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## THE

## GEOMETER'S SKETCHPAD ${ }^{\circ}$

for T1-89
T1-92 Plus
Voyage" 200
User Guide and
Reference Manual

# The Geometer's Sketchpad ${ }^{\circledR}$ for TI-89, TI-92 Plus, and Voyage ${ }^{\text {TM }} 200$ PLT 

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## Introducing Sketchpad



For new users of The Geometer's Sketchpad forTI-89, TI-92 Plus, and Voyage ${ }^{\mathrm{TM}} 200$ PLT, this chapter offers a brief introduction to
Sketchpad and Dynamic Geometry. Two tutorials, called Guided Tours, will give you experience with many of Sketchpad's features. The chapter ends with an overview of the rest of this User Guide.

A cube in two-point perspective. While Sketchpad is twodimensional in nature, you can use it to build mathematical models of higher-dimensional geometry "projected" into two dimensions. This sketch demonstrates a two-point perspective view of a (three-dimensional) cube. In this perspective, vertical edges of the 3-D cube remain vertical in its 2-D image, but other edges recede toward one of two vanishing points on the image's horizon line. Dragging the vanishing points alters the perspective dynamically. (Example from a TI-89.)


The circumcircle of a triangle. This sketch starts with a triangle $A B C$. Construct the midpoints of each side, and then construct side bisectors-lines perpendicular to each side passing through that side's midpoint. The three bisectors concur in a single point. This point, the circumcenter of the triangle, is the center of the unique circle which passes through all of the triangle's vertices. In what sort of situations does a triangle's circumcenter fall outside of that triangle? (Example from a TI-92 Plus.)

## Dynamic Geometry ${ }^{\text {® }}$

This section gives a brief introduction to The Geometer's Sketchpad and to the significance of Dynamic Geometry.

The Geometer's Sketchpad for TI-89, TI-92 Plus, and Voyage ${ }^{\text {TM }} 200$ PLT is a powerful tool for learning concepts from geometry, algebra, and other math subjects. Geometric relationships can be hard to visualize from the static, unchanging figures in a text, or even from figures you draw yourself with traditional tools like a compass, ruler, and protractor. With Sketchpad, you can construct and measure figures easily and more precisely than with traditional tools. But more importantly, Sketchpad's Dynamic Geometry lets you move parts of your constructions to see which properties change and which don't. With Dynamic Geometry at your fingertips you can put the problems and theorems you encounter in your math class in motion, revealing relationships and giving you a whole new way of seeing mathematics.

Try the following two Guided Tours for an overview of Dynamic Geometry and how to use Sketchpad on your TI-89 or TI-92 Plus /Voyage 200 PLT. Or explore on your own, and turn to the reference sections if you need help learning how to do something.

## Installing Sketchpad

This manual assumes that Sketchpad has already been installed as a Flash application on your handheld device. Detailed Flash application installation instructions are available from education.ti.com/guides.

## Introductory Tour

This is the first of two Guided Tours, which are geometric investigations with step-by-step instructions designed to introduce you to many of Sketchpad's features.

Drawing

The square brackets in the instructions at right mean that text this appears as Sketchpad on the TI-89 and as The Geometer's Sketchpad on the TI-92 Plus/Voyage ${ }^{\text {TM }} 200$ PLT.

The $(\cdot)$ symbol refers to the large multidirectional cursor pad on the TI-92 Plus and to the four cursor keys on the TI-89.

The Straightedge tools are used for making segments, rays, and lines. When you choose one, it remains active until you choose a different tool or arrow.

On the TI-89, you can use CLEAR instead of ENTER for easier keyboard entry when using tools or selecting objects.

With Sketchpad's drawing tools and menus, you can construct figures that are very flexible, so that you can drag any parts of them wherever and however you want; or you can construct them with properties that limit how you can change them. In this tour, you'll draw a triangle that can be any size or shape. Then you'll construct an isosceles triangle, which is constrained so that two sides are always equal. You'll also discover some other properties of isosceles triangles. In the process, you'll learn some of the basics of using Sketchpad's drawing tools, as well as the F1 Edit,
F2 Display, F3 Construct, and F5 Measure menus.
Follow the steps below to construct a general triangle.
 edge.
2. Press F8 (TI-89: 2nd [F8]) to enter the toolbox. Then press $\odot$ until the Segment tool blinks. (The Segment tool appears at the left of three pop-up Straightedge tools.) Press ENTER to choose it. Once the Segment tool is active, the cursor becomes a cross-hair.
3. Press ENTER to construct a segment endpoint. Press $\odot$ to
 move away from the first endpoint, constructing a segment as you move. Press ENTER again to construct the segment's second endpoint.
4. Construct the first of two more segments to complete the triangle. With the cursor still positioned on the endpoint of your first segment, construct a second segment by pressing ENTER, $\odot$ to drag away, and ENTER again to complete the second segment.

You might notice a "snapping" action when the cursor gets close to an existing point.
5. With the cursor still positioned on the endpoint of your second segment, press ENTER, $\odot$ back to the original endpoint, and ENTER to construct the third segment. Be sure this segment connects back to the starting point before you press the final ENTER that completes the triangle.

You can drag parts of your triangle to change its size and shape.
Steps Display

1. Press ESC to quit the Segment tool and choose the Arrow tool.

2. Position the Arrow over a vertex. (The cursor changes to a sideways arrow when it points to an object.) Press and hold 图
 (TI-89: alpha), then press $\odot$ to drag the vertex. Drag it around the entire screen.
3. Practice dragging the other vertices of the triangle. Try dragging a side, too. Notice how you can make this triangle any size or shape.

Dragging is at the heart of Dynamic Geometry. It lets you generalize from a single construction. For example, in the steps above, you drew a single triangle. But by dragging, you can look at many possible sizes and shapes for a triangle. You can also drag to check that your construction is holding together properly - that all possible appearances are still triangles. Drag often!

Sketchpad's unlimited undo lets you look back at a previous state of your sketch. You can also use undo if you need to correct a construction mistake.

## Steps Display

1. Press F1 Edit: 1 Undo to undo the last step. Repeat to undo a few more steps.

2. Press F1 Edit: 2 Redo to redo the steps you've just undone. Throughout these tours, use undo (and redo) to revisit your work, or to correct any mistakes you make while constructing objects.

## Saving

## Constructing

A shortcut for activating the segment tool is $\square$ [F4 (TI-89: $\quad 4$ ). (There are comparable shortcuts for each of the other tools in the Toolbox based on position. For example, the Point tool is the second tool down, so its shortcut is - F2
[TI-89: ©2].)

This is tricky! Be sure you start your circle on one segment endpoint and finish it with the cursor directly over the second segment endpoint. Otherwise, the circle and segment will not be connected. Look for the snapping action when the cursor gets close to the point you want.

You can save your work in case you want to return to it later, after you've worked on something else.
Steps Display

1. Press $\mathbb{F 1}$ Edit: 9 Sketch: 3 Save to save the sketch. Sketchpad presents a dialog box that lets you specify a folder and a variable name.
2. To save this sketch in the main folder, press $\bigcirc$ to move the cursor into the Variable field and type triang1. Then press ENTER twice to save.

Congratulations! You've constructed your first geometric figure with Sketchpad: a triangle.

In this part of the investigation, you'll construct an isosceles triangle and investigate some of its properties.
 ENTER to choose the Segment tool. Then press ENTER $\odot$ ENTER to construct a segment.
3. Press F8 (TI-89: 2nd [F8]) $\bigcirc$ until the Compass tool blinks, then press ENTER to choose it.
4. To construct a circle with this segment as radius, move the Compass tool's cursor to one segment endpoint, press ENTER. Then press $\uparrow$ to start drawing a
 circle. Position the cursor over the second segment endpoint and press ENTER to finish your circle there.

## Animating

You've probably noticed that pressing ENTER with the Arrow over an object changes the object's appearance to indicate you've selected it. In general, you need to select objects that you want to act upon with menu commands.

There should be only two points in your sketch that define both the segment and the circle. If the circle separates from the segment when you drag, undo ( $\mathbb{1} 1$ 1) until the circle is gone and try again.

Remember: On the TI-89, you can use CLEAR as a shortcut for ENTER when drawing or selecting objects. (The CLEAR key might be easier to reach.)
5. Press ESC to return to the Arrow, then press and hold (TI-89:alpha) while using $\uparrow$ to drag the circle's two defining points to be sure the circle and segment are connected.
6. Use the Segment tool to draw another radius of the circle. Be sure to place one endpoint with the cursor positioned at the circle's center (ENTER) and the
 other endpoint with the cursor anywhere on the circle's circumference (ENTER).
7. Construct a segment to connect the two points on the circle and complete the triangle.
8. Press ESC to return to the Arrow. Then position it on a vertex, press and hold (TI-89 : alpha), and press $\odot$ to drag the vertex. Drag the other vertices too, and observe how pher each changes the triangle.

Did you notice that two sides of this triangle always appear to stay equal? Why? Did you notice that dragging one of the vertices didn't change the length of these equal sides? What's different about that point?

You can automate dragging using animation. Follow these steps to animate one vertex traveling around the circle.
Steps Display

1. Press ESC to deselect all objects. Press ENTER on the point on the circle that does not change the circle's radius to select it. If you're not sure you have the right point, drag it first.

2. Press F2 Display: 8 Animate Point.

3. To stop the animation, Press F2 Display: (TI-89: alpha) A Stop Animation.

You can hide parts of your construction to change its appearance without affecting its geometric properties.

## Steps Display

1. Press ESC to deselect all objects. Then press ENTER on the circle to select it. Press
 F2 Display: 1 Hide [Circle]. The circle disappears.
2. Drag vertices of the triangle. Notice that even when hidden, the circle determines the
 behavior of your triangle (it stays isosceles).

## Constructing with the F3 Construct Menu

If the Midpoint command in the F3 menu has a padlock icon (i) next to it, it's not available because the selection is not correct. Press ESC to deselect all objects and try again.

You constructed the isosceles triangle using only Sketchpad's Compass and Straightedge tools. You can create almost any Euclidean construction using just these tools, but the Construct menu offers shortcuts for many constructions.

| Steps | Display |
| :--- | :--- |

1. Construct the midpoint of the base of the triangle. To do this, first deselect all objects (ESC),
 then select the base. (Press ENTER on the side that's not necessarily equal to another side). Then press
F3 Construct: 2 Midpoint.
2. In order to construct a perpendicular line through this midpoint, you'll have to select two objects-a point to go through (the midpoint, in this case), and a straight object to be perpendicular to (the base segment, in this case). The just-constructed midpoint is already selected. Select the base segment as well, by moving the Arrow to it and pressing ENTER.
3. Construct the perpendicular line defined by your selections by choosing F3 Construct:


## 8 Perpendicular Line.

Sketchpad constructs and displays the line perpendicular to the selected segment, passing through the selected midpoint.

In this case, because it passes through the segment's midpoint, your perpendicular line is a perpendicular bisector. What do you notice about the perpendicular bisector of the base of an isosceles triangle? Is this true for any isosceles triangle?

## Conjecturing

## Measuring

Just as you use three points to name an angle-such as angle ABC-in Sketchpad you select three points to measure an angle.

## Sketchpad names the

 measurement using the labels of the points that define the angle. You'll learn how to show or change these labels in the next chapter.Dynamic Geometry's real power lies in how it enables you to make generalizations about figures you construct and manipulate.
Steps Display

1. Drag different vertices of the triangle and observe the perpendicular bisector.


By observing the perpendicular bisector in your isosceles triangle as you change the triangle's size and shape you can conjecture (make an educated guess) that the perpendicular bisector of the base of any isosceles triangle passes through the opposite vertex.

Making measurements and observing them while you drag a figure is another way to arrive at geometric conjectures. In this part of the tour, you'll measure the angles in your triangle.
Steps Display

1. Press ESC to deselect all objects, then press ENTER on each of the three vertices of your triangle to select them. Pay attention to the order in which you select them. The second (middle) point you select will be the vertex of the angle you're about to measure.
2. Press F5 Measure: 5 Angle to measure the angle.

3. Press ESC to deselect all, then select the three vertices of the triangle again, with your second selection a different point than the second selection you made in step 1.
4. Press F5 Measure: 5 Angle to measure this second angle.
5. Press ESC to deselect all, then select the points that define the third angle in your triangle, choosing a second selection different from the second selections you made in steps 1 and 3.
6. Press F5 Measure: 5 Angle to measure this third angle.


What do you notice about the three angle measures? Confirm that the two equal measurements are really measures of different angles. (Check that their vertices-the points in the middle of the names displayed in their measures-are different.)

Drag different parts of the figure to verify that the base angles of an isosceles triangle always appear to be congruent.

## Modifying a Sketch

There are various ways to change how things are displayed in a sketchand even to change geometric relationships-besides undoing and hiding. In this part of the tour, you'll learn how to show hidden objects and how to split related objects apart.
Steps Display

1. Press F2 Display: 2 Show All Hidden. The circle you hid earlier should reappear.

2. Drag each of the two points on the circle and observe the perpendicular bisector of the chord between them. Also observe which of the two points
 does not change the circle's radius.
3. Animate the point that does not change the circle's radius. (Select it and Press F2 Display: 8 Animate Point.) Observe the
 chord-the triangle's base segment-and its perpendicular bisector as this point travels around the circle.
4. To stop the animation, Press F2 Display: A Stop Animation (TI-89: alpha A).
5. While the point on the circle is selected, Press F1 Edit: 4 Split Point From Circle (Split From Object on the TI-89). The point is redefined to
 be no longer on the circle.
6. Drag the point to confirm that it's no longer related to the circle.

Now that the point is no longer on the circle, what kind of triangle do you have? Do the conjectures you made about the perpendicular bisector of the base of an isosceles triangle (or of a chord in a circle) and about the base angles of an isosceles triangle still hold for this triangle?

If you wish to "repair" your isosceles triangle, select the point that you split from the circle, and select the circle itself. Then press F1 Edit:
4 Merge Point To Circle (Merge to Path on the TI-89) to redefine the point on the circle.

## Summary

With this relatively simple construction of an isosceles triangle, you sampled many of Sketchpad's features and commands and developed a sense of the power and significance of Dynamic Geometry for making conjectures. The next tour takes you to two menus you haven't seen yet: the F4 Transform menu and the F6 Graph menu.

## Advanced Tour

In the previous introductory tour, you learned how to construct an isosceles triangle using a Euclidean (compass and straightedge) approach. In this section, you'll use transformations to construct another isosceles triangle. Then you'll investigate your triangle on a coordinate grid.

## Transformational Approaches

Reminder: A shortcut for activating the segment tool is $\bullet$ F4 (TI-89: $\bullet$ 4). Repeat the shortcut to scroll through the palette of straightedge tools.

Reminder: On the TI-89,
CLEAR is a shortcut for ENTER when using tools and selecting objects.

