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Introduction to DERIVETM 6

**A book for learning how to use DERIVE 6
with hints on how to teach with it**

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Introduction to DERIVE 6

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Preface

This book resulted from a desire to make DERIVE 6 easily and quickly accessible, especially for teachers.

Many thanks to Albert Rich and Theresa Shelby, the principal authors of DERIVE 6, for their continuous support during the writing of this book.

Many thanks to Patricia Littlefield and David Stoutemyer who polished the language of this text.


Bernhard Kutzler & Vlasta Kokol-Voljc, July 2003

Introduction

DERIVE is a mathematical computer program. It processes algebraic variables, expressions, equations, functions, vectors, and matrices like a scientific calculator processes floating point numbers. DERIVE can perform numeric and symbolic computations, algebra, trigonometry, calculus, and plot graphs in two and three dimensions. The main strengths of DERIVE are symbolic algebra and powerful graphics. It is an excellent tool for doing and applying mathematics, for documenting mathematical work, and for teaching or learning mathematics.

For a teacher and student, DERIVE is the ideal tool for supporting the teaching and the learning of mathematics. DERIVE enables new approaches in teaching, learning, and understanding mathematics by providing seamless integration of numeric, algebraic, and graphic capabilities. You will find that many topics can be treated more efficiently and effectively with DERIVE than by using traditional methods. Many problems that require extensive and laborious training at school can be solved with a single keystroke using DERIVE: it eliminates the drudgery of performing long mathematical calculations. While DERIVE takes the burden of doing the mechanical/algorithmic parts of solving a problem, students can concentrate on the mathematical meaning of concepts. Instead of teaching and learning boring technical skills, teachers and students can concentrate on the exciting and useful techniques of problem solving and on the beauty of mathematics. DERIVE has proven to be highly supportive for the development of advanced mathematical concepts.

For an engineer, DERIVE is the ideal tool for fast effective access to numerous mathematical operations and functions and for visualizing problems together with their solutions in various ways. If you use DERIVE for your everyday mathematical work you will find it a tireless, powerful, and knowledgeable mathematical assistant that is easy to use.

This book teaches you how to use DERIVE 6 by independent study. Install DERIVE 6 on your computer. Starting with the first chapter you will learn step by step how to use the program. Follow all instructions and examples. The text leads you through several mathematical topics to learn how to solve mathematical problems with DERIVE. Many of the examples also provide ideas for using DERIVE during teaching, some of them are explained in more detail in “Educator’s footnotes”. Paragraphs starting with the symbol  give instructions about what you should do on your computer. Hundreds of screen images ensure that you will not get lost on this journey.

Through solving typical mathematical high school level problems, you will learn to handle DERIVE 6 so you may use the program for everyday purposes and for teaching or learning mathematics. You will learn how to use the major commands, keys, and functions. At the end of each chapter you will find a summary of the features learned in the chapter. The Reference Guide accompanying this book is an overall summary of commands, keys,

functions, and utility files. The index at the end is useful if you need to locate a particular portion of the text.

All you need to run DERIVE 6 is a PC compatible computer with WINDOWS 2000 or WINDOWS XP. Compatibility with WINDOWS 98 and WINDOWS ME will be added soon.

It is assumed that you know how to use the computer and WINDOWS operating system. The screen shots in this book were produced from DERIVE 6 running on WINDOWS XP. If you are running DERIVE 6 on WINDOWS 2000, some of the screens may appear slightly different.

This book introduces those features and functions that are required for routine use of DERIVE 6. There is more functionality than can be described here. This book is *not* a reference manual for DERIVE. A complete reference to all features is included with the software as online help. Some of the chapters give examples of how to use the online help.

This text was written for DERIVE Version 6.00. If you use a later (updated) version of Derive 6, some of the screen images may be different.

Note to DERIVE 5 users:

If you are familiar with DERIVE 5 and have read our book “Introduction to DERIVE 5”, you will find both the software and this manual an (upward-compatible) extension. Here is a list of the major new features and where they are described in this text:

- Display Step Mode (Ch 13)
- Interconnectivity with TI-89, TI-92+, and Voyage 200 (Ch 16)
- Customizability of menus, toolbars, and short-cut keys (Ch 17)
- Slider bars to animate expression plots (Ch 6)
- Automatic plot labeling (Ch 6)
- Rotate 3D-plot with mouse (Ch 7)
- Improved and extended online help (Ch 9)
- Multi-line editing (Ch 9)
- Derive Unicode font (Ch 1)
- Parentheses matching (Ch 4)
- New options for connected point plots (Ch 9)
- New options for 3D data-point plots (Ch 9)
- Control 3D plot mesh lines (Ch 7)

Chapters 13, 16, and 17 are new. Most other chapters contain updates or additions. For your convenience we added vertical bars to these parts of the text. (See the vertical bar added to this paragraph as an example.)

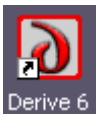
Have fun reading and discovering.

Chapter 1: First Steps

DERIVE makes it easy to perform mathematical operations: enter an expression, apply a command, and a new expression is obtained. All expressions can be used for new computations – just like on a piece of paper. This chapter teaches the basic techniques of using DERIVE 6. For simplicity we will abbreviate this as DERIVE throughout this text.

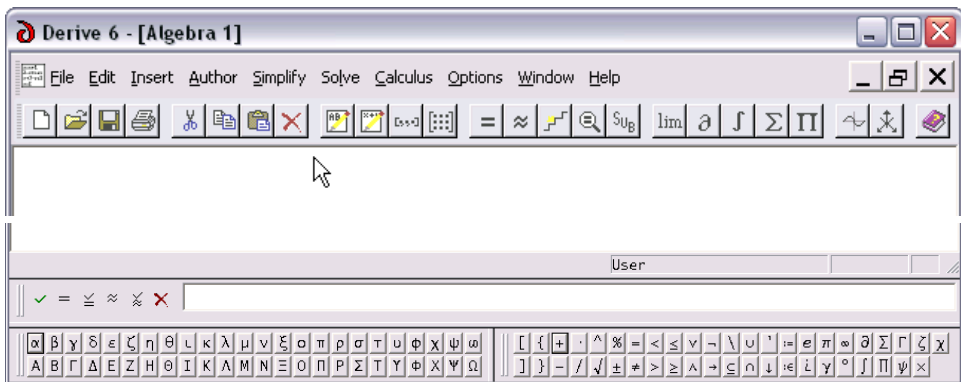
This text assumes that you use a factory default DERIVE. Only then (and when you use WINDOWS XP) will your screen images fully match those in this book. If you just installed DERIVE, it is a factory default version. If you use a version of DERIVE that was used by someone else earlier, we recommend that you turn it into a factory default version now.

Appendix B gives instructions on how to do this.




- Start DERIVE by double clicking on the DERIVE icon. If there is no DERIVE icon on your computer's desktop or task bar, you probably will find DERIVE on the **Start** menu or via **Start>Programs**.

The following screen appears after a few seconds:

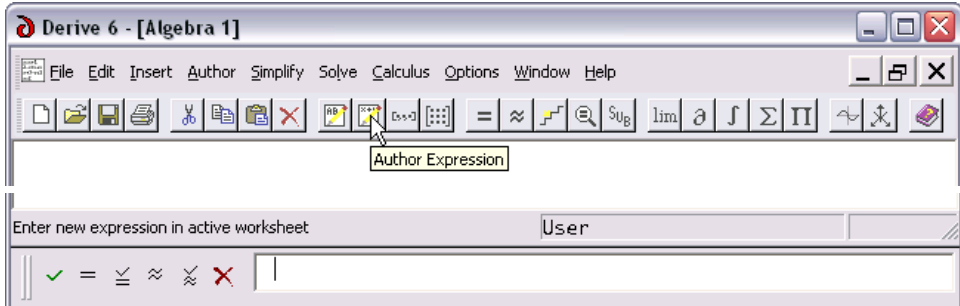


The DERIVE screen comprises (from top to bottom):


- the Titlebar
- the Menubar
- the Command Toolbar
- a (currently empty) Algebra Window, also called the View
- the Status Bar
- the Expression Entry Toolbar, also called the entry line
- the Greek Symbol Toolbar and the Math Symbol Toolbar

Work with DERIVE by entering expressions and applying commands, thus creating a worksheet. After starting DERIVE, the system is ready to accept user input via the Expression Entry Toolbar, as is indicated by the blinking cursor in the toolbar's entry field. Input mode can be activated with the Command Toolbar's tenth button from the left, labeled .

- Learn more about the button  by moving the mouse pointer onto it.



The message **Author Expression** below the cursor is the button's title. The Status Bar message Enter new expression in active worksheet is the button's function description.

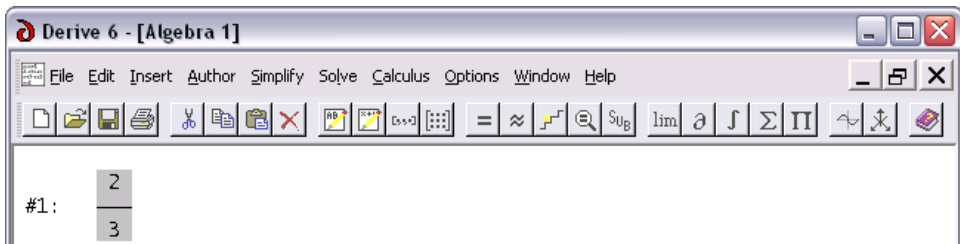
- Prepare for entering an expression: move the mouse pointer onto , and then click (i.e. press and release) the left mouse button.



- Enter the fraction: $2/3$



- End the input with the 'Enter'-key (C).

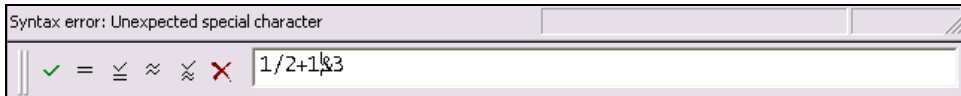


DERIVE displays this expression as a fraction with a horizontal line, a numerator, and a denominator, i.e. in “2-dimensional” output format, as opposed to the “1-dimensional” or “linear” input format used for entering the number. The expression's unique label number, #1, is shown to the left of the expression. DERIVE is again ready to accept the next input, i.e. input control (the *focus*) is still in the entry line. Also observe that a copy of the input is still in the entry field and is completely highlighted. This has the same meaning as in text editors and word processors. You can remove the highlighting with the right arrow key, and then edit the string of symbols, or you can replace the marked string by typing new symbols.



Replace the last input by $\frac{1}{2} + \frac{1}{3}$ with an intentional typographical error:

Enter $1/2+1&3$ (C).



When a syntax error is detected, the cursor is moved to the location of the error and the cause of the error is displayed in the Status Bar's first pane. In the above example DERIVE discovered an unexpected special character "&". In some cases (for example, when entering an opening parenthesis instead of the division symbol) there are several errors possible, and DERIVE can only guess.

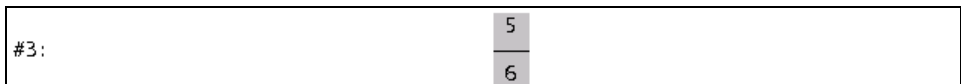
Update the input to $1/2+1/3$: use the (Del) key (or the right arrow key (Æ) followed by the backspace key (æ_)) to delete the incorrect character, and then type the division operator. Conclude with (C).



The expression and its label, #2, are displayed. The new expression is highlighted in reverse video. Expression #1 is no longer highlighted.

If you mistyped the input and want to delete the highlighted expression for a retry, use (Esc) to move the focus into the algebra window, use the 'Delete' key (Del) to delete the highlighted expression, and then use the **Author Expression** button to move the focus back into the entry line. An alternative technique for replacing an expression will be explained in Chapter 2.


Simplify expression #2 using the Command Toolbar's **Simplify** button .

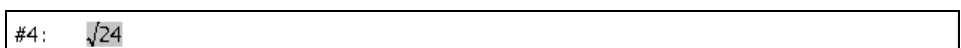


The result becomes the next expression with the label #3. By default, simplified expressions are displayed centered. This makes it easy to distinguish between an entry and a result. As with many other behaviors of DERIVE, this can be customized if desired.

Even after using the **Simplify** button, the focus still is in the entry line. Enter the next expression, $\sqrt{24}$. To enter the square root symbol, use the respective button on the Math Symbol Toolbar:



Enter $\sqrt{24}$ as:  24 (C)



- ☞ Simplify using $\boxed{=}$.

#5: $2\sqrt{6}$

This is different from what an “ordinary” calculator would produce. A mathematician once asked: “*How do you recognize a mathematician?*” and suggested the following answer: “*A mathematician considers expression #5 a beautiful result.*” Most students strive to replace such an expression by the corresponding floating point approximation. DERIVE can do this as well. Highlight expression #4 so that you can apply a different command to it:

- ☞ Highlight expression #4 by moving the mouse pointer anywhere in the row occupied by the expression, and then click the left mouse button.

