Calculus:
Differential Equations, Data Analysis

Overview:

The students will analyze temperature data in the context of Newton's Law of Cooling.

Newton’s Law of Cooling

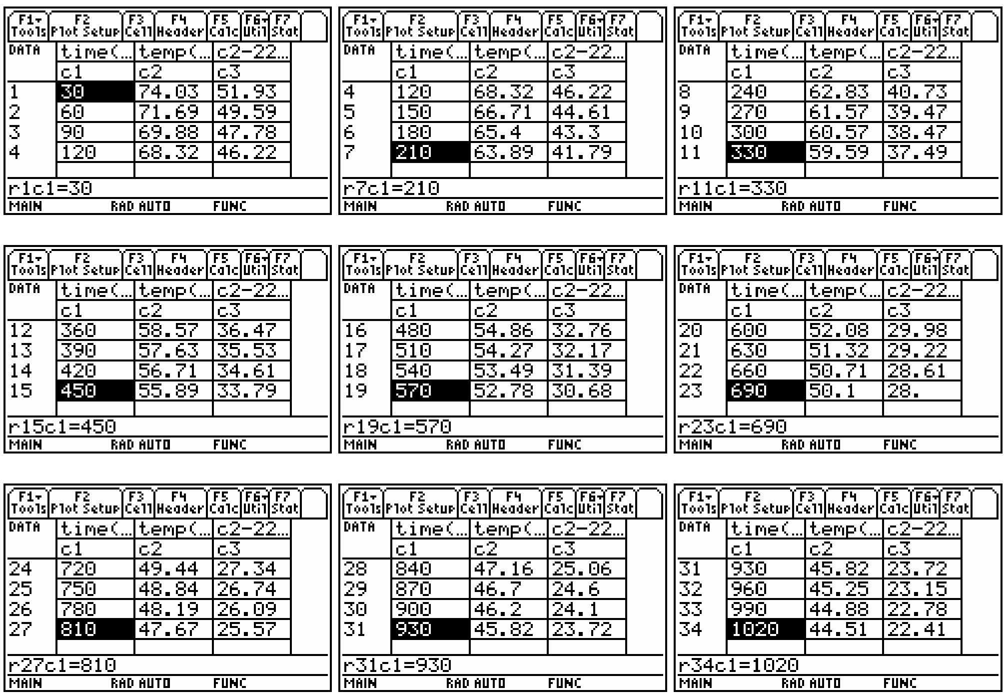
You are planning to serve hot cider at a party. How long will it take the hot cider to reach room temperature?

Exercise 1.

Use Newton's Law of Cooling to model the problem.

Solution:

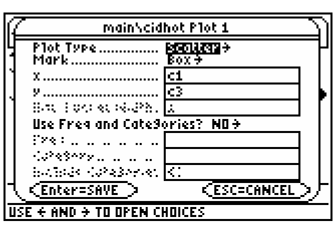
1. With your TI-89 (or TI-92 Plus), a Calculator Based Laboratory (CBL or CBL2), and a heat probe, you are ready to experiment. First you measure the room temperature (ambient temperature) at 22.1 C degrees. Then you set the CBL to measure the temperature every 30 seconds for 34 samples. When the hot mulled cider is ready you insert the temperature probe in it and collect the data in the TI-89 (or TI-92 Plus).

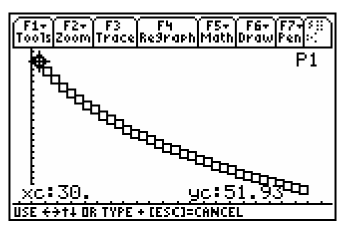


DATA	time(...	temp(...	c2-22...
	c1	c2	c3
1	30	74.03	51.93
2	60	71.69	49.59
3	90	69.88	47.78
4	120	68.32	46.22
4	120	68.32	46.22
5	150	66.71	44.61
6	180	65.4	43.3
7	210	63.89	41.79
8	240	62.83	40.73
9	270	61.57	39.47
10	300	60.57	38.47
11	330	59.59	37.49
12	360	58.57	36.47
13	390	57.63	35.53
14	420	56.71	34.61
15	450	55.89	33.79
16	480	54.86	32.76
17	510	54.27	32.17
18	540	53.49	31.39
19	570	52.78	30.68
20	600	52.08	29.98
21	630	51.32	29.22
22	660	50.71	28.61
23	690	50.1	28.
24	720	49.44	27.34
25	750	48.84	26.74
26	780	48.19	26.09
27	810	47.67	25.57
28	840	47.16	25.06
29	870	46.7	24.6
30	900	46.2	24.1
31	930	45.82	23.72
31	930	45.82	23.72
32	960	45.25	23.15
33	990	44.88	22.78
34	1020	44.51	22.41