Sketchpad's F4 Transform menu lets you translate (slide), rotate (turn), dilate (shrink or stretch), or reflect (flip). In this part of the tour, you'll use a reflection to construct an isosceles triangle.

## Steps Display

1. Start Sketchpad, or, if it's started already, press F1 Edit: 9 Sketch: 1 New to open a new sketch.
2. Press F8 (TI-89: 2nd [F8]) $\odot$ until the Line tool (as opposed
 to the Segment or Ray tool) blinks, then press ENTER to activate it.
3. Press ENTER © ENTER to draw a vertical line.
4. With the line still selected, Press F4 Transform: 2 Mark Mirror to mark the line as the mirror across which you'll reflect objects. Sketchpad confirms this mark with a brief animation.
5. Choose the Point tool from the toolbox: F8 (TI-89: 2nd [F8]) © ENTER.
6. Move the cursor to one side of the line and press ENTER to construct a point nearby.

7. With the point still selected, Press F4 Transform:
A Reflect (TI-89: alpha A).


Sketchpad constructs and displays the image of your point reflected across the marked mirror.
8. Press ESC to return to the Arrow, then drag either point to see how its mirror image behaves. Try dragging the mirror and the points defining it too.

## Analytic Approaches

The unit point of a coordinate system appears beside the origin, and determines the magnitude of that coordinate system's units. In a square coordinate system, the unit point appears on the horizontal axis at (1, 0).
9. Press ESC to deselect all. Press ENTER on three points: the two points not on the line and one of the points that defines the line.
10. Press F3 Construct:

4 Segments to construct segments connecting the points
 you selected.
11. Drag some more. What kind of triangle do you have? Notice that it's Drag some more. What kind of triangle do you have? Notice th
controlled by any of four points. One of the points is not on the triangle, but it controls the mirror line.

In this part of the tour, you'll analyze your triangle on a coordinate grid.
Steps Display

1. Press ESC to deselect all objects.
2. Press F6 Graph:
(TI-89: 2nd [F6])
3 Graph Form:
2 Square Grid. Sketchpad
 displays a coordinate system (two axes and a grid).
3. The coordinate system is defined by its origin and a unit point to the right of the origin. To shrink the axes' scales, press
 ESC to deselect all, then drag the unit point toward the origin. Drag it just until you can see the 5 on the $x$-axis.
4. Press ESC to deselect all, then press ENTER on each of the three triangle vertices. Press
F5 Measure: Analytic:

(TI-89: alphal D )
D1 Coordinates.
5. Press ESC to deselect all, then press ENTER on the mirror line. Press $F 5$ Measure: Analytic: (TI-89: alpha D) 2 Equation.

6. Press F66 Graph: (TI-89: 2nd [F6]) 5 Snap To Grid.
7. Drag the points that control the mirror line. Observe how they snap to the grid.
8. Drag the triangle's base vertices. Observe their coordinates as one or both of them snap to the grid. Notice that one point will have whole-number coordinates, while the mirror-image point may not, depending on the angle of the mirror line.


## Conjectures and Challenges

## Summary

Next, you'll make some conjectures about the coordinates of the vertices of your isosceles triangle when it's located in different ways on the coordinate grid. You'll also try to solve some challenges about the sketch.

1. Drag the mirror line's control points so that they lie on the $y$-axis. Drag the triangle base vertices around and observe the coordinates. If a point has coordinates $(a, b)$, what are the coordinates of its mirror image across the $y$-axis?
2. Locate the mirror line on the $x$-axis. Drag the triangle base vertices around and observe the coordinates. If a point has coordinates $(a, b)$, what are the coordinates of its mirror image across the $x$-axis?
3. Drag the mirror line's control points so that the line's equation is $y=x$. Drag the triangle base vertices around and observe the coordinates. If a point has coordinates $(a, b)$, what are the coordinates of its mirror image across the line $y=x$ ?
4. See if you can locate the three vertices of the triangle so that each one has 0 as one of its coordinates. Describe where you located the points.
5. See if you can locate the three vertices of the triangle so that one vertex is at the origin and each of the two other vertices has 0 as one of its coordinates. Describe where you located the points and describe the triangle.
6. Locate the mirror line so that its equation is $y=2 x$. See if you can locate the base vertices so that they both lie on the grid-that is, so that they both have whole-number coordinates. Describe how you did this. (Hint: You might find this easier if you construct the midpoint of the base first.)
In this tour and the previous introductory tour, you've seen two approaches to constructing and investigating an isosceles triangle. In the introductory tour you took a Euclidean approach to the construction, using the drawing tools and the Construct menu. In this tour you took a Transformational approach, using a reflection in your construction. You also analyzed your figure in this tour using a coordinate system. In the process, you were introduced to the power of Dynamic Geometry as well as all of Sketchpad's menus and most of its tools. You may also have discovered something new about geometry!

The answers to Conjectures and Challenges follow.

1. $(-a, b)$

2. $(a,-b)$
3. $(b, a)$
4. Each point must lie on an axis. If the vertex opposite the base is on the $y$-axis, the two base vertices must be on the $x$-axis.

. The vertex opposite the base must be at the origin, with each base vertex on an axis. The triangle is a right isosceles triangle.
5. This is tough! The midpoint of the base is the point where the base intersects the mirror line. The base vertices will both be grid points if and only if this midpoint is a grid point. So, locate a point on the grid that the mirror line passes through, such as $(0,0),(1,2),(2,4)$, and so on. Try to position a base vertex so that the midpoint of the base lands on one of these points. You can use the fact that the slope of the base is $-1 / 2$.

## Where to Go from Here

If you successfully completed the Guided Tours, you've made a good start at learning how to use Sketchpad. At this point, you may choose to explore Sketchpad more on your own, referring back to this documentation as you need it, or you may prefer to read on to learn more about the software's many features. This section gives a brief overview of what you'll find in Sketchpad and previews the other sections of the documentation so that you can more easily find the specific information you're looking for.

## The Sketch

## Tools

See the Tools chapter for more detail about tools.

## Choosing a Tool

Press F8 (TI-89: 2nd [F8]) to enter the toolbox.
Use the $\uparrow$ cursor controls to highlight the tool you wish to use.
Press ENTER to choose a highlighted tool.
Alternately, use the keyboard shortcuts $\rightarrow$ F1- F6 (TI-89: $\bullet$ - $-\boxed{6}$ ) to activate tools directly. The number of the shortcut corresponds with the position of the tool-for example, the Compass tool, which is the third tool down, can be activated using (TI-89: $\square$ F3). For a palette of tools (the Arrow and Straightedge tools), type the shortcut repeatedly to scroll through the palette.

## Drawing

Once a drawing tool-the Point, Compass, Straightedge, or Custom toolis chosen, use the $\uparrow$ cursor controls to move to the location where you wish to start drawing an object. This can be a blank area, an existing point, or on an existing object.

Press ENTER to construct a point. (On the TI-89, you may use CLEAR as a shortcut for ENTER.)

For objects defined by two points-segments, rays, lines, and circles-use () to move to the location where you wish to finish constructing the object, and press ENTER to construct the second point.

The chosen tool remains active until you choose a different tool.

## Dragging

Choose the Arrow tool. You can do this by entering the toolbox F8, (TI-89: 2nd [F8]) highlighting the Arrow, and pressing ENTER, or you can simply press ESC to quit the current tool and choose the Arrow.
Use $\odot$ to position the cursor over the object you wish to drag. When it's over an object, the cursor will turn into a horizontal arrow.

Press and hold down (TI-89: alpha) as you move the object with $\odot$.

## Commands

Again, on the Tl-89, CLEAR may be used as an alternative to ENTER when drawing or selecting objects.

See the Commands section for more details about using menu commands. Sketchpad's six menus are F1 Edit, F2 Display, F3 Construct,
F54 Transform, F5 Measure, and F6 Graph.

## Selecting

To act on objects using menu commands, you'll need to select them:

1. Choose the Arrow tool.
2. Using the $\uparrow$ cursor controls, position the cursor over an object. When it's over an object, the cursor will turn into a horizontal arrow.
3. Press ENTER. Sketchpad selects the object. Select additional objects by repeating steps 2 and 3 .


Selected objects appear thickened. Previously selected objects will remain selected as long as you're selecting objects. Many menu commands depend on multiple selections. If you press ENTER with the cursor in a blank area, everything will become deselected. You can also press ESC to deselect everything.

## Choosing Commands

To open a menu, press its assigned function key. To choose a menu item, press its assigned number or letter, or highlight it using the cursor controls and press ENTER.


The F1 Edit menu (TI-92 Plus/Voyage ${ }^{\text {TM }} 200$ PLT)

These commands allow you to modify your sketch in various ways. Use the 9 Sketch submenu to open a new sketch, open a previously saved sketch, or save the current sketch. The 8 Properties and (A) Preferences (TI-89: alpha A) commands contain many powerful options for tailoring individual objects and general program operations to your needs.

## The F2 Display Menu



Use these commands to hide and show objects and their labels, to trace objects, and to animate objects.

On the TI-89, command A is called "Circle thru Point" and command $B$ is called "Circle by Radius."

This menu provides a large assortment of Euclidean constructions, as well as interiors and loci. Some of these commands construct the same objects as some of the tools. Others are shortcuts for commonly performed constructions that could be done with the tools but would be more time-consuming that way. Each command depends on particular selections in your sketch in order to be available. Unavailable commands appear with a padlock icon next to them.


The F4 Transform menu (TI-89)

Transformations offer powerful methods for establishing relationships in geometric figures. The Mark commands set parameters for the transformation commands in the menu: Translate (slide), Rotate (turn), Dilate (stretch or shrink), and Reflect (flip). In general, you will first mark appropriate parameters, then select objects to transform, and finally choose one of the transformation commands.

On the TI-92 Plus
/Voyage 200 PLT, command
1 is called "Define Coordinate System" and command 2 is called "Mark Coordinate System."

The (F5) Measure Menu


The F5 Measure menu (TI-92 Plus/Voyage ${ }^{\text {TM }} 200$ PLT)

## The F6] Graph Menu



These commands let you work in an analytic geometry environment, as well as explore many of the connections between geometry and algebra.

## Advanced Features

Once you've started to get the hang of making and acting on basic geometric figures, you can use the F2 Display menu to make action buttons. Action buttons can hide, show, and automatically move objects in your sketch, turning it into a presentation that someone else can interact with and learn from. See Action Buttons (page 56) for details.
Using the Custom tool at the bottom of the toolbar, you can turn constructions that you've made, and that you think you might want to make again, into new tools. Custom tools allow you to expand the power and flexibility of Sketchpad, and customize it to your needs. See Custom Tool (page 34) for details.

Finally, as you become familiar with the software, be sure to review Tips and Techniques starting (page 80) for pointers about how to work efficiently and successfully with Sketchpad.

## Tools



This chapter describes the toolbox and explains how to use the various tools to construct and drag geometric objects, as well as to create and edit labels and captions for your figures.

Designing a typeface with compass and straightedge. Before the digital era, typeface designers often used compass and straightedge constructions to guarantee harmonious proportions in their alphanumeric characters. This sketch-depicting a character from the Times Roman typeface-shows arc constructions used to balance the serifs (small "tails") of the character $\mathbf{R}$. The character itself is constructed in Sketchpad as a polygon interior, where the polygon $\mathbf{R}$ has many hidden vertices. Today's computers perform similar constructions
 thousands of times a second, generating polygons from mathematical definitions to display as alphanumeric characters.


## A presentation of the Pythagorean

 Theorem. For this sketch, a Custom Tool that creates squares was defined from a square constructed by compass and straightedge techniques. This tool was then used to build three squares sides on a right triangle. The areas of the two smaller squares were measured and added together using the Calculate command. This sum remains equal to the area of the larger square even when you drag the vertices of right triangle.
## Using Tools

## Choosing a New Tool

Note: Pressing ESCC is a shortcut for choosing the Translate (standard) Selection Arrow tool.

The toolbox appears on the right side of the screen and displays the available tools. The active tool appears highlighted. The active tool determines the function of the cursor in your sketch.


When using a specific tool, the $\odot$ cursor controls position the active tool within the sketch. To change which tool is active, you must first enter the toolbox to the right of the sketch, as described here:
Steps Display

1. Press F88 (TI-89: 2nd [F8]) to enter the toolbox. The active tool begins blinking.

2. Move to the desired tool. Press $\bigcirc$ and $\bigcirc$ to change active (blinking) tools.
3. Some tools have a palette of choices, which appear to the left of the tool. $\bigcirc$ and $\bigcirc$ move through the palette of choices.

4. When you've moved to the desired tool, press ENTER to leave the toolbox. The chosen tool becomes active in your sketch and remains active until you choose another tool.

## Repositioning the View

## Toolbox Shortcuts

A sketch offers a view of the Euclidean plane, which is unlimited in size. Objects that you draw or construct may extend outside the portion of the plane visible in the sketch's window. To reposition the sketch's view of the plane, hold down $\square$ while using the $\oplus$ cursor controls. The sketch scrolls in the appropriate direction. (For example, holding $\square \bigcirc$ moves the view of the plane to the right, scrolling objects visible in the view to the left.) You may reposition the view at any time, regardless of which tool is active in the toolbox.

1. $\square$ F1 (TI-89: $\square$ ) activates the first tool (the Arrow) without having to enter and leave the toolbox. F2 (TI-89: $\square 2$ ) activates the second tool (the Point tool), and so forth. For a tool with a palette of choices, such as the Straightedge tools, pressing the tool shortcut ( $\boxed{\square}$ or $\square 4$ in this case) repeatedly cycles through the tool's choices.
2. On the TI-92 Plus/Voyage ${ }^{\text {TM }} 200$ PLT only, - F8 shows or hides the toolbox. (You may want to hide the toolbox to reclaim space for your sketch.) When the toolbox is hidden, entering the toolbox using F8 will temporarily show the toolbox. When you leave the toolbox again, it will be rehidden. (Use F8 to show it again permanently.)

## Selection Arrow Tools

This section describes the Selection Arrow tools, which you'll use to select and drag objects. Selecting objects is an important technique for using Sketchpad's menu commands. Dragging objects-to explore properties of a construction under a variety of transformations-is the heart of Dynamic Geometry.

## About the Selection Arrow Tools

## Selecting Objects

Many of Sketchpad's commands require that one or more objects be selected before the commands become available. These commands then apply only to the selected objects.

The three Selection Arrow tools can be used to select objects in the same manner, but offer different techniques for transforming objects' positions. The most commonly used tool is the Translate Arrow.


Use any of the Arrows to select objects in your sketch. Sketchpad displays selected objects by highlighting them.
Deselected

To select an object:

## Steps

## Display

1. Make sure that an Arrow is the active tool. If not, see Choosing a New Tool (page 22).
2. Using the $\dagger$ cursor controls, move the Arrow to the object you wish to select.