#4: $\sqrt{24}$

Selecting an expression with the mouse button is one technique of highlighting it. An alternative technique is first to move the focus into the algebra window (if necessary) using the (Esc) key, and then use the cursor keys ($\frac{1}{2}$) or ($\frac{1}{4}$) to move the highlighting one expression up or down.

- ☞ Approximate using the Command Toolbar’s **Approximate** button $\boxed{\approx}$.

#6: 4.898979485

While an expression is highlighted, the Status Bar’s second pane shows the automatically generated expression annotation. The third pane shows the computing time in case the expression was obtained as a result of a computation. For expression #6 this is:

Approx(#4) 0.000s

The automatically generated annotation explains how the expression was obtained. Approx(#4) means that the expression was obtained by applying the **Approximate** button (or command) to expression #4. The computation time displayed in the third pane, 0.000s, indicates that the calculation took less than 0.001 seconds. (The time may be different on your computer).

- ☞ Highlight expression #4, ...

User

- ☞ ... and then expression #5.


Simp(#4) 0.000s

The annotation of expression #4, User, means that it was entered by the user; the annotation of expression #5, Simp(#4), indicates that the expression was obtained by applying the **Simplify** button (or command) to expression #4. The first pane is always available for messages associated with a menu item, button, or command status.


DERIVE worksheets also can include text and other objects. The easiest way of entering text is via the Command Toolbar’s **Insert Text** button \boxed{AB} . New expressions are added at the end of the worksheet. Other objects (including text objects) are added after the highlighted object. To insert a text object above $\sqrt{24}$, first highlight the object that is now above it.

- Highlight expression #3.

#3: $\frac{5}{6}$

- Display a function description of the **Insert Text** button  by moving the mouse pointer onto it.

Insert new text object in active worksheet S imp(#2) 0.000s

- Insert a text object by clicking on the **Insert Text** button .

#3: $\frac{5}{6}$

#4: $\sqrt{24}$


Highlighting of a text object is indicated by a frame around it. The blinking cursor inside indicates text editing mode.

- Enter the text: We compute the square root of 24:

We compute the square root of 24:

A text object allows simple text editing similar to what you can do in standard text editors. Later you will learn how to change the font size, alignment, color, etc.

As the next example compute 1234^{56} . Due to the previous activity, the focus now is in the algebra window. Before you can enter another expression, move the focus into the entry line and delete any characters therein:

- Enter 1234^{56} by using the **Author Expression** button , and then type the respective string of digits followed by (C) . The exponentiation operator \wedge can be found on both the keyboard and the Math Symbol Toolbar (it is the fifth symbol from the left in the first row.)

#7: 1234^{56}

- Simplify using .

#8: 12991190255487145194103208439623513775465782010127392384379012704624~
259433055094648925678485362472902010613951564738491094492118652386~
5849056275359066262352911682504769929216

This is a very big number. There are at least two methods to find out the number of digits, first, you can count them, second, you can approximate the number.

- Approximate using .


#9: $1.299119025 \cdot 10^{173}$



The answer is displayed in scientific notation. Since the count of whole digits is one more than the power of 10, the number has $173+1 = 174$ digits.

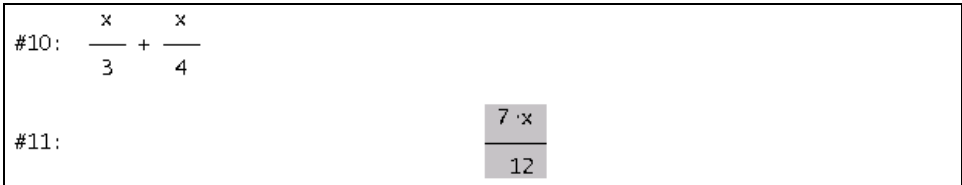
In the next exercise, you will learn a different technique of entering expressions by using the buttons preceding the entry field.

- ☞ Type into the entry line $x/3+x/4$, this time **without concluding with (C)**.




Note the six buttons left of the entry field. The usual technique of moving the mouse pointer onto a button reveals the first one, , as the **Author Expression** button. Selecting this button has the same effect as concluding the input with the (C) key. Try it:


- ☞ Enter the above expression with , then simplify as usual using the Command Toolbar's **Simplify** button .

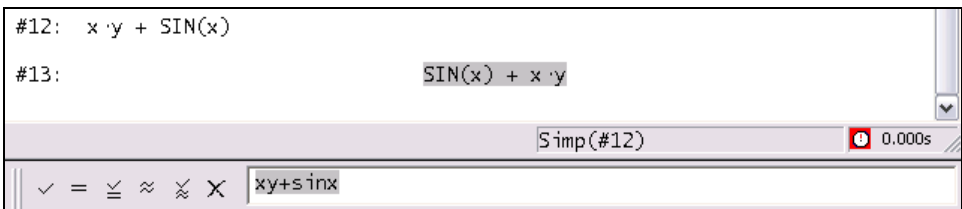




Unlike ordinary calculators, DERIVE can perform non-numeric (symbolic, algebraic) computations such as simplifying expression #10 into expression #11.







To simplify an expression immediately, i.e. without displaying the unsimplified expression, type the expression into the entry line, and then select the entry line's **Simplify** button . The annotation of such an entry is `Simp(User)`.

For the next example use the Expression Entry Toolbar's third button, :

- ☞ Enter and simplify $xy + \sin x$ by typing $xy+\sin x$ and then use the entry line's **Author and Simplify** button .



This button produced two expressions, #12 and #13, and has the same effect as entering the unsimplified expression with (C) or , and then simplify it with . It is, therefore, a convenient shortcut for the frequently used “enter and simplify.” This example also shows how convenient fast input is in DERIVE. You can enter expressions just as you would write them on paper. For ‘x times y’ simply enter xy . No multiplication operator is needed between x and y . For ‘Sine of x ’ simply enter $\sin x$. No parentheses are needed around x .

The Expression Entry Toolbar has buttons for entering () , simplifying () , entering & simplifying () , approximating () , and entering & approximating expressions (). The sixth button  is for deleting all characters in the entry line.

The simplified expression #13 differs from the unsimplified expression #12 only in the order its terms are displayed. While unsimplified expressions are displayed similar to how they were entered (except for the 2-dimensional pretty print format), simplified expressions are displayed in a standardized format using a certain term ordering.

•

Back to how simple it is to enter expressions. A consequence of the convenient fast input, such as $xy + \sin x$ for $x \cdot y + \sin(x)$, is that variable names can consist of only one character (such as x and y). This suffices most of the time, but if you need multicharacter variable names, DERIVE allows this, too (for example *time* or $x12$). Multicharacter variable names are controlled via **Options>Mode Settings** and will be explained in Chapter 15.

Clearly, you cannot omit all parentheses. For example, you will need to parenthesize the denominator to enter a rational expression such as $\frac{2}{x+1}$. If the parentheses are omitted in

this example, the resulting expression has a different meaning.

Enter: $2/x+1$



Oops – the expression on the screen looks different from the intended expression! DERIVE applies operations in the conventional order, for example multiplication and division before addition and subtraction. As you can see from the example, the 2-dimensional screen display of an input provides you with valuable feedback about the correctness of your input.¹

When correcting the most recent input, you can take advantage of the fact that a copy of the most recent input and the focus are still in the entry line.

To edit the expression use the right arrow key (\rightarrow) to remove the highlighting. Change the input to $2/(x+1)$ by adding the parentheses, and then enter the expression with (C).



Now it looks correct. Since you don't need expression #14 any more, delete it:

¹ **Educator's footnote:** A very simple educational exercise with DERIVE consists of asking the students to input expressions given to them on the chalkboard or a piece of paper. Because DERIVE features two-dimensional output of expressions, the students obtain an immediate feedback. If the expression on the screen looks different from the one on the chalkboard or paper, then the input was incorrect, and they must try again. When the teacher lets students input expressions of increasing complexity, they learn how to “linearize” expressions by trying and experimenting, and learn to understand the structure of expressions. In this way, they improve their competence in recognizing structures, which is one of the basic mathematical skills important in many areas.

- ☞ Prepare for deletion: highlight expression #14 either with the mouse or with the keyboard's arrow keys after moving the focus into the algebra window using (Esc).


#14: $\frac{2}{x} + 1$

- ☞ Delete expression #14: use the **Delete Object** button  or press the (Del) key.

#14: $\frac{2}{x + 1}$

The expression that was expression #14 disappears. The expression that was expression #15 has become expression #14. By default, automatic renumbering adjusts expression numbers so that they begin with #1 and have no gaps. (This default can be changed via the **Options>Display>Renumber Expressions** command.)

Errors such as omitting a whole pair of parentheses may change the meaning of an expression, as was the case in the previous example. If only one parenthesis is omitted, the input becomes a meaningless character string, and DERIVE will issue a warning in form of an appropriate syntax error message:

- ☞ Enter $4x-1/x-5)$ after moving the focus into the entry line with .

Syntax error: Unexpected delimiter User

✓ = ≤ ≈ ≉ ✗ $4x-1/x-5)$

DERIVE attempts to position the cursor in front of the expected error. Since a superfluous closing parenthesis is obvious, whereas an intended position of a missing opening parenthesis is not, the cursor is placed before the extra closing parenthesis, with a corresponding message. Depending on how the expression should look, you either have to delete the closing parenthesis or insert an opening parenthesis somewhere before it. For the above example there are six possible repairs:

input	$4x-1/x-5$	$4x-1/x-(5)$	$4x-1/(x-5)$	$4x-(1/x-5)$	$4(x-1/x-5)$	$(4x-1/x-5)$
output	$4x - \frac{1}{x} - 5$	$4x - \frac{1}{x} - 5$	$4x - \frac{1}{x-5}$	$4x - \left(\frac{1}{x} - 5\right)$	$4\left(x - \frac{1}{x} - 5\right)$	$4x - \frac{1}{x} - 5$

To choose the third variant insert an opening parenthesis after the division operator.

- ☞ Edit the input string to $4x-1/(x-5)$ and then press (C).²


#15: $4 \cdot x - \frac{1}{x - 5}$

² **Educator's footnote:** This is another example for an elementary educational use of DERIVE. Ask students how many different expressions they can generate by inserting 1, 2 (or more) pairs of parentheses into a valid string of characters. This is another excellent exercise to help students gain an understanding of the structure of expressions.

When working with DERIVE, focus can be either in the entry line or in the algebra window (also called the “View”). When focus is in the entry line, (Esc) will move focus into the View. When focus is in the View, the **Author Expression** button or its hot key equivalent, (F2), moves it into the entry line. Another method to move focus is using the mouse. Focus is where one last moved the mouse pointer to and then pressed the left mouse button. Focus is visually indicated by the presence or absence of the cursor in the entry line.


- ☐ Ensure that focus is in the entry line by moving the mouse pointer into the entry line’s entry field, and then click with the left mouse button.




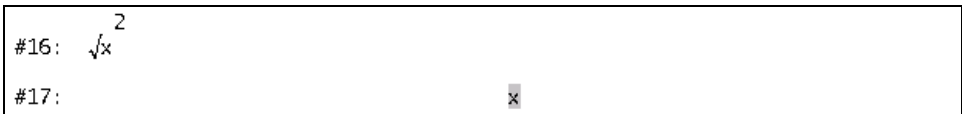
The disadvantage of this method is that it removes highlighting if there was any, so now you cannot simply replace the old input with a new one by starting to type the new input string. You can clear the entry line with the Expression Entry Toolbar’s **Delete All** button  or its hot key equivalent, (Ctrl)+(Del). If you don’t want to delete all characters but merely highlight them, perform the following:

- ☐ Highlight the contents of the entry line with the tab key (\dot{y}).



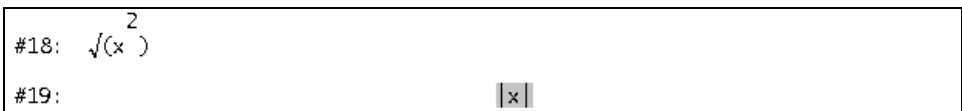
Enter and simplify $\sqrt{x^2}$. It is up to you to either use the ‘Enter’ key followed by the **Simplify** button or to use the entry line’s **Enter and Simplify** button. The square root symbol $\sqrt{\quad}$ can be obtained from the Math Symbol Toolbar  or entered as (Ctrl)+(Q).

- ☐ Type $\sqrt{x^2}$ and then press (Ctrl)+(C). This is the same as , i.e. this is a simple way to perform an “enter and simplify” operation without using the mouse.