r1c1=30, r7c1=210, r11c1=330, r15c1=450, r19c1=570, r23c1=690, r27c1=810, r31c1=930, r34c1=1020

2. Make a scatter plot of the data.





3. Newton's Law of Cooling states that the rate of change of the cooling body with respect to time is directly proportional to the difference between the temperature of the cooling body and the ambient temperature (room temperature = 22.1 C degrees in this case). This statement is represented by the differential equation y' = k(y-22.1) and initial condition y(30) = 51.93 where y' = the rate of change of the cooling body with respect to time;
y = the temperature of the cooling body;
t = time in seconds; and
k = the constant of proportionality.

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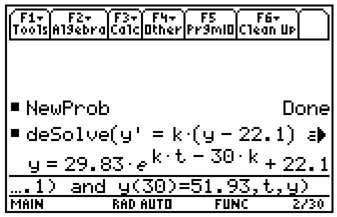
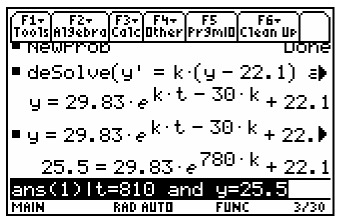
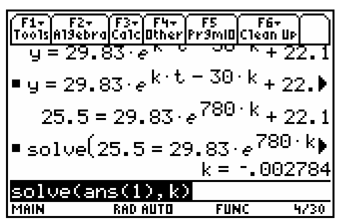
a) Use the command **deSolve** on the Home screen to solve the differential equation.



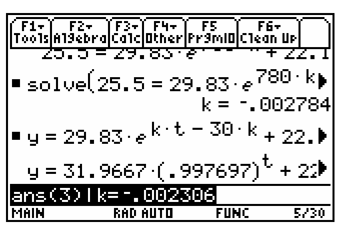
b) Use another data point to obtain an equation with one variable, k.



c) Solve for k.



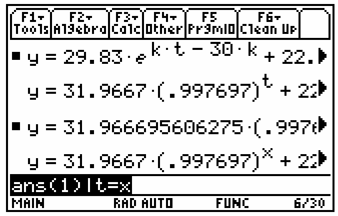
d) Substitute this value for k in the solution to the differential equation.



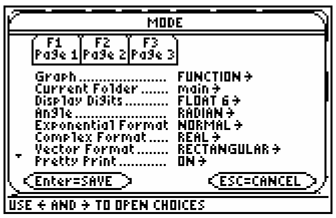
Answer: The mathematical model is $y1(t) = 31.9666956(.9976966)^{t} + 22.1$

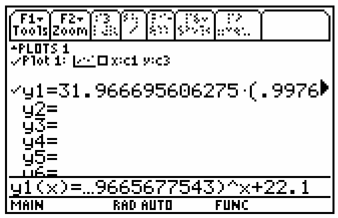
Exercise 2:

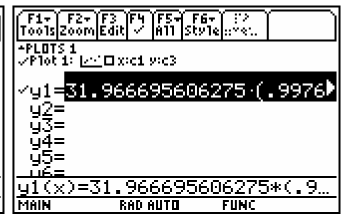
1. Substitute x for t so that you can enter the solution of the differential equation in **Function MODE.**



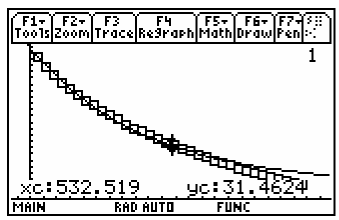
2. Copy this equation into the **Y=** editor. Make sure the TI-89 is in **FUNCTION MODE**.