3. The Arrow changes appearance when it points to the object.

4. Press ENTER. The object becomes selected.

5. Repeat steps 2 through 4 to select additional objects.

## Deselecting Objects

To deselect a single object, without changing your other selections, point the Arrow to the selected object and press ENTER a second time. (ENTER selects deselected objects, and deselects selected objects.)

To deselect all selected objects at once, point to white space in your sketch (away from any object) and press ENTER.

Another way to deselect selected objects is to press ESC.

## Selecting Objects with a Marquee

Multiple objects may also be selected by a marquee-a rectangle drawn around a group of nearby objects which selects them as a group.

To select with a marquee

## Steps Display

1. Make sure that an Arrow is the active tool. If not, see Choosing a New Tool (page 22).
2. Using the $\uparrow$ cursor controls, move the Arrow to white space near one corner of the objects you wish to select.
3. Press and hold :
(TI-89: alpha). Then move the Arrow to the opposite corner of the objects you wish to select. Sketchpad displays a selection marquee rectangle between the two corners.
4. Release (TI-89: alpha).

Every object inside the marquee, or intersecting the


## ฝ

 marquee, becomes selected.

For convenience, Sketchpad deselects previous selections when starting a new marquee. Press and hold 2nd before beginning a marquee to prevent this deselection-that is, to create a marquee which adds objects to the current selection, rather than creates a new selection consisting only of objects that lie within or intersect the new marquee.

## Dragging Objects

Use any of the Arrows to drag objects and thereby change their size and orientation. How an object's size and orientation changes depends on the Arrow that is used-Arrows can translate, rotate, or dilate objects. The Arrow's transformation applies only to the object or objects that are dragged; objects that are geometrically related to the dragged objects will adjust their positions automatically according to the changes to the dragged object(s).

## Steps (Translation Example) Display

1. Make sure the Translate Arrow is the active tool. If not, see
® Choosing a New Tool (page 22).
2. Using the $\uparrow$ cursor controls, move the Translate Arrow to the object you wish to drag.

3. Press and hold ©
(TI-89: alpha). Then use the $\odot$ cursor controls to drag your object. As you drag, all related
 objects adjust automatically.
4. Release (TI-89: alpha) to stop dragging.

To dilate or rotate an object, follow the same steps, but use the Dilate or Rotation Arrow instead of the Translate Arrow.

## Translate Arrow 5alin

Rotate Arrow CH0

Dilate Arrow
A $x_{0}$ 相

## About Marked Centers

## Dragging Labels

Dragging an object with the Translate Arrow translates that object, moving it in the same distance and direction as your cursor. Translation does not change the size or orientation of an object. Related objects adjust automatically.

Dragging an object with the Rotate Arrow rotates that object in a circle about the point in your sketch currently marked as center. Rotation changes the orientation but not the size of an object. Related objects adjust automatically.

Dragging an object with the Dilate Arrow dilates that object toward or away from the point currently marked as center. Dilation changes the size but not the orientation of an object: as you dilate toward the marked center, the dragged object(s) becomes smaller; as you dilate away from the marked center, the object(s) becomes larger. Related objects adjust automatically.

The Rotate and Dilate Arrows drag your object with respect to a point that is marked as the center of transformation. If you have not yet marked a center point, Sketchpad will mark one for you when you attempt to use these Arrows. To mark a different point as the center of dilation, see Mark Center (page 65). If you attempt to drag the center point itself with the Rotate or Dilate Arrow, nothing will move-a point dilated or rotated about itself remains fixed.

When an Arrow is positioned over an object's label, but not over any object, it momentarily becomes the
 Text tool, which repositions labels. See Text Tool (page 31) for more information.

## Point Tool

Use the Point tool to create independent points or to construct points on objects or points of intersection.

## Creating Independent Points

The shortcut for activating the Point tool is $\square$ F2 (TI-89: -2 ).

Use the point tool to create independent points.
Steps Display

1. Make sure the Point tool is active. If not, see Choosing a New Tool (page 22.
2. Using the $\uparrow$ cursor controls, move the Point tool to a blank portion of your sketch. Then press ENTER. Sketchpad creates a point at the tool's location.
3. Repeat step 2 to create as many additional points as you want.
$\circ \quad 0$

When Sketchpad creates a new point, it deselects other objects (if any) and selects that point. If you hold 2nd while pressing ENTER, Sketchpad will not deselect other objects before creating the new point.

## Constructing Points on Objects

See Point on Object (page 58) for a full discussion of points on objects.

## Constructing Points of Intersection

See Intersection (page 59) for a full discussion of points of intersection.

To construct points on other objects, position the Point tool on top of another object-such as a segment or a circle-before pressing ENTER. The object will appear highlighted when you point to it, indicating that the Point tool is ready to construct a point on it.

To construct points of intersection, position the Point tool at the intersection of two objects-such as a segment and a circle-before pressing ENTER. The objects will appear highlighted when the cursor is properly positioned, indicating that the Point tool is ready to construct their intersection.

## Compass Tool

## Constructing Circles

The shortcut for activating the Compass tool is $\square$ F3 (TI-89: $\bullet 3$ ).

Use the Compass tool to construct circles in your sketch. The Compass draws circles from a center point to a second point on the circle's circumference.

The Compass tool requires two points to construct a new circle. These points may already exist or can be created as the tool is used. The first point defines the center of a circle. The second point defines the radius of the circle.
Steps Display

1. Make sure the Compass tool is the active tool. If not, see Choosing a New Tool (page 22).
2. Using the $\uparrow$ cursor controls, move the Compass tool to where you'd like the circle's center to be (at an existing point, on a path, in a blank area of the sketch, or at the intersection of two objects). Then press ENTER.
3. Use the $\uparrow$ cursor controls to move away from the center point. As you move, Sketchpad
 displays a circle growing from the center.
4. Position the Compass tool where you'd like the point defining the circle's radius to be (again, at an existing point, on a path, in a blank area of the sketch, or at the intersection of two objects). Press ENTER a second time.
5. Sketchpad constructs and displays a circle centered at the first point, passing through the second point.


As mentioned in steps 2 and 4, the Compass tool may be positioned over an existing point, over blank space, over a path, or over the intersection of two objects. If the Compass is positioned over an existing point, Sketchpad uses that point to define the circle's center or radius point, as the case may be (in other words, it doesn't construct a new point there). If the Compass is positioned over a path, Sketchpad constructs a point on that path object. If the Compass is positioned in blank space, Sketchpad creates a new point there when you press ENTER. And if the Compass is positioned over the intersection of two objects, Sketchpad constructs the point of intersection.

Note that if you construct a circle so that its circumference appears to pass through an existing point, but the radius (second) point was constructed elsewhere, then when you drag the circle, it will no longer pass through the
existing point. This is because the existing point does not define the circle's radius.

## Examples: Two Different Radius Points

Given two points $A$ and $B$, use the Compass tool to create a circle with $\phi_{\mathrm{A}} \quad \theta_{\mathrm{B}}$ center $A$ and radius point $B$.

This circle is defined as centered at point $A$ and passing through point $B$. Moving $B$ will change the circle's radius.


Imagine, however, that you had pressed ENTER the second time in a blank area (thus constructing point $C$ as shown at right) in such a way that the circle appeared to pass through
 point $B$. In this case, moving $B$ will not change the circle's radius, because the circle is defined by $A$ and $C, \operatorname{not} A$ and $B$.

Be careful to choose the defining points you want!

## Straightedge Tools

Use the Straightedge tools to construct segments, rays, and lines. Each of these objects is defined by two points that determine its direction and, in the case of a segment, its length.


## Segment Tool

The shortcut for activating the Straightedge tools is $\square$ F4 (TI-89: 4). Type these shortcuts repeatedly to scroll through the various Straightedge tools.

When creating a segmentor any other straight object-holding down the 2nd key while dragging the second endpoint constrains the slope in increments of $15^{\circ}$. You might use the 2nd key when you want to draw a horizontal or vertical segment, or a line at a $45^{\circ}$ angle.

Ray Tool

## Line Tool

The Segment tool requires two endpoints to construct a new segment.

## Steps Display

1. Make sure the Segment tool is active. If not, see Choosing a New Tool (page 22).
2. Using the $\uparrow$ cursor controls, move the Segment tool to where you'd like one endpoint of the segment. Then press ENTER. Sketchpad creates the first endpoint at the tool's location.
3. Use $\odot$ to move away from the first endpoint, and press ENTER again when you've reached the
 second endpoint. Sketchpad constructs and selects the segment.

As with the Compass tool, you may place endpoints at existing points, on existing objects, at intersections of two existing objects, or in blank areas as new, independent points.

The Ray tool acts like the Segment tool, requiring two points to define a ray. The first point determines the start, or initial point, of the ray. The ray passes through the second point, and continues infinitely far in that direction.

The Line tool acts like the Segment tool, requiring two points to define a line. These points determine the line's direction, and the line extends through them infinitely far in each direction.

## Text Tool

With the Text tool, you can show and hide labels, create captions, and edit existing text. Text can be used to annotate your sketches before sharing them with others.

## Showing Labels

The shortcut for activating the Text tool is - F5 (TI-89: 5).

If the Text tool is not shaded when pointing at an object, that type of object cannot display a label.

For convenience, when you show the label of a polygon with fewer than seven vertices, Sketchpad shows the vertex labels rather than the polygon's label. That way you can refer to "Triangle ABC" instead of "Triangle P1." Use Label Properties (see page 46) if you want to show a polygon's label itself, instead of showing its vertex labels.

Most objects in Sketchpad can display a label-a short name or description of the object. (Some types of objects-such as measurements and action buttons-appear with text as part of the object. These objects don't have separate labels.)
To create, display, or hide a displayed label:
Steps Display

1. Make sure the Text tool is active. If not, see Choosing a New Tool (page 22).
2. Using the $\odot$ cursor controls, point the Text tool at an object in your sketch. The Text tool becomes shaded, indicating that it's pointing at an object.
3. Press ENTER to display the object's label.

4. Point to other objects and press ENTER to display their labels as well. Or point to an object which is already displaying a label and press ENTER to hide that label.

## About Default Labels

When an object's label is displayed for the first time, Sketchpad creates a default label based on the type of object it names and the order in which it is displayed. (The next section describes how to change an object's default label.)

| Type of Object | Label Sequence |  |
| :--- | :--- | :--- |
| 1. | Points | $A, B, C, \ldots$ |
| 2. | Segments, Rays, | $j, k, l, \ldots$ |
|  | Lines |  |
| 3. | Circles | $c l, c 2, c 3, \ldots$ |
| 4. | Polygon interiors | $P 1, P 2, P 3, \ldots$ |

If an object's defining points are labeled but the object itself has not yet been labeled, any measurement will refer to the object indirectly by
naming its defining points. If a label has been created for the object itself, measurements refer directly to the object by its own label.
Indirectly Labeled

## Repositioning Labels

You can also reposition labels using the Selection Arrow by pointing to a part of a label which is not overlapping an object. The Selection Arrow tool turns into the Text tool momentarily to indicate you can reposition the label.

The Text tool also repositions labels to achieve a pleasing arrangement or to avoid overlap between labels and objects.
Steps Display

1. Using the $\uparrow$ cursor controls, point the Text tool at an existing label. The tool changes its appearance (the small letter "A" appears) indicating that you can
 reposition the label to which it's pointing.
2. Press and hold ©
(TI-89: alpha). Then use the $\odot$ cursor controls to reposition the label. You can drag the label a small distance around the
 object's perimeter, but Sketchpad prevents you from dragging it too far.
3. Release (TI-89: alpha).

Captions are blocks of text that are not associated with specific objects. Add captions to your sketches to give them titles, or to explain your constructions.
Steps Display

1. Using the $\odot$ cursor controls, point the Text tool at blank space (away from all objects or labels). The tool's appearance (no letter "A" and not filled)
 indicates that it can create a caption at the indicated position.
2. Press ENTER. Sketchpad opens the Edit Caption dialog box for a new caption.
3. Type the text you wish to appear in your sketch.

4. Press F1 to close the dialog box. Sketchpad displays the new caption at the location where you originally clicked.

Changing Labels, Captions, or Measurements

See page 45 for more information about the Properties dialog box.

The Text tool also allows you to change or edit the existing text of labels, captions, or measurements.
Steps Display

1. Using the $\odot$ cursor controls, point the Text tool at existing text. The tool changes its appearance when pointing at editable text (a small "A" appears).
2. Press ENTER. Sketchpad opens the Properties or Edit Caption dialog box for the indicated object.

3. Edit the existing text.

4. Press ENTER twice to close the dialog box. Sketchpad updates the edited text.


## Custom Tool

This section introduces custom tools-Sketchpad's advanced facility for creating new tools and adding them to the toolbox. Custom tools allow you to enrich Sketchpad's mathematical vocabulary with as many new operations as you wish. You should be familiar with Sketchpad's basic operations before attempting to make custom tools.

## About Custom Tools

If you're familiar with The Geometer's Sketchpad for desktop computers, custom tools are similar to desktop Sketchpad's "script tools."

Custom tools are tools you define yourself for adding new constructions to the toolbox. Just as the basic toolbox comes with tools for constructing segments, rays, lines, and circles given two points, you can make new construction tools that construct any object-or any set of objects-given one or more points (or other objects). For example, you can create tools to construct triangles, special triangles (such as equilateral triangles), or more complex sets of objects. You can also create tools that construct single basic objects (such as lines and circles) in new ways. For instance, you can make a tool that, given a segment, creates the perpendicular bisector of that segment, or a tool that, given three points, constructs the circle passing through those three points. Custom tools allow you to re-use constructions easily and efficiently.
Before creating custom tools, it's useful to review some facts about geometric constructions. All objects in a construction may be divided into three groups. The independent objects, or givens, are objects that are necessary for the construction but that are not defined by the construction. The dependent objects are objects that are defined by the givens-or by other dependent objects in the construction. These dependent objects may be divided into results, which are the objects of ultimate interest in the construction, and intermediate construction objects, which are dependent objects necessary to produce the results but not of particular interest themselves. Here are some examples:

| Construction | Objects |
| :--- | :--- |
| General Triangle | Givens: 3 points |
| Construction Objects: none |  |
| Results: 3 segments |  |
|  |  |

When you create custom tools, you'll define them as constructions that construct results from given objects, possibly using intermediate construction objects. The resulting custom tool will produce the corresponding results when applied from the toolbox to any equivalent set of given objects.

Custom tools can define any geometric construction. For purposes of example, this section will describe the process of creating a custom tool that creates perpendicular bisectors.

## Creating a Custom Tool

To create a new custom tool, first create the general construction that you wish to define as a tool. (This construction will serve as the "example definition" when creating the tool.) Create the construction using any available means, such as existing tools and commands from the Construct, Transform, Measure, and Graph menus. Before creating your tool, drag objects in the example construction to verify that it's correct.