As an alternative, introduce a pair of parentheses around x^2 .

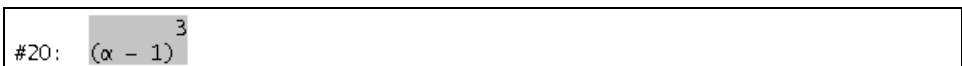
- ☐ Enter and simplify: $\sqrt{(x^2)}$



The last two examples demonstrate the importance of using parentheses to differentiate between $\sqrt{x^2}$ (meaning $(\sqrt{x})^2$) and $\sqrt{x^2}$ (meaning $\sqrt{(x^2)}$). These examples show how carefully DERIVE simplifies expressions.

The third power of $\alpha - 1$ is entered as follows:

- ☐ Enter $(\alpha - 1)^3$. (Insert Alpha with the Greek Symbol Toolbar button )

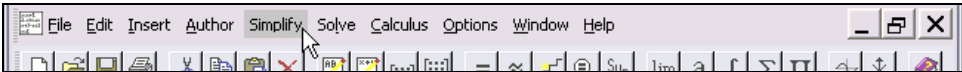


- Try to expand expression #20, first by simplifying with $\boxed{=}$.

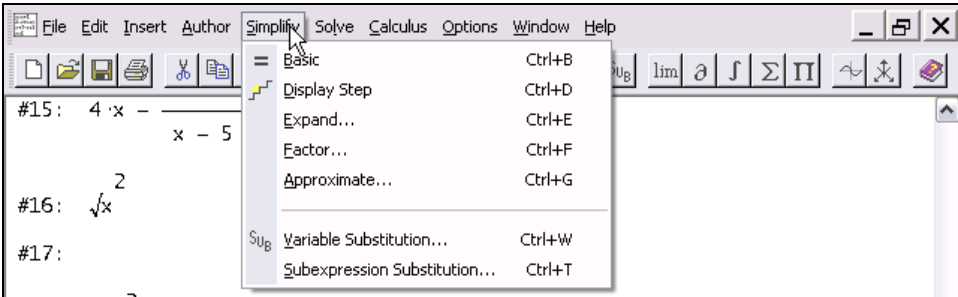
#21: $(\alpha - 1)^3$

This did not change anything. Now you have an opportunity to apply one of those commands for which there is no equivalent Command Toolbar button.

- Prepare for opening the **Simplify** menu by moving the mouse pointer above the Menu Bar's **Simplify** command.

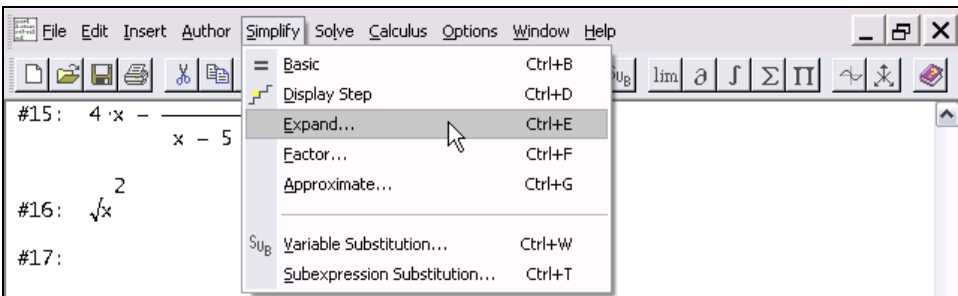


- Open the **Simplify** menu by clicking the left mouse button.

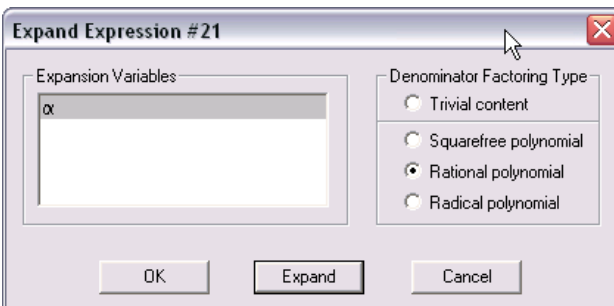


This menu offers several commands. The **Expand** command is appropriate for expanding an expression.

- Select this command by moving the mouse pointer above the word **Expand ...**



- ... and then invoke the command by clicking on it with the left mouse button.



DERIVE opens the **Expand Expression** dialog box. You will obtain similar dialog boxes with all commands that require specification of parameters. The above dialog box requires the specification of the expansion variable and the amount of expansion (determined in the field **Denominator Factoring Type**). Often it is enough to accept the default specifications and immediately exit the dialog box with the ‘Enter’ key or by clicking the default button, which here is (Expand). (The default button is the one prominently displayed.) Use the (Cancel) button or the (Esc) key to cancel the command. Use (OK) if you want an unsimplified application of the EXPAND function.

- ☞ Perform the expansion with the suggested parameters by using (Expand) (either press (C) because this is the default button or click on (Expand)).

#22:	$\alpha^3 - 3 \cdot \alpha^2 + 3 \cdot \alpha - 1$
------	----------------------------------------------------

A keyboard alternative for selecting the **Expansion** command from the **Simplify** menu is the following standard WINDOWS technique: (Alt)+(S) opens the **Simplify** menu (use (S) because of the underscore under the letter S in **Simplify**), and then press (E) (again the letter with the underscore, but now without the (Alt), which is used only to open menus.) This technique works for all menu commands.

For all buttons from the Command Toolbar there exist corresponding menu commands. Use commands for the next example. Enter, simplify, and then approximate $\sin(\pi/4)$:

- ☞ To enter the above expression, select the **Author>Expression** command, then type $\sin(\pi/4)$ (C). (Obtain π from the Math Symbol Toolbar. Please be aware of the difference between the Math Symbol Toolbar button π , which denotes the area of the unit circle, and the Greek Toolbar button π , which denotes the lower case Greek letter. Observe the different appearance of the two toolbar symbols. The area of the unit circle has one slanted vertical “leg”.)

#23:	$\text{SIN}\left(\frac{\pi}{4}\right)$
------	----------------------------------------

- ☞ Simplify expression #23 with the **Simplify>Basic** command.

#24:	$\frac{\sqrt{2}}{2}$
------	----------------------

This is another “beautiful” result. Before computing an approximation, add an appropriate comment to the worksheet in form of a text object.

- ☞ Insert a text object with the **Insert>Text Object** command, and then type:
The following is an approximation of $\sin(\pi/4)$.
(Get the symbol π from the Math Symbol Toolbar.)

The following is an approximation of $\sin(\pi/4)$.

- ☞ (Try to) conclude the input with (C).

The following is an approximation of $\sin(\pi/4)$.

The 'Enter'-key, used from within text editing mode, added an extra line to the text object. This is not what was intended.

- ☞ Delete the extra line using the backspace key (æ_).

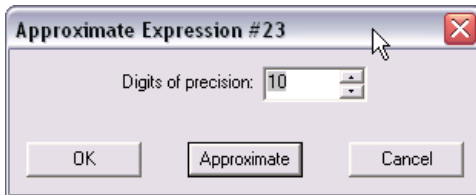
The following is an approximation of $\sin(\pi/4)$.

Note that while DERIVE is in text editing mode, you have no access to certain buttons and menu commands as you can see in the Command Toolbar. The inaccessible buttons and menu commands appear dimmed. For example, the **Approximate** button is not available in text editing mode.



You need to highlight an expression before you can approximate it.

- ☞ Highlight expression #23, and then approximate it with **Simplify>Approximate**.



Unlike the Command Toolbar's **Approximate** button, the **Simplify>Approximate** command invokes a dialog box in which you are asked to specify the number of digits of precision. The currently displayed default value of 10 digits is also used by the **Approximate** button. The **Simplify>Approximate** command allows you to temporarily change the default value for the next computation.

Change the number to 35, and then use the default dialog exit:

- ☞ 35 (_Approximate_)

#25: 0.70710678118654752440084436210484903

In DERIVE you can specify virtually any precision, meaning number of significant digits used for arithmetic. The practical limitations are given by the available memory and your patience. As explained later, computing time increases with increasing precision. (On our computer a 1,000 digits approximation of expression #23 took 0.161 seconds, a 2,000 digit approximation took 0.531 seconds.)

Update your text to indicate the chosen precision:

- ☞ Bring the text object into editing mode by clicking in it. Position the cursor immediately after the word: an

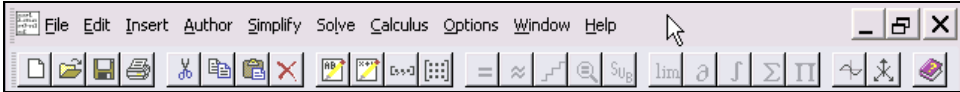
The following is an approximation of $\sin(\pi/4)$.

- Change the text appropriately by using the backspace key (æ_-) to delete the letter n and then adding: 35-digit

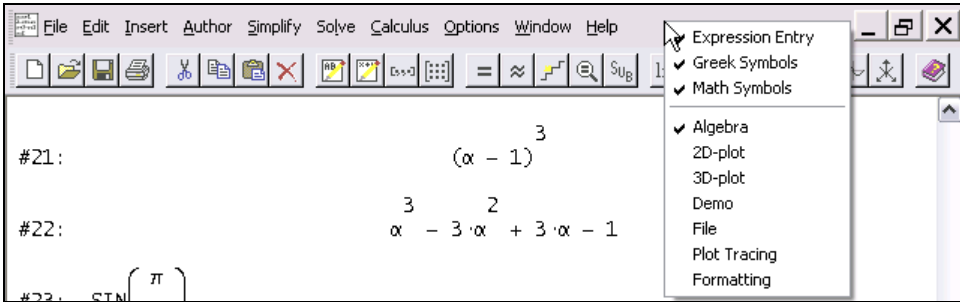
The following is a 35-digit approximation of $\sin(\pi/4)$.

Reducing the text's font size requires the Formatting Toolbar to be on. An elegant way to do this is the following:

- Move the mouse pointer anywhere in the Menubar or Command Toolbar.

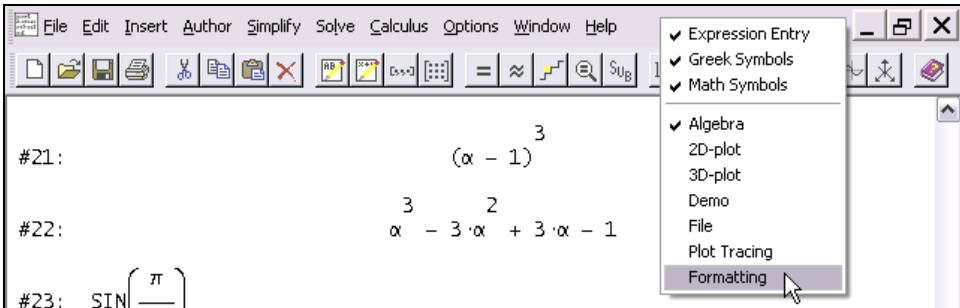


- Open the context sensitive menu by clicking the **right** mouse button.

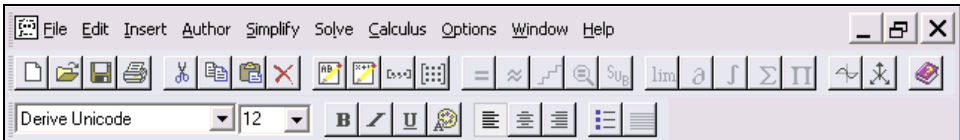


This menu contains switches for all available toolbars. The checkmarks indicate which ones are on. Toggle a switch by moving the mouse pointer over it, then clicking the left mouse button.

- Turn the Formatting Toolbar on by toggling the **Formatting** switch.




The Formatting Toolbar appears under the Command Toolbar.

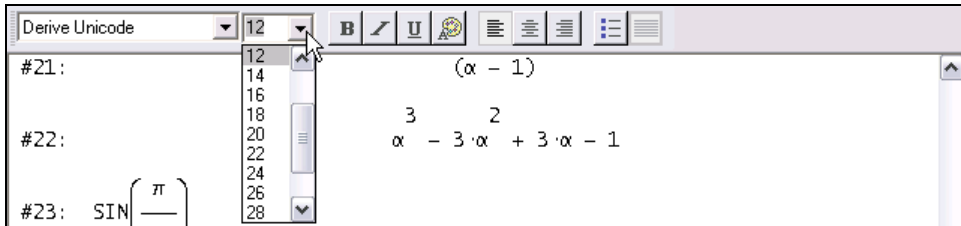


For editing DERIVE texts use the same techniques as in standard word processing programs. This toolbar indicates that the font size is 12 points. Before you can reduce the font size to 10 points, you need to highlight the respective portion of text.