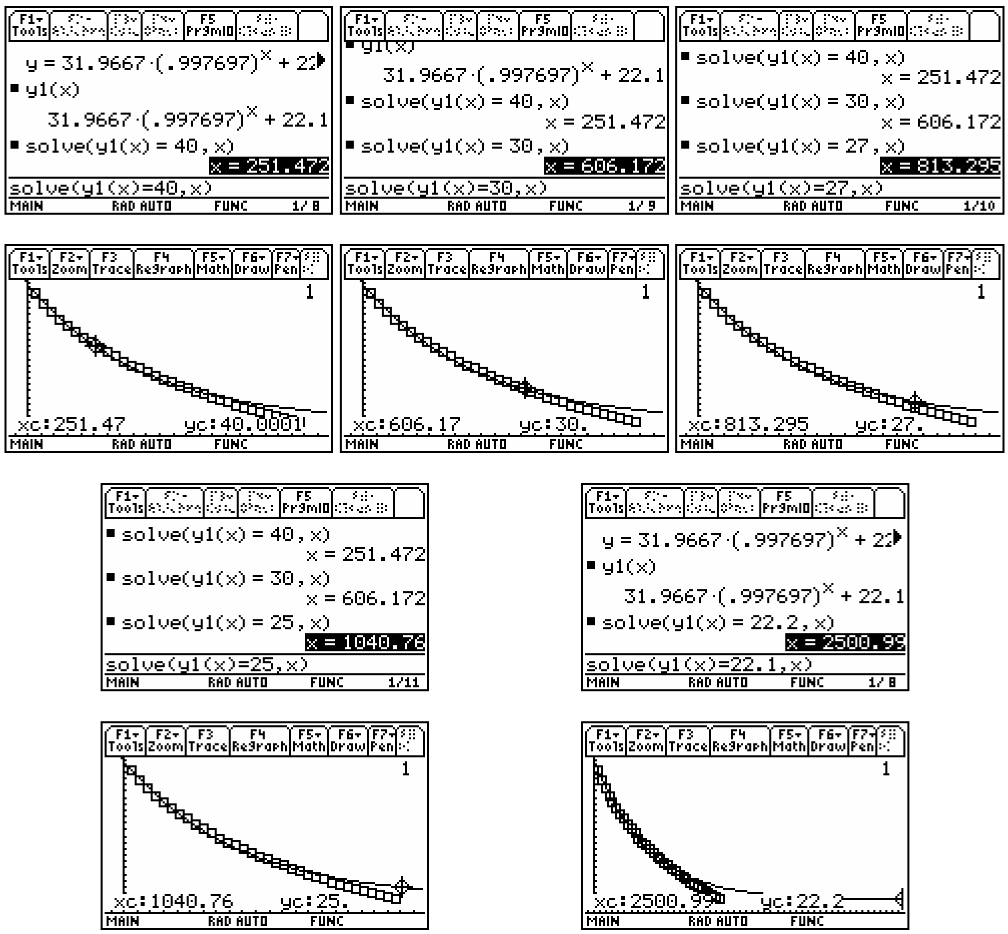
3. Use **F2 Zoom**, **9:ZoomData** to graph the scatter plot of the data and the solution of the differential equation, y1(x) = 31.9666956(.9976966)^x + 22.1 in this case.



Exercise 3.

Use the model, y1(x) = 31.9666956(.9976966)^x + 22.1, to predict when the cider will be
40 C,
30 C,
26 C,
25 C, and
22.2 C degrees.

Solution:



Answer: The model, y1(x) = 31.9666956(.9976966)^x + 22.1, predicts that
the cider will be 40 C degrees after 251.472 seconds or 4.1912 minutes.
the cider will be 30 C degrees after 606.172 seconds or 10.1029 minutes.
the cider will be 27 C degrees after 813.295 seconds or 13.5549minutes.
the cider will be 25C degrees after 1040.76 seconds or 17.346 minutes.
the cider will be 22.2 C degrees after 2500.99 seconds or 41.68 minutes.

Exercise 4.

Compare the model with the data points.

Answer:

It appears that the model is a good fit up to 810 seconds (13.5 minutes). Many factors need to be considered when performing an experiment. Perhaps there was a drop in the room temperature during the experiment.