## Example Construction: Perpendicular Bisector

1. In a new sketch, use the Segment tool to create a new $\quad \stackrel{\square}{\mathrm{A}}$ segment.
2. With the segment selected, Press F3 Construct:


2 Midpoint. Sketchpad displays and selects the midpoint of the segment.
3. Press ESC to activate the Arrow.
4. With the midpoint still selected, point the Arrow to the segment.
 Press ENTER to select the segment.
5. Press F3 Construct: 8 Perpendicular Line.
Sketchpad displays a line perpendicular to the segment passing through its midpoint. (This line is the perpendicular bisector of the segment.)

Once you have created your example construction, select the independent objects that define the custom tool's construction as well as the results that you wish the custom tool to produce. (You do not need to select any intermediate construction objects.) Then create a new tool, using the Custom Tool's pop-up menu.

The shortcut for activating the Custom tool is $\square$ F6 (TI-89: $\square$ )

## Using a <br> Custom Tool

## Example Custom Tool Creation: Perpendicular Bisector

1. Verify that your example construction is correct by dragging.
2. Use the Arrow to select the independent objects that serve as your construction's givens. In this example, select the two endpoints of the original segment.
3. Also select the results you wish the tool to create. These result objects must ultimately be defined in terms of the selected independent objects. In this example, select the perpendicular bisector.
4. Activate the Custom tool in the toolbox by pressing F8
(TI-89: 2nd [F8]), $\odot$ to the Custom tool, and then pressing ENTER to activate your choice.
5. Sketchpad presents a pop-up menu of Custom Tool options. Choose Create New Tool from
 the menu.
6. Sketchpad presents a dialog box asking you to name your new tool. Type PerpBisector and press ENTER twice to close the dialog box.

Congratulations! You have just created and named your first custom tool. To try out this tool immediately, continue to the next section. For additional information on creating other custom tools, skip to More About Tools (next page).

To activate a custom tool that you've previously defined, choose it from the Custom tool's pop-up menu in the toolbox. Then use the tool as you would other drawing tools such as the compass and straightedges, by positioning it in your sketch and pressing ENTER.

## Example Custom Tool Use: Perpendicular Bisector Tool

1. In a new sketch, activate the Custom tool from the toolbox.
2. Sketchpad presents a pop-up menu listing the custom tools you've created. Choose the
 PerpBisector tool you created in the previous steps from this menu.

Your PerpBisector tool requires two endpoints (two presses of [ENTER) because these were the two independent, given objects you selected when creating the tool. You can create tools that require two points, one point, ten points, or any number of other objects-it all depends on the givens of your construction.
3. The drawing tool icon appears in your sketch. Use $\odot$ to move it wherever you wish; then press ENTER. Sketchpad creates the first of the two endpoints of your perpendicular bisector.
4. Use $\odot$ to move the tool away from the first endpoint. As you move, Sketchpad displays the second endpoint and the resulting perpendicular bisector (but not your intermediate construction objects-the segment and midpoint).
5. Position the second endpoint wherever you wish, and press ENTER a second time to place the second endpoint. Sketchpad
 selects the resulting perpendicular bisector.
6. Continue using the PerpBisector tool to construct perpendicular bisectors, moving endpoints with $\Theta$ and pressing ENTER to finalize their positions. After constructing several perpendicular bisectors, press ESC to return to the Arrow tool.

You've now successfully created and used your first custom tool. Take the time to try making a few new tools on your own. Any time you find yourself repeating a construction frequently, consider turning it into a custom tool for greater convenience and flexibility.

## Tool Variations

In the perpendicular bisector example, the tool you created required two points-because you selected two points as givens when defining the tool. In general, the objects required by the tool correspond to the objects you selected as givens when defining it, in the order in which you selected them.

1. Try using the PerpBisector tool, but place the endpoints of the perpendicular bisector on existing points. Like the built-in drawing tools, your custom tools can place independent endpoints at new locations, or place endpoints at existing points, on objects, or at intersections.
2. Try defining a variation of the PerpBisector tool called PerpSegmentBisector, You'll have to recreate your original example construction. But this time, rather than selecting the two endpoints as givens, select the segment itself as your given. Then select the bisector, too, as the result of the construction. Then define a new custom tool. This new tool behaves like the original, except that with this one you can point to an existing segment in a sketch and construct its bisector automatically, without having to point to the segment's endpoints. That's because you've defined the construction in terms of a given segment, instead of in terms of two given points.

You can also use F1 Edit: 6 Select Parents when your results are selected to see the objects that define them. Repeatedly choosing Select Parents will select the independent points that ultimately define your desired results.

## Editing Custom Tools

Periodically delete unused tools to avoid filling up memory.

You only need to (re)open toolkits if you've deleted their tools from the custom tools menu, or if you've received the toolkit from another graphing calculator using VAR-LINK.
3. Try defining a variation of the PerpBisector tool in which you select not only the endpoints and bisector, but the segment and midpoint as well. This tool, given two points, will produce three results instead of one: the segment, its midpoint, and the bisector.
4. Try defining other tools to construct triangles (given three points), squares (given two points defining an edge), centroids or other special points in a triangle (given three points), or a tool for any construction you might use repeatedly.

## What Can Go Wrong

When you choose Create New Tool to define a custom tool, you may see the following message:


This message appears when the selected objects do not define a construction-when the results you've selected are not completely specified (constructed by) the givens you've selected. If you're uncertain which objects you need to select as givens, the easiest approach is to define your example in a new (otherwise empty) sketch. Then press F1 Edit:
5 Select All before creating your new tool. This will guarantee that you've selected appropriate givens. From there, experiment with defining other tools by selecting fewer and fewer objects in the sketch.

Use the Edit Tools command on the custom tools pop-up menu to present the Edit Tools dialog box. This dialog box allows you to delete tools you've created, or to save collections of them as toolkits and reopen saved tools from existing toolkits. By saving collections of tools as toolkits, you can distribute tools you create to others using VAR-LINK. (See the guidebook for your handheld device at education.ti.com/guides for instructions on using VAR-LINK to transmit saved toolkit variables between graphing calculators.)

Once a tool has been defined, it stays in memory and appears in the custom tools pop-up menu until you delete it. Before deleting, you can save individual tools or collections of tools. At any time, you can reopen saved tools and add them to your custom tools pop-up menu. (Like newly created tools, reopened tools remain available until you delete them.)
The Edit Tools dialog box works like the VAR-LINK screen. It displays a list of all open custom tools (i.e., all custom tools in your Custom tool popup menu), with one tool active. Use:

| $\bigcirc$ and $\bigcirc$ | To change the chosen tool in the list. |
| :--- | :--- |
| F2 $V$ | To check or uncheck the current chosen tool. |
| F3 All | To check or uncheck all tools in the list. |
| F1 Manage | To delete or save the currently checked tools, or to |
| re-open saved tools. |  |
| ENTER | To leave the Edit Tools dialog box. |

## Commands



This chapter explains how to use the commands in Sketchpad's six menus: F1 Edit, F2 Display, F3 Construct, F4 Transform, F5 Measure, and F6 Graph. The first section offers general information about using menus. Other sections describe specifically how to use the commands in each menu.

A sine wave emerging from a Sketchpad animation. This sketch began with a horizontal segment and a circle centered at its left endpoint. Points $A$ and $B$ were constructed on the circle and segment, respectively. A horizontal line through $A$ intersects a vertical line through $B$ at point $C$. When $A$ and $B$ are animated at the same speed, $C$ travels along the sine wave. The shaded image is the trace of segment $B C$ as both points animate slowly. How long would the original horizontal segment have to be in order for the sine curve to trace exactly one wave period?


A fishy tessellation. Periodic tessellations are geometric patterns that, when repeated, fill the plane. A starting rectangle was used to construct an irregular polygon whose vertices included the rectangle's corners. This polygon-or tile-was then translated by vectors from each of the rectangle's corners to the corner diagonally opposite, resulting in four new images of the tile. These translations were repeated as necessary to fill the screen, with white spaces left over forming "opposite" tiles. Dragging the vertices of the original tile produces other related tessellations.

## Using Menus

This section explains in general how to choose menu commands and what is meant by the different symbols that sometimes appear with commands. It also describes how to access shortcuts for frequently used menu commands.

## Choosing Menu Commands

## Command Appearances

Besides the numbers and letters assigned to menu commands, you'll see icons and symbols next to some commands. These are explained below.

## Padlocked Commands

A padlock icon ( $\mathbf{(})$ next to a command indicates that the command is not available at the moment because the objects required by the command have not been selected in the sketch. For example, F3 Construct: 2 Midpoint will be padlocked unless one or more segments (and only segments) are selected in the sketch. If one or more segments are selected, the padlock will not appear.

If you think the right objects are selected for a command but it's still padlocked, chances are you have selected something extra. In that case, identify the extra selection and press ENTER on it to deselect it. Or press ESC until nothing is selected, and then make your selections again.

If you choose a command when a padlock appears beside it, Sketchpad displays a message telling you that the command is unavailable and indicating the correct selections required to enable the command.


For example, if you choose a padlocked F3 Construct:
2) Midpoint when no segments are selected, you'll see this message.

## Checked Commands

A few commands-F2 Display: 6 Trace; F6 Graph: (TI-89: 2nd [F6])
5 Snap To Grid; and the commands in the Graph Form submenu in the F66 Graph menu-may appear with a check mark next to them. These are commands that are toggled "on" or "off" as they are chosen repeatedly. Choosing a checked command in a menu unchecks it and turns it off. For more detail about how these commands work, see their descriptions in this chapter.

## Commands with Ellipses

An ellipsis (...) at the end of a command name indicates that choosing that command will bring up a dialog box in which you can make further choices about the command. For example, if you press F4 Transform:
8 Rotate..., Sketchpad presents a dialog box in which you can enter an angle of rotation.


## Commands with Submenus

Commands that appear with $>$ before the command name and $\downarrow$ at the right edge of the menu have submenus of additional commands associated with them.


For example, if you press F1 Edit: 9 Sketch, you'll open a submenu from which you can press 1 New, 2 Open, or 3 Save.

You can invoke certain menu commands directly from the keyboard, without opening a menu. Keyboard shortcuts always start with the $\square$ key on the TI-92 Plus/Voyage ${ }^{\text {TM }} 200$ PLT, and start with the $\dagger$ or the $\square$ key on the TI-89. For example, you can Undo your last action by pressing $\quad$ (TI-89: $\dagger$ ) and then the $Z$ key instead of having to open the F1 Edit menu (and subsequently blocking your view of your sketch). On the TI-92 Plus /Voyage 200 PLT, if a menu command has a keyboard shortcut, Sketchpad displays the shortcut next to the command in the menu (shortcuts are not displayed in the menus on the TI-89). See Menu Command Shortcuts (page 81) for a complete listing of these shortcut key combinations.

## Menu Shortcuts

The F1 Edit menu contains commands for changing and navigating through your sketches, including undoing and redoing actions, deleting objects, redefining relationships, selecting objects, reviewing properties, saving and opening sketches, and creating new sketches. This section explains how to use Edit menu commands.

## Undo

## Redo

Not every action is stored in the Undo history-only constructions that affect objects, their creation, or their location. The text accompanying the Undo and Redo commands in the Edit menu will describe which action is about to be undone (or redone).

1 Edit: 1 Undo revokes the most recent action taken in the sketch. Sketchpad supports unlimited undo; so, by choosing Undo repeatedly, you may undo your construction, step by step, back to the moment at which you began work in the sketch. This command is available only after one or more actions have been taken in a new or newly opened sketch.

F1 Edit: 2 Redo returns the most recently undone action to the sketch. Use this command after undoing one or more actions to trace your steps forward again. This command is unavailable if you haven't undone an action, or if, since undoing an action, you have performed some other action that modifies your sketch.

## Unlimited Undo and Visual Explanations

In addition to helping correct mistakes, Sketchpad's unlimited undo and redo is well-suited for sharing visual explanations of your work, or to inspect a construction created by someone else. Choose Undo repeatedly to return to the beginning of your work. Then choose Redo to proceed one step at a time through the construction, to explore or explain how the construction was assembled.

## Unlimited Undo and Memory

Because Sketchpad stores all of the actions performed in a sketch, the Undo history can take up a considerable amount of calculator memory. Sketchpad refreshes its Undo history whenever you reopen a saved sketch. To save and reopen a sketch while working on it, thereby refreshing the Undo history, press F1 Edit: 9 Sketch: 2 Open and then press ENTER to indicate that you do want to save changes before closing. Type the name of the sketch (either a new name, if this is the first time this sketch has been saved, or the previously entered name of the sketch) in the Variable field of the Save As dialog box that appears, and then press ENTER twice. Navigate the Open dialog box that appears until the name of the just-saved file is highlighted, and once again press ENTER twice. Do this whenever you find yourself running low on memory.

Also, as saved sketches use valuable memory, it's a good idea to periodically delete sketches you no longer wish to keep. To delete sketches, use the 2nd[VAR-LINK] menu. To use the [VAR-LINK] commands, see Delete vs. Undo (page 87) or consult the guidebook for your handheld device at education.ti.com/guides.

## Delete

Delete works best for modifying an existing sketch. If you want to get a blank sketch, press F1 Edit:
9 Sketch: 1 New instead of deleting. To remove the most recently constructed objects, press $\mathbb{F 1}^{1}$ Edit: (1) Undo.

F1 Edit: 3 Delete deletes selected objects, removing them from the sketch. Objects that are not selected but whose definitions depend upon selected objects are also deleted. This command is available only when one or more objects are selected. Use F2 Display: 1 Hide if all you want to do is remove an object from view without permanently deleting it and all the other objects defined by it.


The Split and Merge commands alter geometric definitions in the sketch by separating or combining objects. These commands are available only when one or more objects are selected. The behavior of the command depends upon the geometric definition(s) of the selected object(s). Split and Merge are opposites; anything that can be split apart can be later merged together, and anything that can be merged can later be split.

## Splitting Points from Objects

To separate a point constructed on an object (such as a point on a segment, a midpoint, or an intersection) from that object, select the constructed point and choose Split.

## Example: Splitting a Point from a Segment

1. Select a point constructed as a

Point on Segment or as a
Midpoint of a segment.

2. Press F1 Edit: 4 Split Point From Segment (Split From Object on the TI-89). The point separates from the segment.


When a point is split from its object, it becomes an independent point that can be dragged anywhere in the sketch.

## Splitting Points Apart

To split an independent point that constructs more than one dependent objects into multiple points, select the independent point and choose Split Point.

## Example: Splitting a Point Apart

1. Select an independent point that defines more than one dependent objects.

2. Press F1 Edit: 4 Split Point. The point separates into multiple points.


If you want to merge two points, neither of which is independent, first split one of the dependent points from its parent. This will make it an independent point. Then you can merge that independent point to your other dependent point.

Use Merge Text if you want to create single phrases that contain more than one measurement. For instance, you could merge two measured lengths into a sentence that reads "The model is 5.2 cm high and 3.0 cm wide."