- Highlight the entire sentence. Either use the technique of dragging the mouse pointer with the left mouse button held down from one end of the text to the other, or put the cursor at the text's end (or beginning), and then repeatedly use the left (or right) arrow key together with the shift key, or place the cursor anywhere in the text and then triple-click.

The following is a 35-digit approximation of $\sin(\pi/4)$.

- Prepare for changing the font size: open the **Font Size** field's dropdown selection menu by clicking on 




- Select the number 10.

The following is an approximation of $\sin(\pi/4)$.

Alternatively, you could make the **Font Size** field active, and then overwrite 12 with 10.

Now, announce the next example with an appropriate text:

- Prepare for entering text using the **Insert Text** button .

The following is an approximation of $\sin(\pi/4)$.

#25: 0.70710678118654752440084436210484903

Oops – this is the wrong position. The new text should appear at the end of the document. Since the **Insert Text** button (as well as the **Insert>Text Object** command) adds the text object after the highlighted object, you need to highlight expression #25 first. (Alternatively you could move the empty text object to the end of the document. You will learn how to move objects within a worksheet later.)

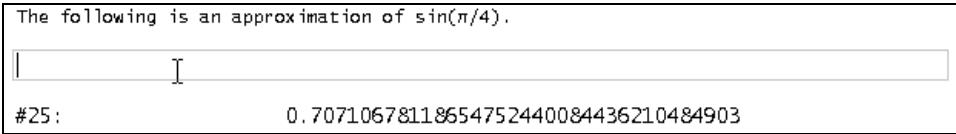
- Select expression #25.

The following is an approximation of $\sin(\pi/4)$.

#25: 0.70710678118654752440084436210484903

Although the frame around the empty text object disappeared, it is still there. It can be deleted like any other object only after it is highlighted.

- Highlight the text object by clicking into it.



- Try to delete it, using the (Del) key.

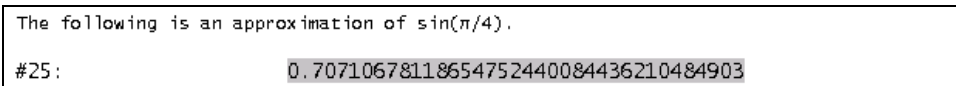
This has no effect. Remember: clicking inside a text object starts text editing mode. To select a text object for deletion, copying, or moving, click (precisely) on the frame or the narrow space left or right of the text object, or press (Esc) from within text editing.

- Select the text object for deletion using (Esc).



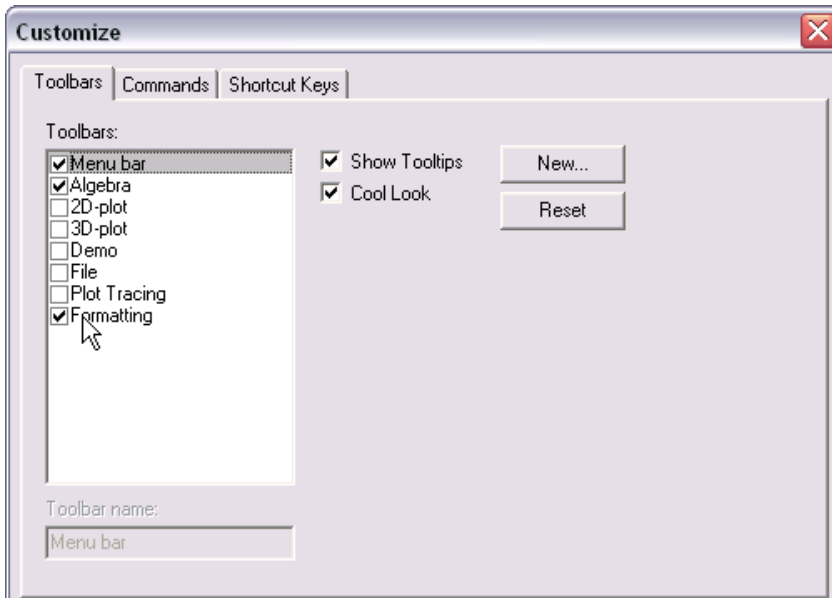
The text object is selected now as is indicated by the frame around it and no cursor in it. If there is still a cursor, press (Esc) again.

- Delete the empty text object using the (Del) key.

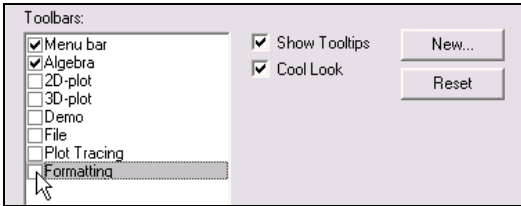


You will not need the Formatting Toolbar any more in this session, so switch it off to provide more space for other purposes. You can switch a toolbar off the same way you switched it on. But now we invite you to apply an alternative technique for controlling the display of toolbars:

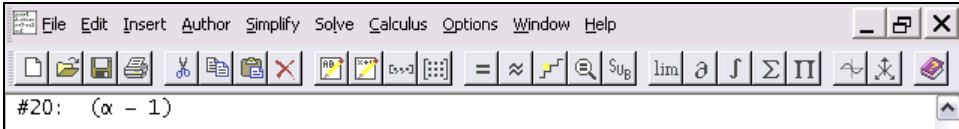
- Open the **Customize** tab dialog box using the **Window>Customize** command.



- Turn the Formatting Toolbar off by clicking with the left mouse button on the check mark left of the word “Formatting”.



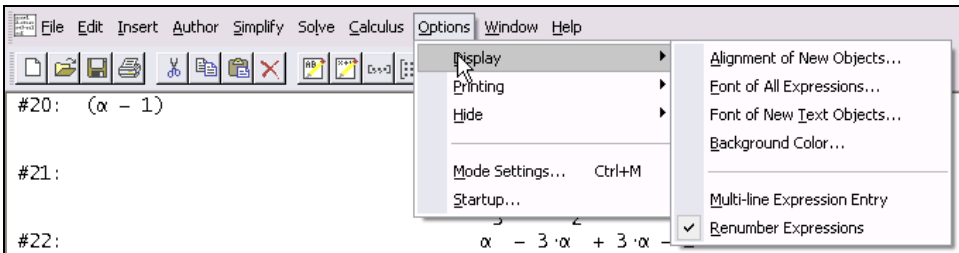
- Leave the dialog with (OK) to make the Formatting Toolbar disappear.



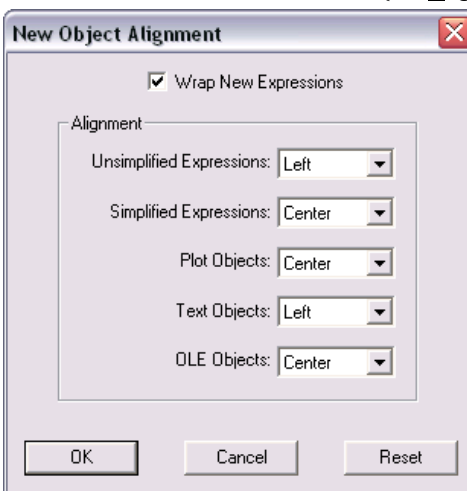
Learn more about customizing your DERIVE (and the various options of the **Window>Customize** command) in Chapter 17.

Experiment with the commands from the **Options>Display** submenu to become familiar with changing the “look” of a DERIVE worksheet:

- Open the **Options>Display** submenu.



- Select this submenu’s first choice (i.e. **A**lignment of New Objects)



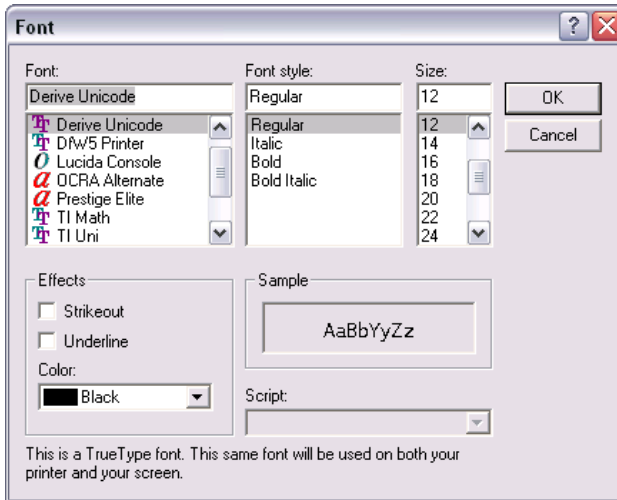
This invokes a dialog box that allows you to control the alignment of all the objects that can be in a DERIVE worksheet. **Unsimpilified Expressions** are expressions entered by you or expressions obtained by applying an operator to an expression without simplifying.

Simplified Expressions are expressions obtained from simplifying or approximating an expression. It is helpful to display user input left justified and the answers centered, as is done by the default setting.

☞ To keep the settings as they are, exit the dialog with (_Cancel_) or the (Esc) key.

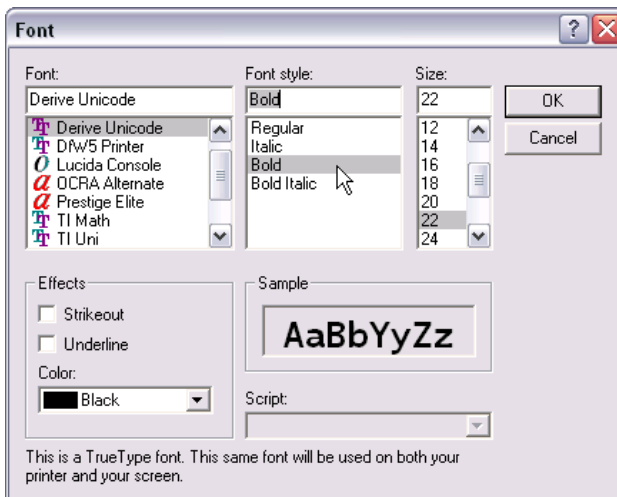
Try the next command in the **Options>Display** submenu:

☞ Try the menu's second choice, **Options>Display>Font of All Expressions**.



The language used in this and similar dialogs is determined by the language setting of your operating system.

☞ Choose **22 point and Bold ...**



..., then carry out the change by leaving the dialog box with (OK).

#24:

$$\frac{\sqrt{2}}{2}$$

The following is an approximation of $\sin(\pi/4)$.

#25: 0.70710678118654752440084436210484903

A large font is useful for demonstration purposes, especially when using a projector. For personal work a smaller font may be preferable. The large font will be used for the last two examples in this chapter.

Earlier you entered the constant π via the Math Symbol Toolbar. There are several methods for entering special constants such as π , the base of the natural logarithm e , or the imaginary unit i . Also, it is important to distinguish between variables with names e , i , and (the lower case Greek letter) π and the famous constants usually denoted with these letters.

To enter a sum of three π 's (the constant), first move the focus into the entry line using (F2). Enter the first π from the Math Symbol Toolbar, the second one by typing π , and the third one as (Ctrl)+(P). (The pluses in between are all entered via the keyboard.) Finally add the lower case Greek letter Pi from the Greek Toolbar.




These are the three methods of entering the constant π . While the second and fourth terms in the entry line look different, all three terms look and mean the same thing in the worksheet. Note the different appearance of the lower case Greek letter Pi.

Conclude the input with (C).

#26: $\pi + \pi + \pi + \pi$

There are also three methods for entering the base of the natural logarithm e . Use all three of them to enter a sum of three e 's, and then add the ordinary letter e to see the difference between a variable with this name and the famous constant. There is also another method of simplifying an expression.

Enter the first e from the Math Symbol Toolbar using e , the second one by typing $\#e$, and the third one as (Ctrl)+(E). Then type: $+e=$ (Note the use of the postfix equals operator.)



- ☞ End the input of the sum of three e 's and the variable e with (C).

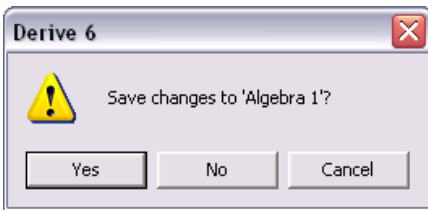
#27:	$e + e + e + e = e + 3 \cdot e$
-------------	---------------------------------

The postfix “equals” operator causes an automatic simplification and the generation of an equation whose left side is the unsimplified expression and whose right side is the simplified expression. This method displays both the unsimplified and simplified expression on the same line, saving lines on the screen.

Similarly there are three methods for entering the imaginary unit. You can obtain i from the Math Symbol Toolbar, type $\#i$, or enter it via the key combination (Ctrl)+(I).

Conclude this chapter by exiting DERIVE. The **Exit** command can be found in the **File** menu:

- ☞ Exit DERIVE using the **File>Exit** command.