## Merging Two Points

To combine two points into a single point, select the two points and choose Merge Points. This command is available only when at least one of the two selected points is an independent point (and the second point is not dependent on the independent point).

## Example: Merging Two Points

1. Select two points.

2. Press F1 Edit: 4 Merge

Points. The points converge into a single point.


## Merging a Point to an Object

To merge an independent point to a path object (such as a segment, circle, point locus, or polygon perimeter), select both the point and the path object and choose Merge [Point] To Path. This command is available only when one independent point and one path (not constructed by that independent point) are selected. After merging a point to a path, the point's location will be constrained to possible locations on the path, as if you had constructed the point using Point on Object (see page 58).

## Example: Merging a Point to an Object

1. Select an independent point and a path (in this case, a circle).

2. Press F1 Edit: 4 Merge Point to Circle (Merge to Path on the TI-89). The point moves to merge with the path.

## Merging Text

To merge separate pieces of text together into a single sentence, select one or more captions, as well as any measurements or other text objects that you wish to merge. Then press F1 Edit: 4 Merge Text. Sketchpad consolidates the separate pieces of text into a single caption.

## Splitting Merged Text

To return a single caption of merged text to its component parts, select the merged caption and press F1 Edit: 4 Split Text. Sketchpad splits the caption into the parts that were originally merged.

## Select All

When a Selection Arrow is the active tool, Select All selects all visible objects in the current sketch.
Before

When a drawing tool is active, this command selects all objects of the same type as the active tool. For example, when the Compass tool is active, this command becomes Select [All] Circles.

## Select Parents

## Select Children

## Properties...

Select Parents selects the objects that geometrically define the currently selected objects. Use this command to determine which objects were used in the construction of a given selected object.
Before

Select Parents does not change the selection if the currently selected object is independent-that is, if it has no parents.

Select Children selects the objects that are geometrically defined by the current selected objects. Use this command to determine the set of objects that were constructed from a given selected object.
Before

Select Children does not change the selection if the currently selected object has no children.

Properties allows you to change a variety of properties of the selected object. Select a single object and press F1 Edit: 8 Properties to display a dialog box in which you can alter the object's properties.

The Properties dialog box is arranged into separate pages of related properties. Flip between different property pages by using the function keys listed at the top of the dialog box. (The current page's name appears in capital letters.) Not all pages will appear in the dialog box for every object; Sketchpad displays only the property pages appropriate to the selected object as described below.


This page contains properties shared by all objects and appears for every object. The object's geometric definition is stated at the bottom of the page, frequently in terms of its relation to its parental objects (the objects that geometrically define this object). You can switch to the Properties of these related objects using this page's Parents and Children menus.

| Visibility | Determines whether the object is showing or hidden <br> in the sketch. You can show and hide objects by <br> changing their visibility in Object Properties or by <br> using the Hide Object and Show All Hidden <br> commands in the F2 Display menu. |
| :--- | :--- |
| Parents | This pop-up menu lists all of the parents of the <br> selected object- that is, all objects that <br> geometrically define the selected object. (No parents <br> will appear if the selected object is independent.) <br> Choose a parent from the menu to switch to the <br> Properties dialog box for that parent. |
| Children | This pop-up menu lists all of the children of the <br> selected object-that is, all objects that are <br> geometrically defined by the selected object. (No <br> children will appear if you have not constructed any <br> objects using the selected object as a parent.) <br> Choose a child from the menu to switch to the <br> Properties dialog box for that child. |

## Label Properties



This page contains properties related to the chosen object's label. It appears in the Properties dialog box only for objects that can have labels.

Label | Displays current label of the selected object. (This |
| :--- |
| may be blank if a label has not yet been assigned to |
| the object.) Type in this field to set or change the |
| object's label. See Text Tool (page 31) for more |
| information about labels. |

See Custom Tool (page 34) for more information.

Changing a measurement's prefix and suffix can turn it into a descriptive sentence. For instance, you could change a measurement that reads Length (AB) = 1.2 cm to read The triangle's base is 1.2 cm long. See Measurement Display (page 70) for more information.

Label Visibility Determines whether the object is currently displaying its label in the sketch. (This property is unavailable for objects that cannot display labels.) You can also control label visibility using the Text tool or with the Show/Hide Labels command (page 53) in the F2 Display menu.
Use in Custom Tools Determines whether the object's label will be used exactly by any custom tool based upon the object. Normally, custom tools assign new and unique labels to the objects they create. However, if you are defining a custom tool that includes an object with a special label—such as "hypotenuse" or "orthocenter"-that you wish to duplicate whenever the tool is used, you may wish to set the Use in Custom Tools property to Yes.

## Measure Properties



This page contains properties about a measurement's display. It appears only when a measurement or calculation is selected.

## Prefix

## Suffix

## Precision

 digits you wish to display in the current measurement. Sketchpad initially chooses a measurement's precision based on your Preferences (see page 52), but you can choose a different precision for a measurement here.
## Locus Properties



The default number of samples assigned to newly constructed loci is determined by Preferences (see page 51).

The default number of samples assigned to newly constructed function plots is determined by Preferences (see page 51).

This page contains properties about loci. It appears only when a locus is selected. (See Locus, page 62, for more information about loci.)

| \# of Samples | Determines how many samples Sketchpad will plot <br> when displaying the locus. The greater the number <br> of samples, the more geometrically accurate the <br> depiction of the locus and the smoother its <br> appearance on the screen. However, the greater the <br> number of samples, the slower Sketchpad will be in <br> calculating and displaying the locus. You can <br> change between a higher and a lower number of <br> samples using Locus Properties depending on <br> whether accuracy or speed is more important. |
| :--- | :--- |
| Display | Determines how Sketchpad displays the locus of a <br> point. (This property is available only when a locus <br> of points is selected.) When continuous, Sketchpad <br> connects all of a locus's sampled points into a curve. <br> When discrete, Sketchpad displays the locus's <br> samples as individual points and does not connect <br> them. |

Continuous Sample Points $\quad$ Discrete Sample Points


## Graph Properties

This page contains properties about graphs of functions. It appears only when a function plot is selected. (See Plot Function, page 78, for more information about function plots.)


Display Determines how Sketchpad displays the function plot. When Continuous, Sketchpad connects all of a plot's sampled points into a curve. When Discrete, Sketchpad displays the plot's samples as individual points and does not connect them.

You can also set the domain of a plotted function by dragging the arrowheads that appear at the endpoints of the function plot.
When entering limits for the domain, you can use mathematical expressions as well as numbers. For instance, if your function's equation was $r=f(\theta)$ and your current angle unit was radians, you might enter $2 \pi$ (or $\pi / 4$ ) as the upper limit of the domain instead of an approximation such as 6.28.

By changing an animation's properties, you can specify more complex or precise motions than you can create using F2 Display: 8 Animate.

## Domain

Determines the domain of the function's independent variable. By default, Sketchpad plots new functions over a domain equal in size to the screen. You can change the domain by entering a minimum and maximum value in the From and To fields of Graph Properties. For graphs of $y=f(x)$, $x=f(y)$, and $\theta=f(r)$, the domain should be specified in units on the $x$-, $y$-, or $r$-axis. For graphs of $r=f(\theta)$, the domain should be specified as an angle in the current units (as determined by Preferences).

## Button Properties

This page contains properties for action buttons and appears only when an action button with changeable properties is selected. (See Action Buttons, page 56, for more information.) The properties that appear on this page depend upon the type of action button.

## Button Properties for Show Buttons

A Show button has a single property:
Show As Determines-when the button is pressed-whether the button selects the objects that it shows. By default, Sketchpad shows all objects as Selected. If you would rather the Show button not select the objects that it shows, switch this property to Deselected.

## Button Properties for Animate Buttons

The properties for Animate buttons give you control over the various animated points that are set in motion by the button when it is pressed. (See Animate, page 55, for a discussion of animation.)


Options For Point Determines which animated point the other properties on this page apply to. This pop-up menu lists all of the points animated by the action button; choose a specific point from it before setting that point's speed and direction. (Note that the points that appear in this list may be different from the objects you selected to animate when you created the Animate button. Sketchpad shows the set of points that must move in order to animate your selected objects, even if these points are different from the selected objects themselves.)

Speed Determines how fast the chosen point moves in comparison with other points animated by the action button. Choices are Slow, Medium, and Fast.

One-Way is the default direction for animating points around circles, because circles form "closed" paths. Bidirectional is the default direction for animating points along segments and other "open" paths. Random motion along a path causes the point to jump to random positions on the path. You might use it to explore topics of randomness that come up in probability and statistics.

## Direction

Travel

## Travel

Determines the direction in which the chosen point moves. Independent points-points not constructed on other objects-may move only in Random directions. Points constructed on other objects may move in Forward, Backward, Bidirectional, or Random directions.
Determines whether the chosen point should travel Once Only or Until Stopped-that is, if it should stop animating after it has moved once along its path or whether it continues indefinitely. This option applies only to points constructed on other objects that are not moving in Random directions.

## Button Properties for Move Buttons

The properties for Move buttons allow you to choose how quickly the moving points reach their destinations. (See Movement, page 57, for a description of Move buttons.)

Speed Determines how fast the moved points travel to their destination. Choices are Slow, Medium, Fast, or Instant. Instant movements cause points to leap immediately to their destinations when the button is pressed, rather than travel there at a fixed speed.

## Sketch <br> New...

Press F1 Edit: 9 Sketch: 1 New to close the current sketch and create a new, blank sketch. If the previous sketch has been changed since it was last saved, Sketchpad asks if these changes should be saved before creating a new sketch. Use New when you wish to start an entirely new diagram or construction.

## Open...

Press F1 Edit: 9 Sketch: 2 Open to close the current sketch and open a previously saved sketch or a sketch that has been downloaded to your calculator from another graphing calculator or from a personal computer. If the current sketch has been changed since it was last saved, Sketchpad asks if these changes should be saved before opening a new sketch.

## Example: Opening a Saved Sketch

1. Press F1 Edit: 9 Sketch: 2 Open. Sketchpad displays the standard variable chooser, listing the folders you've created on your graphing

| Dfen |  |
| :---: | :---: |
| Forder: |  |
|  |  |
| (Enter=0K EESC=CANCEL |  | calculator and any sketch variables you've saved or downloaded into those folders.

2. Choose the folder and variable name of a saved sketch. (Refer to the guidebook for your handheld device at education.ti.com/guides for
 instructions on using the variable chooser.) Sketchpad displays the new sketch.

If you attempt to switch to a new sketch-or to quit Sketchpad-before saving unsaved work, Sketchpad will ask you whether you wish to save your current sketch before continuing.

## Preferences...

Keep in mind that the text sizes you choose in Preferences only apply to newly created objects-to objects created after you close the Preferences dialog box. Use F2 Display:
5 Text Size to change the size of existing text.

## Save...

Save stores the current state of a sketch as a system variable for later recall. Saved sketches can be reopened with the Open command or can be transmitted between graphing calculators or between a calculator and a computer using VAR-LINK. (Refer to the guidebook for your handheld device at education.ti.com/guides for instructions on using VAR-LINK.)

Note that saved sketches occupy memory on your calculator; occasionally you may wish to free up occupied memory by using the VAR-LINK menu to delete sketches that are no longer in use.

## Example: Saving the Current Sketch

1. Press F 1 Edit: 9 Sketch:

3 Save. Sketchpad displays the standard variable chooser.

2. Choose the folder in which you wish to save the sketch and type a unique name for the sketch. Names are limited to eight characters.
3. Sketchpad saves the current state of your sketch with the given name. You can open this sketch again later by choosing Open and locating the folder and variable name you just saved.

Use Preferences to configure Sketchpad's overall behavior. Some Preferences, such as units, are system-wide settings that take effect immediately. Others, such as Text Sizes, determine how new objects are created and therefore don't take effect until you create new objects.
Use the function keys (F1-F3) at the top of the Preferences dialog box to choose different Preference pages.

## Text Preferences

Text Sizes
Label Measured
Objects

Determines the size of text for newly created object labels, action buttons, captions, or measurements. Choices are Small, Medium, or Large.
Label Measured Determines whether labels appear automatically for previously unlabeled objects when you measure their geometric properties. Setting this preference to Yes makes it easier to determine which objects a given measurement refers to, without requiring you to label them individually.

Label New Points Determines whether labels appear automatically for newly created points. Setting this preference to Yes guarantees that all points in a new construction will be labeled automatically, without requiring you to label them individually with the Text tool.

> Angles measured in degrees range from $0^{\circ}$ to $180^{\circ}$ (or $360^{\circ}$, for major arc angles) and are always positive in sign. Angles measured in directed degrees range from $-180^{\circ}$ to $180^{\circ}$, and are positive if the angle is measured counterclockwise and negative if the angle is measured clockwise.
> Degrees are common in Euclidean geometry, where they indicate how "wide" an angle is. Directed degrees are common in transformational geometry, where they indicate not just how much an angle turns, but in what direction in turns.

## Units Preferences

Units Preferences determine the precision (number of displayed digits) and units in which Sketchpad shows all measurements and calculations. Changes to units take effect immediately on all measurements in the sketch; changes to precision only affect new measurements. (To change the precision of an existing measurement, select it and press F1 Edit: 8 Properties: F3 Measure. See Measure Properties, page 47, for more information.)

## Distance

Angle

Other

Plot Preferences
Point Loci
Point Loci
Function Plots

Display

Determines the units and precision of distance measurements such as distance, length, circumference, radius, and so forth. Choices include cm, inches, and pixels (screen units).

Determines the units and precision of angle measurements. Choices include radians, degrees, and directed degrees.

Determines the precision of any measurement or calculation that is neither a distance nor an angle. These measurements usually appear without units.

Determines how many samples Sketchpad will plot when displaying a new locus of points. The greater the number of samples, the more accurate the depiction of the locus and the smoother its appearance on the screen. However, the greater the number of samples, the slower Sketchpad will be in calculating and displaying the locus.

Determines how many samples Sketchpad will plot when displaying a new function plot.

Determines how Sketchpad displays a new function plot or locus of points. When continuous,
Sketchpad connects all of the locus's or plot's sampled points into a curve. When discrete,
Sketchpad displays the locus's or plot's samples as individual points and does not connect them.

## About Sketchpad...

About Sketchpad displays version and copyright information about The Geometer's Sketchpad for TI-89 and TI-92 Plus.

## F2 Display Menu

> Commands in the F2 Display menu let you change how objects in your sketch appear without changing geometric relationships in the sketch. The Display menu is also where you'll find commands for starting and controlling animations. This section explains how to use Display menu commands.

## Hide Objects

## Show All Hidden

You can use Show All
Hidden to show just one hidden object. See Showing One Object (page 85) for more information.

## Show/Hide Labels

Choosing Show Labels and Hide Labels is equivalent to pressing on each of the selected objects with the Text tool. See Showing Labels (page 31) for more information.

## Line Width

Hide Objects hides selected objects, removing them from view without removing them from the geometric construction of the sketch. Hide objects when their appearance is unimportant or undesired, even though they play an important role in constructing other objects in the sketch. This command is available only when one or more objects are selected. (Note that any selectable object may be hidden, including captions, measurements, and geometric objects such as points and lines.)
Before

Show All Hidden reveals and selects all objects formerly hidden by the Hide Objects command. This command is available only when one or more objects have previously been hidden.
Before

Show Labels displays the labels of any selected object that is not already showing its label. Show Labels also assigns labels to any object that was previously not labeled. When all selected objects' labels are showing, the command becomes Hide Labels, which removes selected objects' labels from display. Hiding an object's label never changes that label.

Show Labels and Hide Labels are available only when you have selected objects for which labels can be displayed. (Most geometric objects can display labels, but objects like measurements and captions do not display additional labels.)
The Line Width settings, Thin and Thick, determine the appearance of selected circles, segments, rays, lines, axes, point loci, and function plots. By default, Sketchpad creates new objects with thin lines. Use thick lines to add visual emphasis to important objects in your sketch. These commands are available only when one or more objects that can display thin and thick line widths are selected.

Traced objects produce visible traces on the screen only when they are moved. Likewise, traces on the screen can remain even after you've stopped tracing a previously traced object. Use Clear All Traces to remove visible traces of objects.

The Text Size settings, Small, Medium, and Large, determine the appearance of selected captions, measurements, and object labels. By default, Sketchpad creates new objects with a text size determined by your Preferences (see page 51). Use different text sizes to add visual emphasis to your sketch.
Objects with the Trace command activated leave a "trail" behind them whenever they are dragged. Choosing Trace has no immediate effect on the sketch, but affects how objects display when they are moved. Use tracing to investigate the path (the locus) of an object over the course of a dynamic transformation (such as dragging or animating).
When you choose Trace, tracing is activated for selected objects, and it remains activated until you select them and choose Trace again. When tracing is activated for a selected object (or all selected objects), the Trace command appears in the menu with a checkmark beside it. Choosing Trace a second time removes the checkmark and deactivates tracing for the selected object(s).

## Example: Using Trace

1. Select an object (or objects) whose path or locus you are interested in.

2. Press F2 Display: 6 Trace [Object]. Tracing is now activated for the selected object, and it will leave behind traces when it's moved.
3. Drag any part of your construction that causes the previously selected object to move. Sketchpad displays traces
 of that object's former locations.
4. To stop tracing the traced object, select it again and press F2 Display to show the menu. Trace now appears with a checkmark, indicating that tracing is activated for the selected object. Press 6 Trace to deactivate tracing for the object.
5. Continue dragging your construction. The previously selected object no longer leaves traces.

6. To remove the previously displayed traces from the screen, press F2 Display: 7


Clear All Traces (or hit ESC twice). Sketchpad removes all visible traces from the display.

Clear All Traces removes any visible object traces from the screen. This command is only available when object traces are visible. Clearing all traces does not alter whether tracing is activated for the objects themselves; it only affects the display. During an animation of traced objects, you can Clear All Traces to unclutter the display without interrupting the animation or causing traced objects to no longer trace. See the description of the Trace command, above, for information about tracing objects and stopping objects from tracing when they are moved.

You can gain more control over how Sketchpad animates objects by using animation action buttons. See Animation, page 57, for more information.

If you animate an independent point and it wanders off the screen, don't worry! You can always Undo to return it to where it started.

Animate sets selected objects into motion. Use animation to investigate the properties of a construction as parts of it move, to illustrate mathematical models and geometric designs, or just for fun. Use animation in conjunction with the Trace command (see Trace, page 54) to investigate the position of one or more objects through a transformation.

## About Animation

To animate an object, select that object and press F2 Display:
8 Animate. Sketchpad instantly begins animating the object. You can continue interacting with the sketch while an animation is in progress. You can change the speed of the animation or pause and resume the animation with other commands in the Display menu. You can alter the geometry of your sketch by dragging and constructing new objects, or you can select and animate other points while an animation is active. An animation continues until you explicitly stop it or until you choose a command that ends the animation automatically (such as switching to a New Sketch).

How Sketchpad animates an object depends upon how the object is constructed:

1. Independent points move freely in the plane, in random directions.
2. Points defined on paths animate at a constant rate along these paths. For instance, a point constructed on a circle animates at a constant rate around the circle. (See Point on Object, page 58, for more information about points defined on paths.)
3. Other objects animate by moving the parental points that define them. (These points will be independent points, points defined on paths, or a combination of the two.)

Animate is available only when one or more geometric objects are selected. (You cannot animate non-geometric objects such as captions or measurements.)

## Pausing and Resuming Animations

While active, animations may be paused and resumed. (Pausing an animation differs from stopping it in that the animation may be resumed without having to reselect and reanimate all the objects that were initially animated.) Pause animations in order to easily select objects, construct objects, or otherwise manipulate your sketch.

## Example: Using Animate, Pause, and Resume

1. Construct and select a point on a path.

2. Press F2 Display: 8 Animate. Sketchpad begins animating the point along its path.
3. With the Arrow, press ENTER in white space to deselect all objects (or press ESC). When there are no objects selected and an animation is active, the Animate command becomes the Pause Animation command.
4. Press F2 Display: 8 Pause Animation. The animation pauses.
5. Press F2 Display: 8 Resume Animation. Sketchpad resumes animation of the point, even though its path has been hidden.

## Speed <br> Stop Animation

## Action Buttons

The Speed commands, Faster and Slower, change the speed of all active animations by a fixed small increment. Choose Faster or Slower repeatedly in order to speed up or slow down an animation by larger increments. These commands are available only when one or more animations are active.

To reset all animations to default speeds, stop the animations and start them again.

Stop Animation stops all active animations and selects the points that were previously animating, if they are visible. This command is available only when one or more animation is active.

You may also press ESC to stop animations when the Arrow tool is active and nothing is selected in the sketch. (Because ESC switches to the Arrow if the Arrow is not active, or deselects objects if any objects are selected, pressing ESC repeatedly will eventually stop an active animation.)

The Action Buttons menu creates action buttons in your sketch. When pressed with the Arrow tool, these buttons perform actions similar to those that could be performed using menu commands. Use action buttons to make frequently used commands easily accessible in your sketch.

## Using Action Buttons

Action buttons are objects like any other in your sketch-they can be selected, moved, hidden, or deleted like other objects. In addition, they can be pressed, which causes them to perform their action.

## Using Action Buttons

- To press an action button, move the Arrow over any part of the button except the small handle on the left. The cursor becomes a finger, indicating the button can be pressed. Press ENTER to press the button (and perform its action).
- To select an action button, move the Arrow over the black handle
 at the left of the button. The cursor becomes a sidewayspointing arrow, indicating the button can be selected. Press ENTER to select the button.

When a Show button is pressed, the objects it shows are usually then selected. If you'd rather that a Show button not display its objects selected, change this default behavior in Button Properties (see page 49).

Animate buttons let you set up more complex animations than you can with F2 Display:
8 Animate. Once you've created a complex animation, you can restart it any time you want by pressing its Animate button.

## After creating a Move

 button, you may want to hide the destination point so it doesn't intrude in your sketch. When you press the button, points will move to their destinations even if the destinations are hidden.
## Show/Hide Toolbox

- To relabel an action button, point the Text tool at it and press
 ENTER. Sketchpad will display the button's Label Properties (see page 46), where you can edit the existing label or type a new one.


## Creating Action Buttons

Use the F2 Display: B Action Buttons menu commands to create the following types of action buttons.

## Hide/Show

Use Hide and Show buttons to selectively conceal or reveal portions of a construction. When pressed, Hide and Show buttons hide or show a specific object or group of objects. (See Hide Objects, page 53, for more information on hiding objects.) To create these buttons, select the object or objects you wish to have hidden and shown by action buttons, then press F2 Display: B Action Buttons: (TI-89: alpha B) 1 Hide/Show. These buttons are always created in pairs (one Hide button and one Show button), though you can move, rename, or delete them independently.
Animation...
Use Animate buttons to begin a prescribed animation for one or more objects in the sketch. To create an Animate button, select points or other objects to animate, just as you would for the Animate command (see page 55). Then press F2 Display: B Action Buttons: (TI-89: alpha B) 2 Animation. Sketchpad creates an Animate button, and then displays the button's Properties, where you can choose the speed and direction of each of the points that will be animated when the button is pressed. See Button Properties (page 49) for more information.

## Movement...

Use Move buttons to move one or more points directly to specific destinations, specified by other points in the sketch. To create a Move button, select a point to move and a second point as its destination. (If you want to move more than one point, continue selecting pairs of points: the first to move, the second as destination.) Then press F2 Display:
B Action Buttons: (TI-89: alpha B) 3 Movement. Sketchpad creates a Move button and then displays the button's Properties, where you can choose the speed with which points will move to their destinations. See Button Properties (page 49) for more information.

This command alternates between Hide Toolbox and Show Toolbox and controls the visibility of the toolbox on the right edge of the screen. When the toolbox is hidden, more screen area is available for displaying geometric constructions. Even when hidden, the toolbox can still be accessed by pressing [F8 (TI-89: 2nd [F8]). (The toolbox will temporarily display itself as you choose a new tool.) Keyboard shortcuts for accessing specific tools continue to work when the toolbox is hidden. Use Show Toolbox to return the toolbox to the right edge of the screen.

## F3 Construct Menu

The [F3 Construct menu contains commands for many useful compass and straightedge constructions, including many you can make using the tools. Each command requires specific selections to make it available. These selections and other details about each command are described in this section.

## Point on Object

You can also construct points on objects with the Point tool-see Constructing Points on Objects (page 27).

A path is the directed geometric extent of an object or object perimeter. Not all objects are paths. (For instance, points have no dimension or extent, so points cannot be paths.)

## Midpoint

To construct a point on an object, first select an object (such as a segment or circle) on which to place the point. Then press F3 Construct: 1 Point on Object. Sketchpad constructs and selects a randomly positioned point on the selected object. To construct points on more than one object simultaneously, select multiple objects before choosing Point on Object.


A point constructed on an object may be dragged anywhere along that object's path. When you animate a point constructed on an object (see About Animation, page 55), the point moves along the object's path.

Points may be constructed on the following paths:
Path Object Location of Point

1. Segment, Ray, Line, Along the object Axis, Circle, Locus of Points, Function Plot
2. Polygon Interior Along the polygon's perimeter
3. Circle Interior Along the circle's circumference

To construct the midpoint of a segment, first select a segment. Then press
F3 Construct: 2 Midpoint. Sketchpad constructs and selects the midpoint of the selected segment. To construct multiple midpoints simultaneously, select multiple segments.


## Intersection

You can also create intersections with the Point tool-see Constructing Points of Intersection (page 27). Or simply choose the Arrow at an intersection of two objects to construct an intersection point there.
When you drag objects so they no longer intersect, their points of intersection vanish. Drag them back to see the points again.

## Segment

Constructing a segment, ray, or line using the Construct menu is equivalent to drawing a segment, ray, or line between two points with the Straightedge tools.

To construct the intersection of two objects, select both intersecting objects. Then press F3 Construct: 3 Intersection. Sketchpad constructs and selects one or both points of intersection of the selected objects.
Before

Intersections can be constructed between segments, rays, lines, circles, and axes.

To construct a segment, select two endpoints. Then press F3 Construct: 4 Segment.


Construct more than one segment at a time by selecting three or more points before choosing F3 Construct: 4 Segments. Segments will connect the points in the order in which the points are selected, and the last-selected point will connect back to the first-selected point.


To construct a ray, select two points: the endpoint of the ray, and a point determining its direction in that order. Then press F3 Construct: 5 Ray. Construct more than one ray at a time by selecting three or more points before choosing F3 Construct: 5 Rays.

To construct a line, select two points through which the line should pass. Then press F3 Construct: 6 Line. Construct more than one line at a time by selecting three or more points before choosing F3 Construct:
6 Lines.
To construct a parallel line, select a point through which the line should pass, and a segment, ray, line, or axis to which the line should be parallel. Then press F3 Construct: 7 Parallel Line.


## Perpendicular Line

## Angle Bisector

Sketchpad's bisectors always bisect the minor angle determined by the three points. This is the angle that measures less than $180^{\circ}$.

## Circle by <br> Center+Point (TI-92 Plus /Voyage ${ }^{\text {TM }} 200$ PLT)/ Circle thru Point (TI-89)

The second selected point defines the radius of the circle-drag it, and the radius changes. But if you place another point on the circumference and drag it, the radius will not change, as this other point does not define the circle.

To construct two or more parallels to a given straight object-a segment, ray, line, or axis-select the straight object and two or more points. Then press F3 Construct: 7 Parallel Lines (Parallels on the TI-89).

To construct lines passing through a single point parallel to two or more straight objects, select the given point and two or more straight objects. Then press [F3 Construct: 7 Parallel Lines (Parallels on the TI-89).

To construct a perpendicular line, select a point through which the line should pass and a segment, ray, line, or axis to which the line should be perpendicular. Then press F3 Construct: 8 Perpendicular [Line].


To construct two or more perpendiculars to a given straight object-a segment, ray, line, or axis-select the straight object and two or more points. Then press F63 Construct: 8 Perpendicular Lines (Perpendiculars on the TI-89).

To construct lines passing through a single point perpendicular to two or more straight objects, select the given point and two or more straight objects. Then press F3 Construct: 8 Perpendicular Lines (Perpendiculars on the TI-89).

To construct an angle bisector, select three points defining the angle to bisect. (Select the vertex of the angle second.) Then press F3 Construct: 9 Angle Bisector.
Before

To construct a circle, select two points. The first selected point defines the center of the circle; the second point defines the radius of the circle. Then press F3 Construct: A Circle by Center+Point (Circle thru Point on the TI-89).


Constructing a Circle by Center+Point (or Circle thru Point) is equivalent to drawing a circle from one center point to another radius point using the Compass tool.

## Circle by [Center+]Radius

You may also use a selected distance measurement or measurements instead of a selected segment to specify the length of the radius.

## Interior

On the TI-92 Plus /Voyage ${ }^{\text {TM }} 200$ PLT, if you select from three to six points, Sketchpad will display a more meaningful command name than Polygon Interior. For instance, if you select three points, Sketchpad will display Triangle Interior.

To construct a circle of a given radius, select a point to become the center of the circle and a line segment to specify the length of the radius. Then press F3 Construct: B Circle by [Center+]Radius.
Before

To construct two or more concentric circles, select a point as the center of the circle and two or more line segments to specify the radii. Then press
F3] Construct: B Circles by [Center+]Radii.
To construct two or more circles with the same radius, select a line segment to specify the radius for each circl and two or more points to define the center of the circles. Then press $\mathbb{F 3}$ Construct: B Circles by [Centers+]Radius.
The Interior command allows you to construct the filled interior of various objects. Its name changes based on the current selection.