DERIVE asks if the yet unnamed worksheet should be saved under the suggested file name “Algebra 1”.













- ☞ To exit without saving the worksheet select (_No_).

Summary

Algebra window

	or (Del)	delete highlighted expression
	or I nser T > T ext O bject or (F5)	insert text object after the highlighted object
	or A uthor> E xpression or (F2)	enter expression, move focus into entry line
	or S implif y > B asic	simplify highlighted expression
	or S implif y > A pproximate	approximate highlighted expression
F ile> E xit		exit DERIVE
S implif y > E xpand		expand highlighted expression
O ptions> D isplay		change display settings
W indow> C ustomize: T oolbars		control display of toolbars
$(\frac{1}{2}), (\frac{1}{4})$		move highlighting one expression up, down
(Esc)		cancel command
click left mouse button into row occupied by the expression		highlight expression
click left mouse button into text object		edit contents of text object
click onto text object frame or left or right of it, or press (Esc) from within text editing		highlight text object (without text editing)

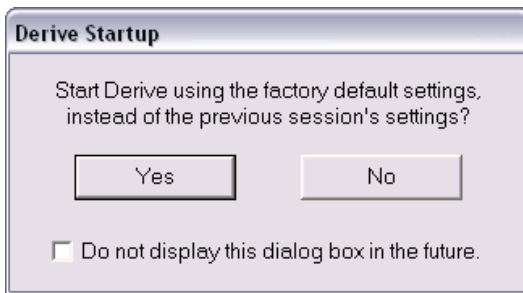
Expression Entry Toolbar

	or (C)	enter expression
	enter simplified expression
	or (Ctrl)+(C)	enter expression then simplify
	enter approximated expression
	or (a)+(C)	enter expression then approximate
	clear entry line
(Esc)		move focus into the algebra window
(\dot{y})		highlight contents of entry field
	or (Ctrl)+(P) or pi	area of unit circle π
	or (Ctrl)+(E) or #e	base of natural logarithm e
	or (Ctrl)+(I) or #i	imaginary unit i
 ,  , etc.		Greek letters
	or (Ctrl)+(Q) or sqrt	square root symbol
= (postfix equals operator)		show unsimplified and simplified expression as an equation

Chapter 2: Documenting Polynomial Zero Finding

The emphasis in this chapter is on creating a simple mathematical document about determining the zeros of a polynomial. At the same time you will learn the corresponding basic techniques of using DERIVE.

- Start DERIVE.



Your first session with DERIVE left information in an initialization file. This file stores information about the status of the DERIVE window before you last exited DERIVE. The **Derive Startup** dialog gives you the choice to either start DERIVE with the factory default settings or start DERIVE with the settings from the initialization file, which are some of the changes from the first chapter. This book is written so that each chapter starts with a factory default DERIVE. You should do the same so that your results will look similar.


- Start with a factory default DERIVE by exiting the dialog with (_Yes_).

Start the new document with an appropriate headline.

- Insert a text object containing the text "*Finding the zeros of a polynomial*".



You will look for the zeros of the polynomial $y = p(x)$, $y = x^3 - \frac{x^2}{2} - \frac{3x}{2} - \frac{1}{2}$.

- Enter the above polynomial by preparing for expression input with , and then type:
 $y = x^3 - x^2 - 3x/2 - 1/2$
 (Intentionally omit the /2 in the second term.)



From here on, the key (C) or the button (OK) will be displayed only in ambiguous situations. It will not be used any more for simple inputs such as the above. It is important for some of the features you are going to study and use in this chapter that you work with the above polynomial. Therefore, make sure you entered it properly.

As you know, it was not! The /2 in the second addend is missing. This is easily repaired by applying the **Edit>Expression** command to the highlighted expression.

- ☞ Edit the highlighted expression by issuing the **Edit>Expression** command.

The screenshot shows a software window with a text entry field containing the expression $y = x^3 - x^2 - \frac{3 \cdot x}{2} - \frac{1}{2}$. The expression is surrounded by a hatched border. Below the entry field, a status bar reads "Editing expression #1; press Esc to cancel" and "User". At the bottom, a toolbar contains icons for mathematical operations (add, subtract, multiply, divide, power, root, etc.) and a text input field containing the same expression: $y = x^3 - x^2 - 3 \cdot x / 2 - 1 / 2$.

This brings a copy of the expression into the entry line with the cursor positioned at its left end. A status bar message indicates editing mode and how to cancel it. A hatched pattern surrounds the original expression until the edit operation is either completed (by entering possible changes to the characters in the entry line) or cancelled (with (ESC)).

- ☞ Insert /2 after x^2 then end the input with (C).

The screenshot shows the same software window as before, but the expression in the entry field is now $y = x^3 - \frac{x^2}{2} - \frac{3 \cdot x}{2} - \frac{1}{2}$. The expression is still surrounded by a hatched border.

The (C) key performed a *replacement* of the old expression with the new one. There is no need to delete the old expression when using the **Edit>Expression** command.


Consider looking at a house from several different positions. From each position you will see details that you can't see from other positions. Based on this idea, mathematicians use a variety of different representations for mathematical objects. The third degree polynomial that you entered is displayed as an *algebraic* representation. Next you will produce a *graphical* representation, because this representation is particularly useful for obtaining information about the zeros. In other words: you will plot³ its graph.

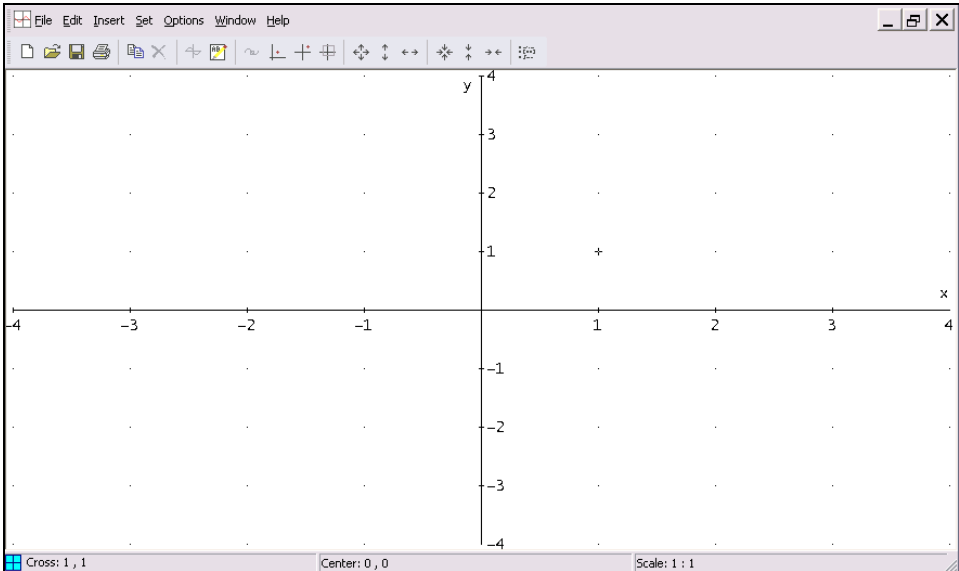
Since the major goal in this session is to properly document the mathematical work, ...

- ☞ ... insert the following text:

We try a graphic approach by plotting the polynomial in a 2D-plot window.

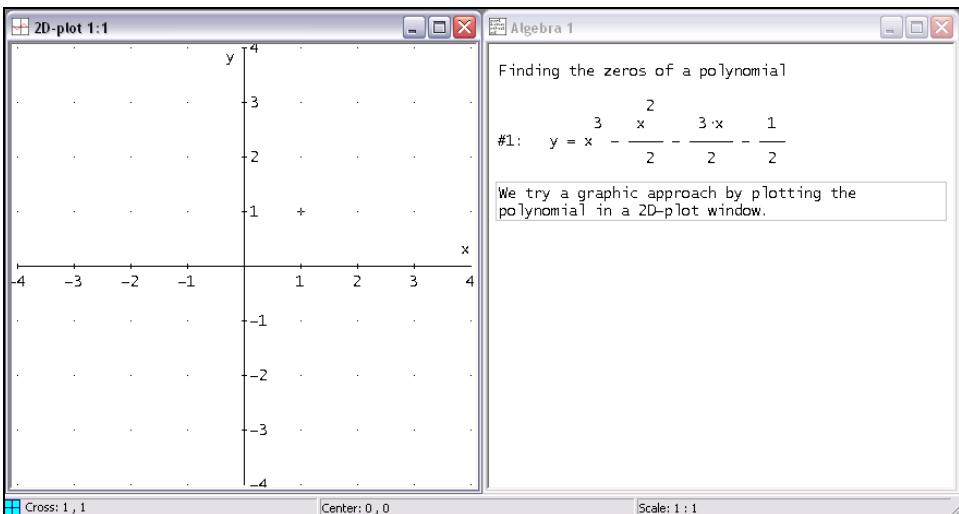
³ "Plot" includes different aspects of drawing and graphical representation. In this book it will be used with three different meanings: for the activity of producing a graphical representation, for a graphical representation as an object, and for the corresponding DERIVE command.

- Prepare for plotting a 2D graph: open a 2D-plot window by clicking on the **2D-plot Window** button  or issuing the **Window>New 2D plot Window** command.



DERIVE created a plot window so that you now have two windows to work with: an algebra window and a 2D-plot window. Use the usual WINDOWS techniques to toggle between windows or change their sizes and positions:


- Put the two windows side by side using the **Window>Tile Vertically** command.

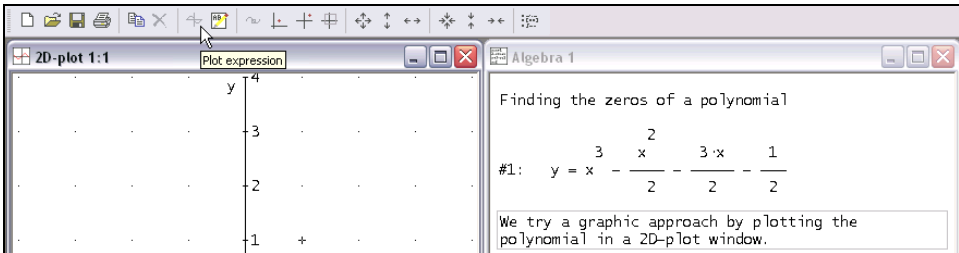


Each window is labeled with the window type in its upper left corner (**2D-plot** and **Algebra**). The active window's Titlebar is dark; the inactive window's Titlebar is dimmed. Since the plot window is active, the Menubar, the Command Toolbar, and the Status Bar are all

different from that of the algebra window. In particular, the Status Bar displays the following graphics information:

- The crossed square icon preceding the word **Cross** indicates Cartesian coordinates,
- **Cross** gives the coordinates of a movable cross,
- **Center** gives the coordinates of the picture center,
- **Scale** gives the scale factors of both axes.

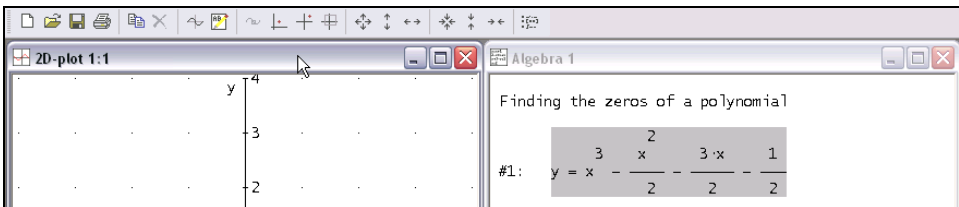
☞ Draw the graph using the **Plot** button .





Oops –the **Plot** button is dimmed/inaccessible (disabled).

The reason is that the **Plot** button (as well as its equivalent, the **Insert>Plot** command) plots the point set given by the algebra window's highlighted expression, but currently the second text object is highlighted and a text object can't be plotted.


☞ Highlight the polynomial by clicking on it. This makes the algebra window the active window. Therefore, make the 2D-plot window active again by clicking on its Titlebar.