## Polygon Interior

To construct a polygon interior, select three or more points as vertices of the polygon, in consecutive order. Then press F3 Construct: $C$ Polygon Interior.


Note that the order in which points are selected determines the shape of the polygon interior.


## Circle Interior

To construct a circle interior, select a circle. Then press F3 Construct: $\mathbb{C}$ Circle Interior. Select two or more circles to simultaneously construct two or more circle interiors.


## Locus

You can increase the accuracy of displayed loci by increasing the number of samples that Sketchpad uses to determine their appearance. See Locus Properties (page 47) for more information.

To avoid filling the screen with opaque objects, Sketchpad displays the loci of polygon interiors and circle interiors as outlines (i.e., as polygons and circles) rather than as filled interiors.

A locus describes a set of positions to which an object is capable of moving. To construct the locus of a geometric object, select the objectthe sampled object-and any point - the driver point -whose position on a path determines the location of the sampled object. (The driver point should be a point constructed on a path.) Then press F3 Construct: (TI-89: alpha D) Locus. Sketchpad displays and selects the locus of the sampled object as the driver point moves along its path.


You may construct the locus of points, segments, rays, lines, circles, axes, circle interiors, or polygon interiors. Most commonly, you'll construct the locus of a point. Sketchpad allows you to display this type of locus either as a discrete set of points, or as a continuous curve-see Locus Properties (page 47). If a point locus is infinitely large-as it may be if the path of the driver point is infinitely long-Sketchpad will display a finite portion of the locus with arrowheads to suggest that it continues infinitely far. Alter the displayed portion of the infinite locus by dragging the arrowheads with the Arrow tool. Dragging arrowheads in the direction their arrow points makes the displayed portion of the locus longer, dragging them in the opposite direction makes the locus shorter.

In some cases, you may wish to construct the locus of a sampled object as if some driver point that geometrically determines the sampled object's position were constructed on a path when the driver itself is not constructed on a path. In that event, select the sampled object, the desired driver point that affects the sampled object, and the desired path for the driver point. Then press F3 Construct: Locus. Sketchpad constructs and displays the locus of the sampled object as it would appear if the driver moved along the selected path.

## F4 Transform Menu

Commands in the F4 Transform menu allow you to construct translations (slides), rotations (turns), dilations (expanded or contracted images), and reflections (flips) of selected objects. Before you perform a transformation, you must identify (mark) your transformation's parameters. This section starts with some general information about transformations and then explains how to use the marking and transforming commands.

## About Transformations

Transformations in Sketchpad create translated, rotated, dilated, or reflected images of selected pre-image objects. You can construct the transformed image of points, segments, rays, lines, circles, axes, circle interiors, and polygon interiors. The type and shape of a transformed image depends on the type and shape of its pre-image-for example, a point's rotated image is a point, and a triangle's rotated image is a triangle. The location, size, and orientation of a transformed image is determined by the type of transformation as well as various other transformational parameters that determine the transformation, such as the distance by which an image is translated or the angle by which it is rotated.
The next sections describe Sketchpad's four types of transformations as well as their differing transformational parameters. To construct a transformation in Sketchpad, you'll first mark any necessary transformational parameters, then select the pre-image object(s) to transform, and then choose a transformation command from the F4 Transform menu. These steps are described in more detail in the following sections.

## Translation

Translations slide objects in the plane without turning them; their location is affected, but not their size or orientation. In the following illustration, the selected triangle is the image of the unselected triangle translated by some distance to the right:


The parameters of a translation specify a vector indicating how far, and in which direction, the image is translated from the pre-image. Sketchpad allows you to specify these parameters in a number of ways.
Rectangular Vector The translation vector is specified by a pair of distances, representing horizontal and vertical offsets from the pre-image. (A positive horizontal distance translates to the right of the pre-image and a positive vertical distance translates above the preimage. Similarly, negative distances translate to the left and below the pre-image.) An image translated by a rectangular vector of 1 cm horizontally and -2

Positive angles result in counter-clockwise rotations, and negative angles result in clockwise rotations.
cm vertically appears to the right $(1 \mathrm{~cm})$ and below ( 2 cm ) the pre-image.

## Polar Vector

## Marked Vector

The translation vector is specified by a distance (by which to translate) and an angle (the direction in which to translate). An angular direction of $0^{\circ}$ points due east, and a direction of $90^{\circ}$ points due north. An image translated by a polar vector of 5 cm at $45^{\circ}$ appears above and to the right of its pre-image. (In this polar vector example, the diagonal distance between pre-image and image would be 5 cm .)
The translation vector is specified by the previously marked distance and direction between two points. Thus if an object were translated by a vector from point $A$ to point $B$, where $B$ was above and some distance to the left of $A$, the translated image would be defined above and to the left of its pre-image by the same distance and direction.

## Rotation

Rotations turn objects in the plane, affecting their location and their orientation but not their size. In the following illustration, the selected triangle is the image of the unselected triangle rotated by $90^{\circ}$ about center point $D$.


The parameters of a rotation specify a center point, about which the image turns, and an angle of rotation.

## Dilation

Dilations expand and contract objects, affecting their size and location. In the following illustration, the selected triangle is the image of the unselected triangle dilated by a factor of one-half toward center point $D$.


The parameters of a dilation specify a center point, toward or away from which the image contracts or expands, and a scale factor or ratio that determines how much the image expands or contracts. If the magnitude of the scale factor is less than 1 , the image is smaller than the pre-image-a ratio of $1 / 2$, for instance, defines an image only half as big as its pre-image. If the magnitude of the scale factor is greater than 1 , the image is larger than the pre-image-a ratio of $2 / 1$, for instance, defines an image twice as big as its pre-image. (If the scale factor is negative, the image is dilated

## Mark Center <br> Mark Mirror Mark Angle Mark Ratio/Scale Factor Mark Vector Mark Distance

Think of marking parameters as similar to selecting prerequisites for a
Construct commandexcept marking is more durable. Sketchpad remembers parameters you've marked until you mark new ones-even if they're no longer selected. Thus, you don't have to mark parameters again and again each time you choose a Transform command.
"through" the center point, and appears inverted on the opposite side of the center.)

## Reflection

Reflections create mirror-images of objects-objects are flipped across a mirror, changing their orientation and location but not their size. In the following illustration, the selected triangle is the image of the unselected triangle reflected across the mirror segment $D E$.


The transformational parameter of a reflection is the mirror across which the reflection flips. In Sketchpad, any segment, ray, line, or axis may act as a reflection mirror.

The Mark commands (Mark Center, Mark Mirror, and so forth) are used to establish transformational parameters before constructing transformed images of selected pre-images. In general, each transformation must be defined in terms of at least one marked parameter (required marks). Some transformations may take advantage of additional marked parameters (optional marks). You must mark the required parameters-and any optional parameters you may be using-before choosing one of the transformation commands (Rotate, Dilate, Translate, or Reflect).

To mark a transformational parameter, select appropriate objects to define that parameter and then choose the corresponding Mark command, as described in the following table. If you select more objects than necessary, Sketchpad will mark the most recently selected object as the parameter. For instance, Mark Center requires that you select a point to mark as the center for subsequent rotations or dilations. If more than one point is selected, Mark Center will mark the most recently selected point as center. Sketchpad confirms the Mark command with a brief animation of the marked object.

Once a parameter is marked, it remains marked until you mark a different object as that parameter or until you delete or undo the marked object(s). For example, you need to mark a center only once in order to dilate or rotate about that marked center many times.

If you attempt to transform a selected pre-image and have not marked a required transformation parameter, Sketchpad will attempt to mark one for you based on objects available in your sketch. (You'll see a brief animation indicating the marked parameter.) If no object exists that can be marked, Sketchpad displays a warning indicating you must construct and mark the appropriate parameter. For example, if you attempt to rotate without first marking a point as center, Sketchpad will mark an existing point as center before displaying the Rotate dialog box.

The following table describes the various Mark commands, as well as the objects to select in order to make the commands available.

| Command | Selection | Transformational Role |
| :---: | :---: | :---: |
| 1 Mark Center | One point | Required center of rotation; required center of dilation |
| (2) Mark Mirror | One straight object (segment, ray, line, or axis) | Required mirror of reflection |
| 3 Mark Angle | Three points (selected in directed order: point, vertex, point) | Optional angle of rotation; optional angle of translation by a polar vector |
| 3 Mark Angle | One angle measurement (a measurement in degrees or radians) | Optional angle of rotation; optional angle of translation by a polar vector |
| 4 Mark Ratio | Two segments (selected in order: numerator $j$, denominator $k$ ) | Optional ratio of dilation (scale factor is equal to the ratio of segment lengths $j / k$ ) |
| 4 Mark Ratio | Three collinear points (selected in order: $A, B, C$ ) | Optional ratio of dilation (scale factor is equal to the ratio of the directed lengths $A C / A B$; this ratio is negative if $B$ and $C$ are on opposite sides of $A$ ) |
| 4 Mark Scale Factor | One scalar measurement (a measurement with no units) | Optional scale factor of dilation |
| 5 Mark Vector | Two points (selected in order: foot $A$, head $B$ ) | Optional vector of translation (translation by the vector from $A$ to $B$ ) |
| 6.Mark Distance | One distance measurement (a measurement in inches, centimeters, or pixels) | Optional distance of translation by either a rectangular or polar vector |
| 6.Mark Distances | Two distance measurements (selected in order $m 1, m 2$ ) | Optional distances of translation by a rectangular vector (translation by the rectangular vector of $m 1$ horizontally and $m 2$ vertically) |

The next table summarizes the same information in a different format. Here each type of transformation is listed with the required and optional marks used to specify transformations of that type.

| Transformation | Required Marks | Optional Marks |
| :--- | :--- | :--- |
| Translation (rectangular vector) | none | Distance(s) |
| Translation (polar vector) | none | Distance, Angle |
| Translation (marked vector) | Vector | none |
| Rotation | Center | Angle |
| Dilation | Center | Ratio/Scale Factor |
| Reflection | Mirror | none |

## Translate...

To construct the translated image of an object or objects, select the preimage object(s) and press F4 Transform: 7 Translate. Sketchpad displays the Translate dialog box.


Sketchpad displays constant distances and angles in the transformation dialog boxes using the current units. You can change the units in Preferences (see page 51). For example, if Sketchpad displays an angle in degrees and you'd rather work in radians, use Preferences to switch the current angle unit from degrees to radians.

In the dialog box, indicate the vector by which you wish to translate the selected objects. Use

| F1, F2, F3 | To choose between dialog pages for specifying <br> rectangular, polar, or marked point vectors. |
| :--- | :--- |
| F1. Polar | To enter a polar vector-a vector specified by a <br> distance and a direction. For the distance, you may <br> choose between a previously marked distance and a <br> fixed distance (such as 5 cm ). For the direction, you <br> may choose between a previously marked angle and <br> a fixed angle (such as 45 $).$ |
| F2] Rectangular | To enter a rectangular vector-a vector specified by <br> horizontal and vertical offsets. For these offsets, you <br> may choose between previously marked distances <br> and fixed distances (such as 5 cm). |
| F3 Marked | To enter a marked vector determined by two points. <br> You must previously mark two points as a vector <br> (see Mark Vector, page 65). If no vector has been |
| previously marked, this choice is unavailable. |  |

Press ENTER to complete the translation.

## Example Translation Steps Display

1. Select objects to translate.

2. Press [F4 Transform: 7 Translate.

3. Press F1 to switch to polar vector entry, if Polar is not already active (capitalized). Then enter numbers into the dialog box fields to describe a vector 2 cm in length at an angle of $45^{\circ}$.
4. Press ENTER to complete the translation. Sketchpad constructs and displays the image of your selection, translated by 2 cm at $45^{\circ}$ from
 the horizontal.

## Rotate...

Remember: if you've already marked a center, you don't need to mark it again each time you rotate or dilate.

You can also type a brief mathematical expression for your angle such as 360/5.

To construct the rotated image of an object or objects, first mark a point as the center of rotation (see Mark Center, page 65). Then select the preimage object(s) and press F4 Transform: 8 Rotate. Sketchpad displays the Rotate dialog box.

| Rotate |
| :--- |
| Rotate By: Fixed Fingle |
| Angle: 45 |
| Center Point: ${ }^{\circ}$ |
| Enter=0K |

In the dialog box, enter the angle of rotation. You may enter a constant angle (such as $90^{\circ}$ ) or a marked angle if you've marked an angle previously (see Mark Angle, page 65). Press ENTER to complete the rotation.
Dilate...

To construct the dilated image of an object or objects, first mark a point as the center of dilation (see Mark Center, page 65). Then select the preimage object(s) and press F4 Transform: 9 Dilate. Sketchpad displays the Dilate dialog box.


In the dialog box, enter the ratio of dilation. You may enter a constant ratio (such as $1 / 2$ ), or a marked ratio or scale factor if you've marked one previously (see Mark Ratio/Scale Factor, page 65). Press ENTER to complete the dilation.
To construct the reflected image of an object or objects, first mark a segment, ray, line, or axis as the mirror of reflection (see Mark Mirror, page 65). Then select the pre-image object(s) and press F4 Transform: A Reflect. Sketchpad immediately constructs and displays the image of the selected objects reflected about the marked mirror.

Measurements reveal relationships in figures crucial for conjecturing. Using commands in the F5 Measure menu, you can measure and calculate any property you like, then watch measurements and calculations change (or not) as you change your figure. As usual, to create measurements you must first identify, through selection, which object(s) you want to measure. This section explains how to make measurements and use Sketchpad's calculator.

## About Measurements

Sketchpad measures geometric properties of objects (such as the measure of a segment's slope) or of groups of objects (such as the distance between two points). These measurements change dynamically as you drag and resize objects. To make a measurement, select the object or objects you wish to measure and choose the appropriate measurement command from the F5 Measure menu.

Sketchpad supports the following measurements:

| Command | Required Selection | Result |
| :---: | :---: | :---: |
| 1 Length | One or more segments | Length of segment(s) |
| 2 Distance (between points) | Two points | Distance between points |
| 2 Distance (point to line) | One point and one straight object | Shortest (perpendicular) distance from the point to the line extending through the straight object |
| 3 Perimeter | One or more polygon interiors | Perimeter of polygon(s) |
| 4 Circumference | One or more circles or circle interiors | Circumference of circle(s) |
| 5 Angle | Three points, selected in order $A, B, C$ | Measure of angle $A B C$ (vertex is second selected point) |
| 6 Area | One or more circles, circle interiors, or polygon interiors | Area of circle(s) or polygon interior(s) |
| 7 Arc Angle | One circle and two or three points on that circle | Measure of arc angle between the two points (minor arc, less than $180^{\circ}$ ); or arc angle between the first and third points including the second (minor or major arc, less than $360^{\circ}$ ) |

(os
The Ratio (of points)
command measures what is
essentially a ratio of dilation
from the second selected
point to the third, centered
at the first selected point.
Measurement
Display

## Calculate...

A displayed measurement consists of a value, a prefix, and an optional suffix.