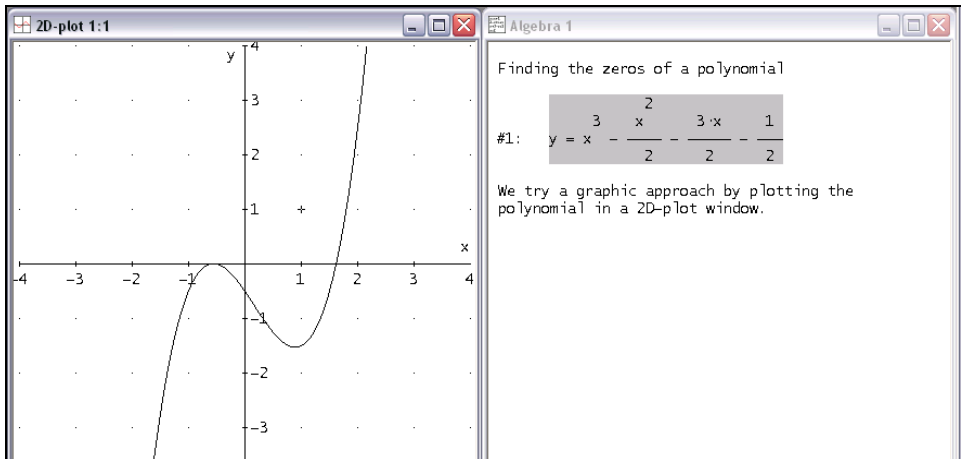


There are several techniques to make a different window active:

- Use (Ctrl) + (F6) to make the other window active.
- From the algebra window use the Command Toolbar's **2D-plot window** button  and from the 2D-plot window use the Command Toolbar's **Algebra window** button .
- Click on the window you want to make active. This method, however, must be used with care: Clicking on an algebra window with the left mouse button is likely to alter the highlighting. Clicking on a 2D-plot window with the left mouse button is likely to move the graphics cross. This might have unexpected effects. Therefore, it is better to click, with any mouse button, in the window's Titlebar.
- From a plot window you can use (Ctrl) + (1) to make the algebra window active.

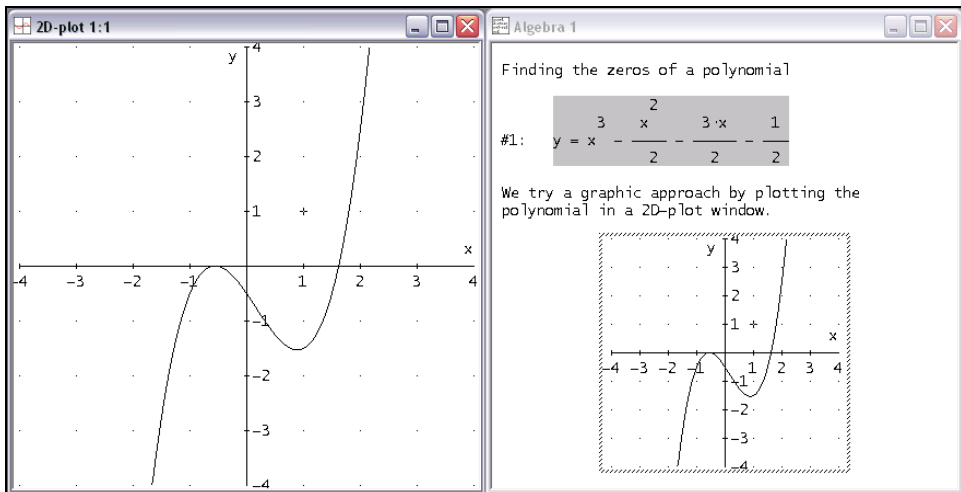
Now that the **Plot** button is available, you are ready to plot the graph of the equation.

- ☞ Draw the polynomial's graph using the **Plot expression** button .



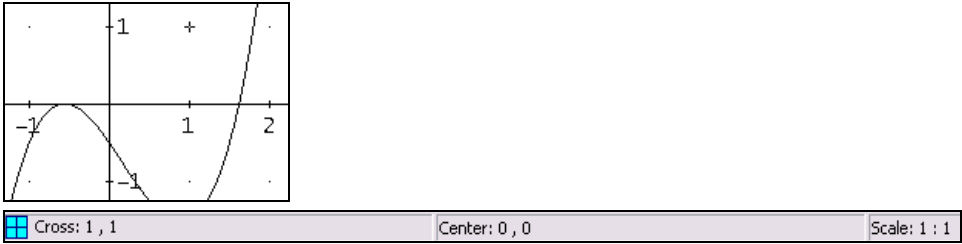
Now we have both an algebraic and a graphical representation of the polynomial available. However, the graphical representation is *outside* the algebra window's worksheet in its own independent plot window.

- ☞ Copy the current plot window into the algebra window's worksheet by using the 2D-plot window's **File>Embed** command.



This “freezes” the current status of the plot window into the worksheet. The plot window is interactive, the embedded plot image is not. The embedded plot image can be brought back into an interactive plot window at any time with a double mouse click.

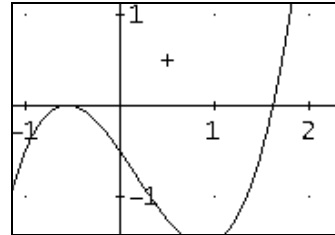
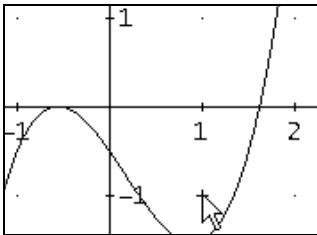
The graphical representation is useful for exploring the polynomial's zeros. However, from the current picture it is not clear whether the polynomial has one, two, or three distinct zeros. An answer can be found by inspecting the graph with the moveable graphics cross. Its coordinates are displayed in the status line, which now shows the cross at the initial position (1,1):




The color of the cross can be changed via the **Cross** tab of the **Options>Display** dialog.

When the plot window is active, the cross can be repositioned by either moving the mouse pointer and clicking with the left mouse button or by using the arrow keys (Æ), (æ), ($\frac{1}{2}$), and ($\frac{1}{4}$).

- Move the mouse pointer to (1,-1) or near it, then click with the left mouse button to move the cross to this position (left picture). Use the arrow keys to move the cross to (0.5,0.5). Try $(\text{Ctrl})+(\text{Æ})$, $(\text{Ctrl})+(\text{æ})$, $(\text{Ctrl})+(\frac{1}{2})$, and $(\text{Ctrl})+(\frac{1}{4})$ to move the cross in bigger steps.

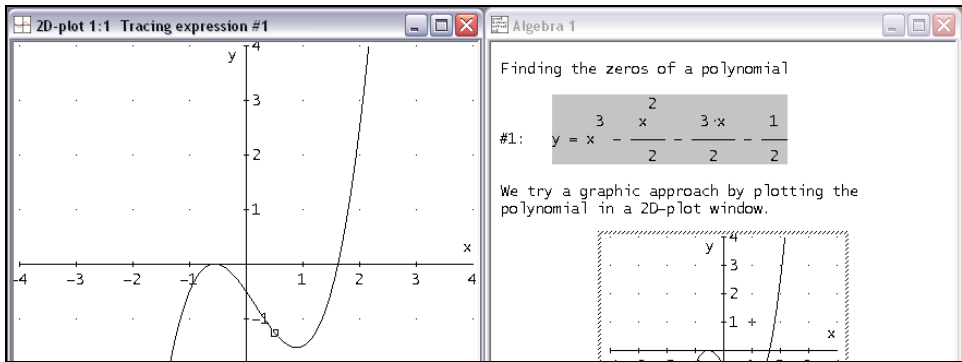


The **(Home)** key moves the cross to the plot window center.

The trace mode is very useful for inspecting curves. This mode can be switched on and off with the **Trace Plots** button , the **Options>Trace Plots** command, or the corresponding hot key (F3). As is customary in WINDOWS programs, a button with the same effect as a command is displayed in the respective menu left of the command, while the hot key is displayed right of the command. Check this for the **Options>Trace Plots** command:

- Open the **Options** menu.

Turn trace mode on by selecting the **Trace Plots** command.



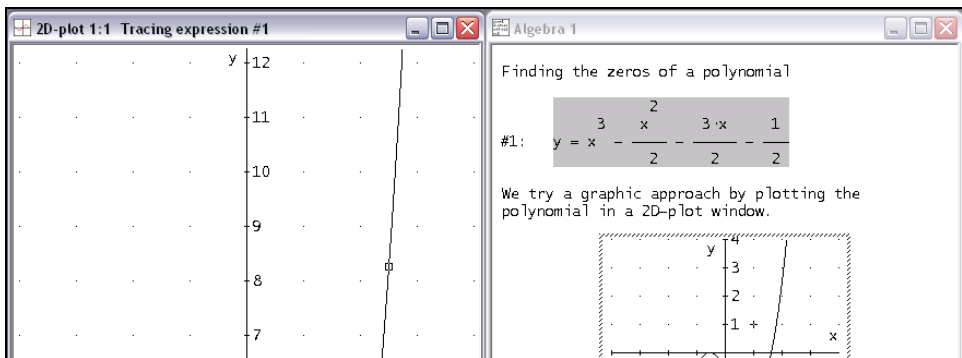
When trace mode is switched on, the cross changes its shape into a square and jumps vertically to the curve with its horizontal coordinate unchanged. The expression number of the traced curve is displayed in the plot window's Titlebar (here: **Tracing Expression #1**⁴). When trace mode is on, the square can be moved only along the curve. This can be done using (Æ) and (æ), or using (Ctrl)+(Æ) and (Ctrl)+(æ) for larger steps. It can also be done by moving the mouse pointer and clicking with the left mouse button to the new position. If there are several graphs displayed, use (½) and (¼) to select another graph.

Become familiar with moving the square. Use the arrow keys and the mouse to move the square. Finally click the left mouse button at the point (2.5,0).



What happened to the square? It disappeared. The status line indicates the reason. The square's vertical coordinate is 8.25, so it is outside the current plot area. You can ask DERIVE to move the plot area where the cross or square is.

Move the plot area where the cross is by flipping the switch **Options>Follow Cross**.




The plot window “follows” the square. This means that the plot ranges for the horizontal and the vertical axes are changed automatically to ensure that the cross is visible. Since

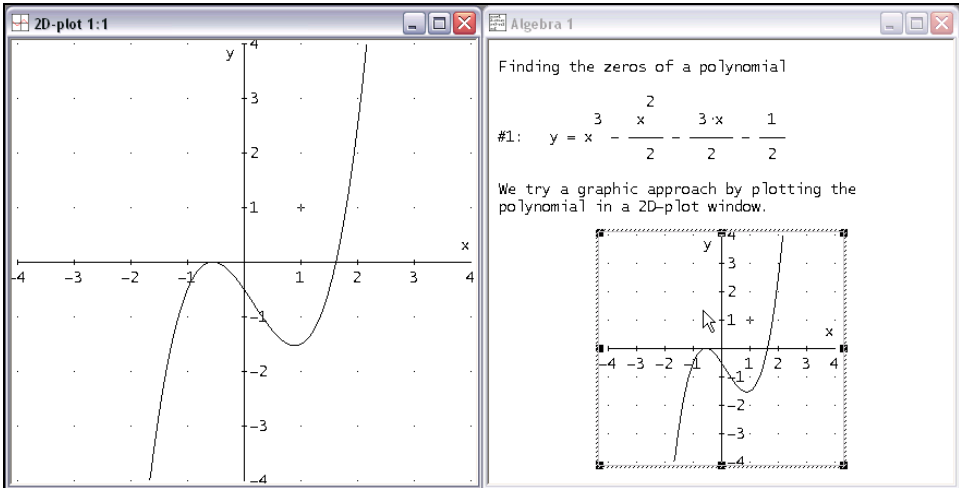
⁴ To conserve space this titlebar message is an abbreviation for “Tracing the graph of the function defined by expression #...”.

this mode can alter a chosen plot range, **Follow Cross** mode should be used carefully and is therefore switched off by default.


☞ Turn **Follow Cross** mode off by issuing **Options>Follow Cross** again.

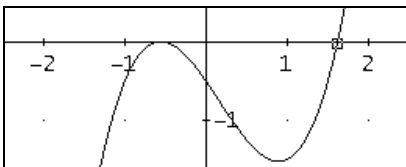
There are several ways to restore a previous range:

- Use the **Center on origin** button .
 - Issue the **Set>Plot Range>Length/center** or the **Set>Plot Range>Minimum/maximum** command, use the (Reset) button, and then leave the dialog with (OK).
 - If available, double click on an embedded version of the original graph. This last option is particularly elegant and convenient.
- ☞ Restore the original graph by double clicking on the embedded graph.



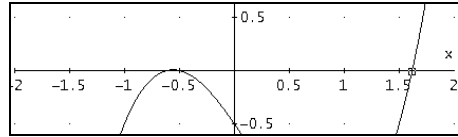
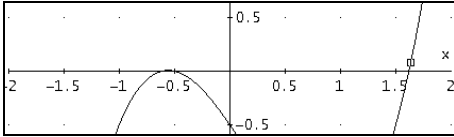
Trace mode is abandoned because the embedded graph was produced before trace mode was turned on. Switch trace mode on again to start looking for the polynomial's zeros:

☞ Switch trace mode on with , then move the square to the rightmost zero, as near as you can get to the horizontal axis.




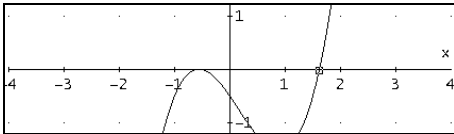
DERIVE displays the square coordinates as **Cross: 1.615385, -0.01251707**. (Your numbers might be different.) Using the right arrow key (\rightarrow) once moves the square to **Cross: 1.634615, 0.07973231**. You have not found a position at which the y -coordinate is zero, but you can say that the polynomial zero must be between 1.615385 and 1.634615, probably being closer to 1.615385. An obvious approach for getting closer is magnification.

- Zoom in using the Command Toolbar's **Zoom in** button  (left picture), then try moving the square closer to the rightmost zero.



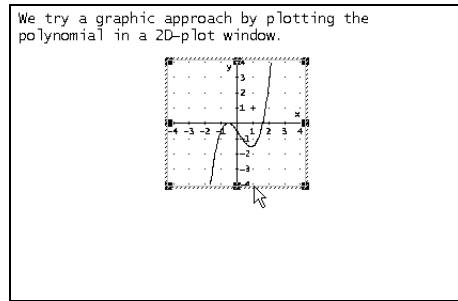
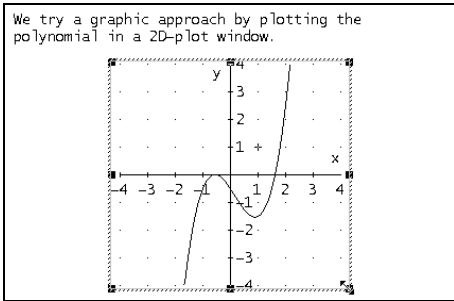
On our computer we couldn't get any closer than we did previously (it may be different on your computer).

- Restore the original scale factors by zooming out with the **Zoom out** button .



Document what you found so far by inserting appropriate text objects:

- Switch to the algebra window. Resize the embedded plot: select the image by clicking on it. The image is surrounded by 8 black squares, which can be used to resize it. Move the mouse pointer to the lower right corner until a double-headed arrow appears. Press and hold the left mouse button. With the left mouse button held down, drag the pointer towards the image center. When a suitable size is reached, release the mouse button.



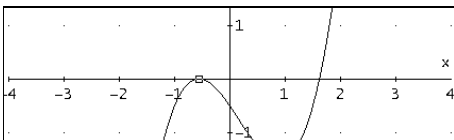
When you choose one of the four corner points, the size changes proportionally, so that the aspect ratio is preserved.

Insert a text object, documenting the method and result of your findings:


- Insert a new text object and enter the following text (use the numbers you found):

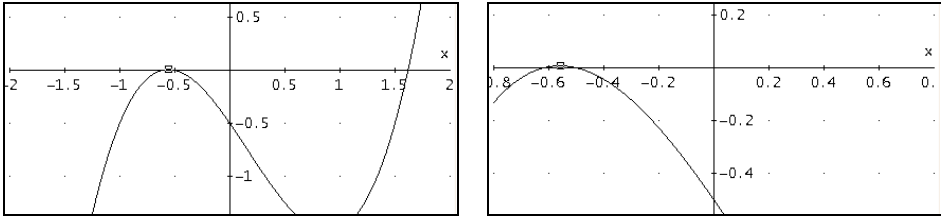
Using trace mode we found $1.615385 < x < 1.634615$.

- Search for more zeros: make the plot window active, and then move the square to the uncertain zero between $x = -1$ and $x = 0$:





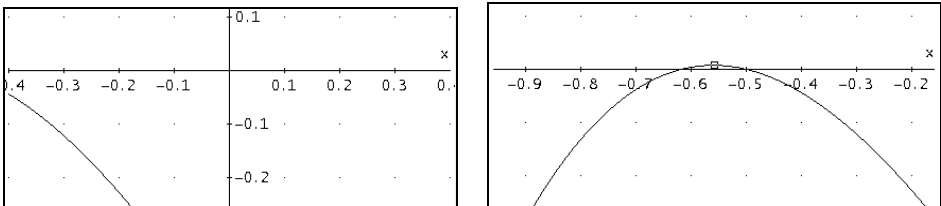
You will find that there is one zero between -0.6346154 and -0.6153486 . (Again, your numbers may be different). Another zero seems to be at exactly $x = -0.5$. To obtain a picture with intersections of the graph, magnify again:

🖨 Zoom in, this time using the **Zoom in** button  twice.



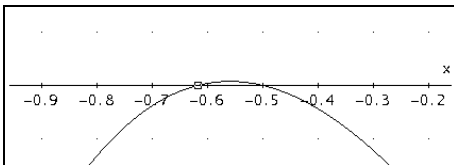
It becomes obvious that there are two zeros. Continue to magnify the graph:

🖨 Zooming in once more with  lets the square leave the plot window because follow mode is switched off (left picture). The very useful **Center on cross** button  shifts the plot range so that the square/cross is in the center of the new plot image.




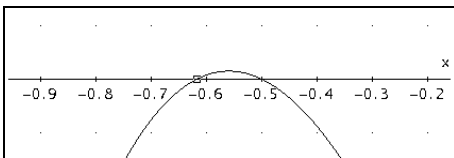
Move the square to get a better approximation of the left zero:

🖨 Move the square near the left zero and note the cross coordinates in the Status Bar.




Now the change of sign happens between $x = -0.6192308$ and $x = -0.6173077$. Produce a graph with steeper intersections to get a more accurate answer:

🖨 Zoom in vertically only, using the **Zoom vertical in** button .

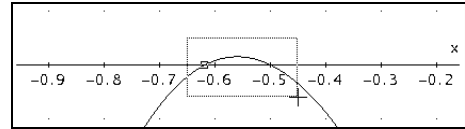
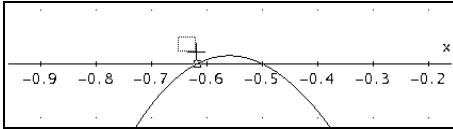


A highly recommended tool is the **Set range with box** button , which allows you to choose a crop rectangle graphically.

🖨 Prepare for choosing a crop rectangle by using the **Set range with box** button .

The mouse cursor turns into a crosshair.

- Choose a crop rectangle: Click and hold the left mouse button at the top left corner of the desired area. Drag the mouse down and to the right until the box encloses the desired area.

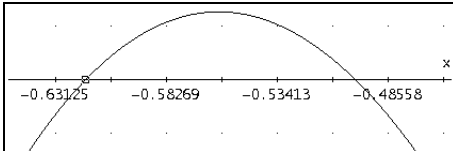


- Release the mouse button.

The 'Set Plot Range Minimum/maximum' dialog box is shown. It has a title bar with a close button (X). The dialog contains three columns: 'Minimum', 'Maximum', and 'Intervals'. Under 'Horizontal', the values are: Minimum: -0.65192307692, Maximum: -0.45769230769, Intervals: 8. Under 'Vertical', the values are: Minimum: -0.025, Maximum: 0.023, Intervals: 8. At the bottom are buttons for 'OK', 'Cancel', and 'Reset'.

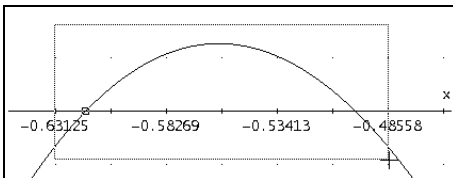
The **Set Plot Range** dialog box is displayed, reflecting the numerical equivalents of the choices you just made with the mouse. This dialog box could be obtained using the **Set>Plot Range>Minimum/maximum** command, but a graphical choice of the plot range is often more convenient.

- See what happens if you confirm with (OK).



Notice the complicated numbers below the tick marks (your numbers are likely to be different) and in the Status Bar scale factors. This is caused by the graphical box selection.

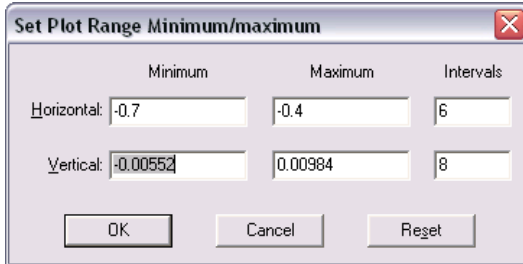
- Zoom in again using the **Set range with box** button .



The 'Set Plot Range Minimum/maximum' dialog box is shown again, reflecting the zoomed-in view. The values are: Horizontal: Minimum: -0.63125, Maximum: -0.48557692308, Intervals: 8; Vertical: Minimum: -0.00552, Maximum: 0.00984, Intervals: 8. Buttons for 'OK', 'Cancel', and 'Reset' are at the bottom.

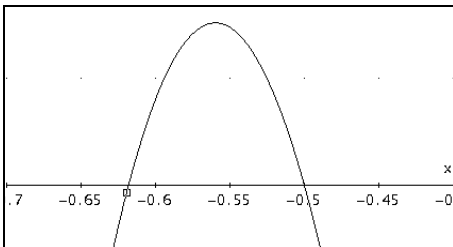
It is helpful to edit the suggested numerical values to the nearest simple values. Start by overwriting the highlighted value of the input field for the **Horizontal Minimum**. Then use the tab key (\dot{y}) to make the next input field active. Enter the following values:

\dot{y} -0.7 (\dot{y}) -0.4 (\dot{y}) 6 (\dot{y})



Make the values for the **Intervals** fields fit the difference of the values for the **Minimum** and the **Maximum** fields. For example, 6 intervals for a horizontal range of length 0.3 (= difference of -0.7 and -0.4) ensures simple numbers below the tick marks.

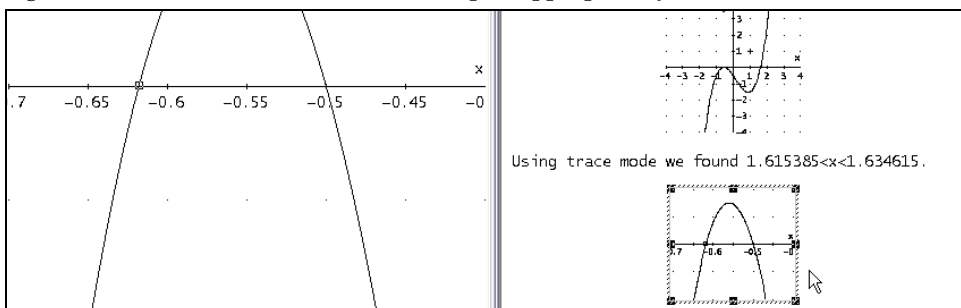
\dot{y} -0.01 (\dot{y}) 0.01 (\dot{y}) 4 (\dot{y}) OK



Use the trace mode square to find approximations of the two zeros.

The left zero lies between -0.6185714 and -0.6178571; and the other zero probably is at -0.5. All the above work now should be documented in the algebra window's worksheet by embedding the graph and adding an appropriate text object.

From the 2D-plot window issue the **File>Embed** command, and then switch to the algebra window and resize the embedded plot appropriately.



By using the variable names `hcross` and `vcross` you can transfer the current graphic cross coordinates into the Algebra window.




Enter and approximate: hcross

#2: hCross
 #3: -0.6178571428

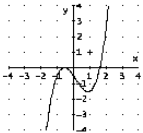
Insert a new text object documenting the method and result of your findings:

Using zooming we found $x = -0.5$ and $-0.6185714 < x < -0.6178571$.

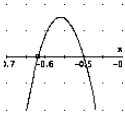
Close the plot window, and then open the algebra window to full size:

Close the plot window by clicking the left mouse button on the  button that is located in the window's upper right corner. Open the algebra window to full size by clicking on the  button, which is located left of the algebra window's  button.

We try a graphic approach by plotting the polynomial in a 2D-plot window.



Using trace mode we found $1.615385 < x < 1.634615$.



#2: hCross
 #3: -0.6178571428

Using zooming we found $x = -0.5$ and $-0.6185714 < x < -0.6178571$


Next compute the zeros by solving the corresponding polynomial equation. Before doing so, enter an appropriate textual description of your approach:

Enter the text:

We apply the SOLVE function to the corresponding polynomial equation.

Generate the corresponding polynomial equation:


Highlight the polynomial #1, move focus into the entry line with (F2) (which is the hot key for authoring expressions), and then auto-paste a copy of the polynomial using the hot key (F3).



 $y = x^3 - x^2/2 - 3 \cdot x/2 - 1/2$

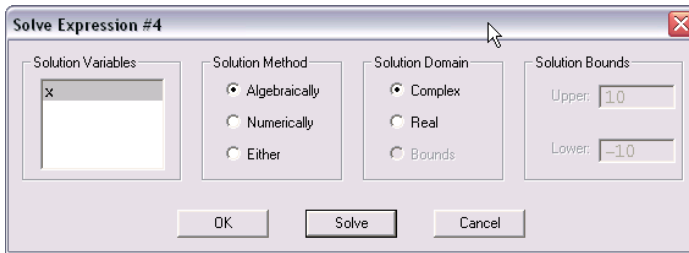
(F2) may become your most frequently used hot key.

Replace y with 0 and then conclude the input with (C).

#4: $0 = x^3 - \frac{x^2}{2} - \frac{3 \cdot x}{2} - \frac{1}{2}$

For solving this equation either use the **Solve>Expression** command or the corresponding toolbar button .

 Prepare for solving the equation by applying the **Solve Expression** button .




 Solve the equation. Accept all suggested parameters by selecting (**Solve**).

#5:
$$\text{SOLVE} \left(0 = x^3 - \frac{x^2}{2} - \frac{3 \cdot x}{2} - \frac{1}{2}, x \right)$$


#6:
$$x = \frac{1}{2} - \frac{\sqrt{5}}{2} \vee x = \frac{\sqrt{5}}{2} + \frac{1}{2} \vee x = -\frac{1}{2}$$

Here \vee is the mathematical symbol for the logical operator OR.

For online help with any DERIVE dialog box, press (F1) when the dialog box is visible.

Similar to the Entry Toolbar's **Author and Simplify** button , (**Solve**) generated both an unsimplified expression (which is the formal application of the SOLVE function to the equation) and a simplified expression (which is the solution of the equation.) The exit (**OK**) would have generated the unsimplified expression only.

You can approximate expression #6 to compare these results with what you found graphically. Before doing so, again add a textual description of your approach:

 Enter the following text:

Expression #6 gives the four exact zeros of the polynomial.
 We approximate #6 so that we can compare it with what we found graphically.
 More about Derive at www.derive-europe.com.

If you type in a URL, DERIVE automatically makes it a link.

 Approximate expression #6 by first highlighting it, and then applying the **Approximate** button .




#7:
$$x = 1.618033988 \vee x = -0.6180339887 \vee x = -0.5$$

To turn this worksheet into a polished mathematical document, do some more editing, then print it out and save it, but first, add a signature documenting author(s) and date:


 Switch the Formatting Toolbar on using **Window>Customize**.

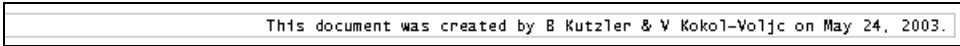


All fields and buttons are dimmed as long as there is no text object in editing mode.


-  Add a text object at the end of the worksheet using . Choose a special format for the signature: in the Formatting Toolbar change the font size to 8 points and click on the **Right Justify** button .

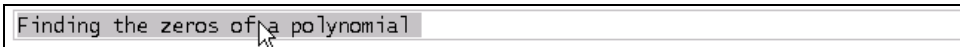


-  Enter the following text:



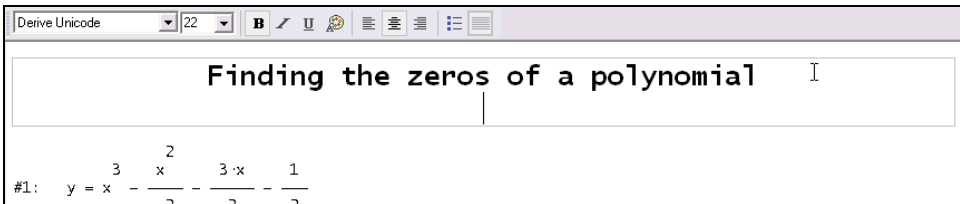
Next, change the topmost text object into an attention-catching title line:

-  Highlight the first text object's contents using the usual text processing techniques.




Choose a format that is suitable for a title line, for example ...

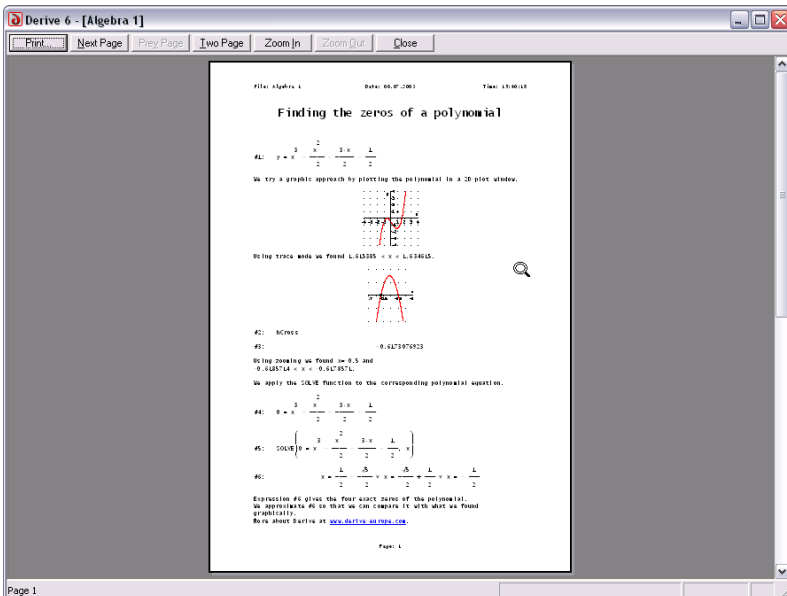
-  ... change to **22** points font size, bold (**B**), centered (), and then add a blank line.




-  Switch the Formatting Toolbar off via **Window>Customize**.

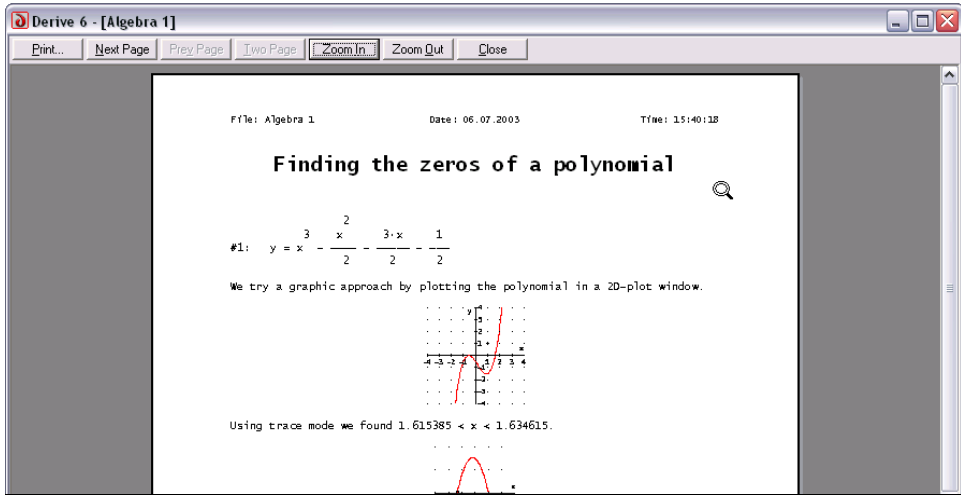
Before printing a document, it is wise to do a print preview.

-  Look at the print preview using the **File>Print Preview** command.



Print preview offers various options including a button for zooming in.


 Zoom in with (ZoomIn_).

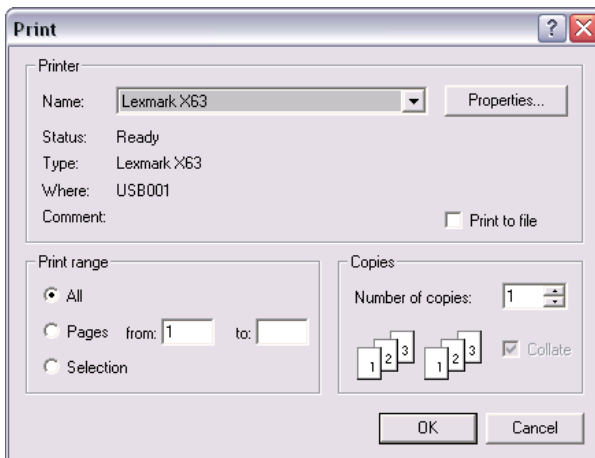


The magnifying glass shaped cursor in the upper right quarter of the page indicates that an alternative to using the (ZoomIn_) button is to click with the left mouse button.

Using the commands from the **Options>Printing** submenu you can make changes to the printed document. For example, the **Options>Printing>Expression Layout** command controls the printing of **Annotations** and **Computation Times**. (By default neither is printed).

The worksheet is now ready to be printed.

 Prepare for printing the document using print preview's (Print_) button.

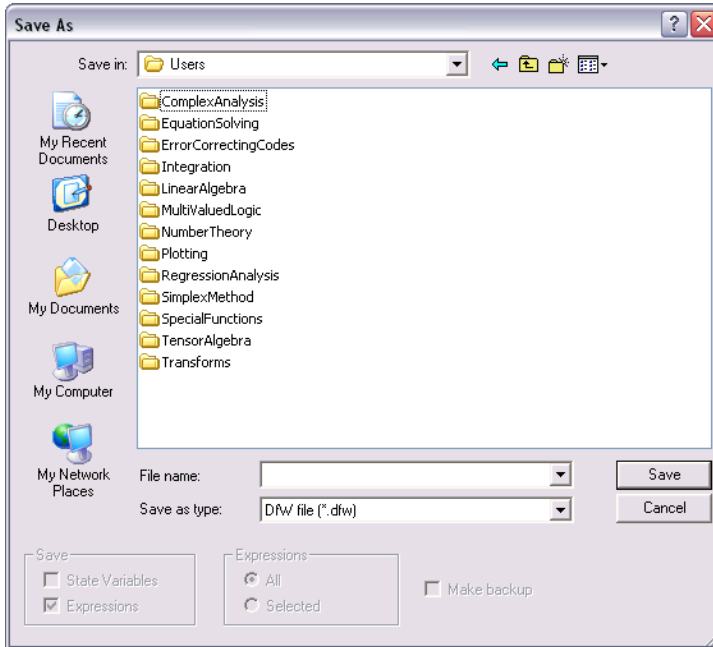


Make sure that the printer is properly connected, switched on, and set. In the **Printing** dialog box you can change the printer or the printing properties, the print range from **All** to either a range of pages or the highlighted expressions, or the number of copies from the default 1 to however many you want.

- ☞ Provided your computer is connected to a printer, send the document to the printer with (OK).

Saving the worksheet preserves your work for later use or modification.

- ☞ Save the worksheet by issuing the **File>Save As** command.



DERIVE suggests storing the file in the subdirectory **Users**. You can choose a different directory by selecting one from the selection menu that is offered for the **Save in** field.

- ☞ Accept the suggestion and enter the file name `chapter02` in the **File name** input field. Close the dialog with (Save).





 Derive 6 - [Algebra 1 chapter02.dfw]

Notice the Titlebar. Previously there was **[Algebra 1]** as the indication of an unnamed algebra worksheet. Now there is **[Algebra 1 chapter02.dfw]**, indicating an algebra worksheet with name `chapter02.dfw`. The suffix `.dfw` is the default that is chosen when you do not specify a suffix as part of the filename.









- ☞ Exit from DERIVE.

Summary

Algebra Window

-  or **Solve>E**xpression solve equation
-  open new 2D-plot window or switch to one
- Window>New 2D** plot Window open new 2D-plot window
-  right justify highlighted object
-  center highlighted object
- File>Save A**s save worksheet using a name
- File>Print** Preview print preview
- E**dit>**E**xpression or double-click left or right of expression edit highlighted expression
- O**ptions>**P**rinting>**E**xpression Layout format expression layout
- double-click left mouse button on embedded plot open embedded plot in plot window

2D-plot window

-  or **I**nsert>**P**lot plot highlighted expression
-  or **O**ptions>**T**race Plots or (F3) toggle trace mode
-  center plot region on cross
-  center plot region on origin
-  or (F9) zoom in
-  or (F10) zoom out
-  or (F7) zoom in vertically
-  graphically choose a crop rectangle
- File>E**mbed copy plot window into algebra worksheet
- S**et>**P**lot **R**ange>**L**ength/**C**enter set length, center, and scale of plot range
- S**et>**P**lot **R**ange>**M**inimum/**M**aximum set minimum, maximum, and scale of plot range
- O**ptions>**F**ollow **C**ross toggle follow mode
- (\AA), ($\frac{1}{4}$), (\ae), ($\frac{1}{2}$) move cross one pixel (one dot) on the screen
- (Ctrl)+(\AA), (Ctrl)+($\frac{1}{4}$), (Ctrl)+(\ae), (Ctrl)+($\frac{1}{2}$) move cross several pixels
- (Home) move cross to plot window center

All windows

- Window>Tile** **V**ertically arrange windows as right-left split (active window on the left)
- (F1) display context sensitive online help