The precision of a measurement-the number of decimal places displayed in its value-is determined by Preferences (see page 51) when the measurement is created. You can change the precision for an individual measurement at any time by editing its Measure Properties (see page 47).

| Value | The value of a measurement changes if the measured <br> property changes as you drag or animate an object. <br> Measurement values are displayed in the current <br> units set in Preferences. (See Preferences, page 51, <br> for information on changing units.) |
| :--- | :--- |
| Prefix | The prefix of a measurement is text displayed before <br> the measurement. Sketchpad assigns a default prefix <br> to each measurement indicating the quantity being <br> measured. You can edit this prefix to any text you |
| Suffix using Measure Properties (see page 47). |  |$\quad$| The suffix of a measurement is text displayed after |
| :--- |
| the measurement. By default, Sketchpad creates |
| measurements with no suffix, but you can add a |
| suffix to any measurement using Measure Properties |
| (see page 47). |


| 8 Arc Length | One circle and two or three points on that circle | Length along the circumference between the two points (minor arc) or length along circumference from the first point to the third including the second (minor or major arc) |
| :---: | :---: | :---: |
| 9 Slope | One or more straight objects | Slope of straight object(s) |
| (A) Radius | One or more circles or circle interiors | Radius of circle(s) |
| B Ratio (of lengths) | Two segments, selected in order $j, k$ | Ratio of segment lengths $j / k$ (select numerator first) |
| B Ratio (of points) | Three collinear points, selected in order $A, B$, C | Signed ratio of directed distances $A C / A B$ (negative if $B$ and $C$ are on opposite sides of $A$ ) | property changes as you drag or animate an object. Measurement values are displayed in the current units set in Preferences. (See Preferences, page 51, for information on changing units.)

The prefix of a measurement is text displayed before the measurement. Sketchpad assigns a default prefix to each measurement indicating the quantity being measured. You can edit this prefix to any text you wish using Measure Properties (see page 47).

The suffix of a measurement is text displayed after measurement. By default, Sketchpad create suffix to any measurement using Measure Properties (see page 47).

Use Calculate to create and evaluate calculations involving measurements. Like all objects in Sketchpad, calculations are dynamic, so as you change geometric objects and thereby change their measured properties, calculations based on those measurements dynamically update as well.

## Display

1. In a sketch containing one or more measurements, press
F5 Measure: C Calculate (TI-89: alpha C).
2. The calculator's menus are displayed at the top of the screen, and the entry line is at the bottom of the screen. The pop-up menus include various mathematical terms you can use in your calculations.
3. Enter a mathematical expression to calculate. Use F1 Values to insert measured values into your expression, and the other menus for functions, constants, and units. Use the keypad to type numbers, operators $(+, /,-, *)$, and parentheses as appropriate. Your expression appears in the calculator entry line at the bottom of the sketch.
4. To calculate the sum of the three angle measurements in this example, press F 1 Values: 1 to enter the first measurement. Then press + on the keypad. Press F1 Values: 2 for the second measure. Press + again. Finally press F1 Values: 3 to insert the final measured value.
5. Press ENTER to complete the calculation. Sketchpad evaluates the calculation, and displays the result in your sketch.


When entering an expression, you may correct mistakes by using $\odot$ and $\bigcirc$ to move the insertion point. (Press $\Delta \odot$ or $\square \bigcirc$ to move the insertion point to the beginning or end of the calculation.) Press $\square$ to remove the term immediately to the left of the insertion point, or $\square \square$ to remove the term immediately to the right of the insertion point. If you press ENTER when an invalid expression is showing on the calculator entry line, Sketchpad displays a warning and moves the insertion point to the part of your expression that is invalid.
Note that the calculator computes with units as well as values. Thus if you add a length to a length, the result is a length-for instance
$1 \mathrm{~cm}+1 \mathrm{~cm}=2 \mathrm{~cm}$. If you multiply a length times a length, the result is an area: $(1 \mathrm{~cm})(1 \mathrm{~cm})=1 \mathrm{~cm}^{2}$. Sketchpad assumes the current units (as set in Preferences) when units are missing from your calculation. Thus, $\sin (90)=1.0$ if the current angle units are degrees-Sketchpad assumes you mean $\sin \left(90^{\circ}\right)$. But if the current angle units are radians, $\sin (90)$ is approximately 0.894 , because $\sin (90$ radians $) \cong \sin \left(5156.62^{\circ}\right) \cong 0.894$. Use explicit units (from the F4 Units menu) in your calculations to avoid dependencies on the current units. That is, a calculation of $\sin \left(90^{\circ}\right)$ will always equal 1.0 regardless of whether your current angle unit is degrees or radians.

You can also use the calculator to enter fixed quantities-for example, $45^{\circ}$. If you define objects in terms of this calculation (for example, a rotation by this marked, calculated angle) you can later use Edit Calculation to change the value of the calculation, thereby redefining the objects that depend on it.

## Analytic Measurements

## Editing Calculations

To alter an existing calculation, select it in the sketch. Then press
F5 Measure: C Edit Calculation (TI-89: alpha C). Sketchpad opens the calculator and displays the current expression. Modify the expression as you see fit and press ENTER to update the expression in your sketch or ESC to cancel editing.

When you edit an existing calculation, Sketchpad lists available measurements in the calculator's F1 Values menu. This list includes visible measurements that do not depend upon the edited calculation itself for their values. That is, you cannot define a calculation in terms of a value that itself is defined in terms of the original calculation.
The Analytic submenu contains analytic measurements. These measurements-of coordinates and equations-are defined in terms of the units of a coordinate system, rather than in terms of physical units such as centimeters and degrees. When you create an analytic measurement, Sketchpad displays it in terms of the active (marked) coordinate system, as described in the F6 Graph menu section. If no coordinate system is active when you make an analytic measurement, Sketchpad creates a coordinate system for you.

## Coordinates

To measure the coordinates of a point or points, select the point(s) and press F5 Measure: (TI-89: alpha D) D Analytic: 1 Coordinates. Sketchpad measures and displays the coordinates of the selected point(s).

| Before | After |  |  |
| :---: | :---: | :---: | :---: |
|  | $\oint_{B}$ | ${ }^{\circ} \mathrm{B}$ |  |
|  | $\theta_{\mathrm{A}}$ | $\begin{aligned} & \text { A: }(1.16,-0.46) \\ & \text { B: }(-0.64,0.6) \end{aligned}$ | ${ }^{\circ} \mathrm{H}$ |

Unlike other measurements, coordinates do not have prefixes or suffixes that can be edited.

By default, Sketchpad measures rectangular coordinates-identified by an ordered pair of numbers $(x, y)$. If the active coordinate system is polar, Sketchpad measures polar coordinates, which are identified by an ordered pair of numbers ( $r, \theta$ ). See Graph Меnи (page 74) for more information about coordinate systems and coordinates.

## Equation

To measure the equation of a line or circle, select one or more lines and circles and press F5 Measure: (TI-89: alpha D) D Analytic:
2 Equation. Sketchpad measures and displays the equations of the selected objects.


Unlike other measurements, equations do not have prefixes or suffixes that can be edited.

## Abscissa (X)

Of course, you can see both the abscissa and ordinate of a point together when you measure that point's coordinates. But you may sometimes wish to measure just the abscissa or ordinate of a point-for instance, if you wish to create a calculation based on just the x - or y -coordinate.

To measure the abscissa-or $x$-coordinate-of a point, select that point and press F5 Measure: (TI-89: alpha D) $D$ Analytic: 3 Abscissa (X).
Sketchpad measures and selects the abscissa of the selected point.

## Ordinate (Y)

To measure the ordinate-or $y$-coordinate-of a point, select that point and press F5 Measure: (TI-89: alpha D) DAnalytic: 4 Ordinate (Y).
Sketchpad measures and selects the ordinate of the selected point.

## F6 Graph Menu

The F66 Graph menu turns your Euclidean plane into a coordinate plane, letting you define square, rectangular, or polar coordinate systems and grids in your sketch. You can plot functions by entering their equations and plot points at fixed or calculated coordinates.

## About Coordinate Systems

Use the F6 Graph menu to create coordinate systems and grids and to plot points and functions. Sketchpad supports three types of coordinate grids:

| Square | Square coordinate grids are defined by an origin and <br> equal horizontal and vertical units, and identify <br> points by coordinate pairs $(x, y)$. |
| :---: | :--- |
| Rectangular | Rectangular grids systems are defined by an origin <br> and unequal horizontal and vertical units, but still <br> identify points by coordinate pairs $(x, y)$. |
| Polar | Polar coordinate grids are defined by an origin and a <br> unit distance, and identify points by coordinate pairs <br> $(r, \theta)$, where $r$ measures the distance of the point <br> from the origin (in the coordinate system's units) <br> and $\theta$ measures the (signed) angle that a ray from the <br> origin through the point makes with respect to a ray <br> extending from the origin to the right. |

Coordinate systems contain the following components:


## Origin

## Unit Point(s)

A point that defines the center of the coordinate system. Drag the origin to change the coordinate system's location in the plane.

A point whose distance from the origin describes the length of a single unit in the coordinate system. Rectangular coordinate systems have two unit points-one on each axis. Drag a unit point to change the coordinate system's scale.

An axis is a horizontal or vertical line passing through the origin. Tick marks on the axis represent the scale of the coordinate system.

Grid
A coordinate system's grid reveals the "shape" of the coordinate system's coordinatization of the plane.

Define Coordinate System (TI-92 Plus /Voyage ${ }^{\text {TM }} 200$ PLT)/ Define Axes (TI-89)

This command creates a new coordinate system defined by existing objects. Usually, you will not need to define a coordinate system explicitly-Sketchpad creates and displays one for you automatically whenever you need it. For instance, when you measure a point's coordinates, Sketchpad displays a square coordinate system centered in the sketch. However, you may occasionally need to define a coordinate system with a given origin, or a given unit length. In either case, select the appropriate defining object or objects and press F6 Graph: 1 Define Coordinate System (TI-89: 2nd [F6] Graph: 1 Define Axes ). The following table lists the ways in which you can define new coordinate systems and the objects you must select to define them.

| Command | Selection | Result |
| :---: | :---: | :---: |
| Define Origin | One point | Square coordinate system centered on the selected point and with a default unit |
| Define Unit Circle | One circle | Square coordinate system centered on the selected circle and with a unit equal to the circle's radius |
| Define Unit <br> Distance (TI-89: <br> Define Unit) | One segment or one distance measurement | Square coordinate system with a default origin and a unit length equal to the selected length |
| Define Unit <br> Distance (TI-89: <br> Define Unit) | One point and either one segment or one distance measurement | Square coordinate system centered on the selected point with a unit length equal to the selected length |
| Define Unit <br> Distances (TI-89: <br> Define Units) | Two segments or distance measurements | Rectangular coordinate system with a default origin, an $x$ unit length determined by the first selected length, and a $y$ unit length determined by the second selected length |
| Define Unit <br> Distances (TI-89: <br> Define Units) | One point and either two segments or two distance measurements | Rectangular coordinate system centered on the selected point with an $x$ unit length determined by the first selected length and a $y$ unit length determined by the second selected length |

## Mark Coordinate System (TI-92 Plus /Voyage ${ }^{\text {TM }} 200$ PLT)/ Mark Axes (TI-89)

## Graph Form

## Show/Hide Grid

See Performance Tips (page 86) for important tips about working with grids.

## Snap To Grid

Choose Mark Coordinate System or Mark Axes to make a non-active coordinate system active. Sketchpad plots points or functions on the active coordinate system. Similarly, Sketchpad measures coordinates and equations in reference to the active coordinate system. In a sketch with multiple coordinate systems, use Mark Coordinate System or Mark Axes to switch between active coordinate systems, as described in the following steps.

1. Show the grid of the non-active coordinate system if it is not already showing. (See Show All Hidden, page 53).
2. Select the non-active coordinate system by positioning the Arrow tool at one of its grid points and pressing ENTER.
3. Press F6 Graph: 2 Mark Coordinate System (TI-89: 2nd [F6] Graph: 2 Mark Axes). Sketchpad activates the selected coordinate system and confirms the activation with a brief animation. Future plotting and measuring will be conducted in reference to the activated coordinate system.
If a sketch has only one coordinate system, there is no need to use Mark Coordinate System or Mark Axes.

Use the Graph Form menu to create a new Square, Rectangular, or Polar coordinate system in a sketch with no coordinate system or to switch the active coordinate system between square, rectangular, and polar shapes.

Note that if you have defined the active coordinate system in terms of a single unit length (for instance, by Define Unit Distance or Define Unit Circle), you can switch it between square and polar shapes, but you cannot switch it to a rectangular shape (because rectangular coordinate systems must be defined by two unit lengths). Similarly, if you have defined a coordinate system in terms of two unit lengths (Define Rectangular Coordinates), you cannot make it square-except by making the two unit lengths equal. Sketchpad makes available only the Graph Form commands that are appropriate to the active coordinate system.

Choose Show Grid to show the grid associated with the active coordinate system. Choose Hide Grid to hide a visible grid.
(You may also show and hide grids as you would other objects-using Show All Hidden from the Display menu or by selecting them and choosing Hide Object from the Display menu. However, it's usually more convenient to show and hide the grid directly with a single command.)

Choose Snap To Grid to force dragged points to snap to grid points. When Snap To Grid is active, a checkmark appears next to it in the Graph menu. Choose Snap To Grid a second time to return to normal dragging.

Snap To Grid is useful when investigating coordinatized constructions in which you'd like dragged points to always have whole number coordinates.

Plot Point...
Plot As ( $\mathbf{x}, \mathrm{y}$ )
Plot As (r, $\boldsymbol{\theta}$ )

## When plotting

measurements or calculations, Sketchpad ignores the units of each measurement and plots a point based only on its magnitude. Thus, plotting two measurements that are both equal to 1 inch in length yields a point at $(1,1)$, where one unit is defined by the coordinate system's units, not inches. If you switch to centimeters in Preferences, the measurements will each change to 2.54 (because 1 inch is 2.54 cm ), and as a result the point will jump to (2.54, 2.54).

Choose Plot Point, Plot As $(\mathbf{x}, \mathbf{y})$, or $\operatorname{Plot} \operatorname{As}(\mathbf{r}, \boldsymbol{\theta})$ to define a point on the active coordinate system given its coordinates.

To define a point at specific numeric coordinates:

## Steps <br> Display

1. Press F66 Graph: (TI-89: 2nd [F6]) 6 Plot Point.
Sketchpad displays the Plot Point dialog box.

2. Enter the values of the coordinates you wish to plot. You may type numbers or short mathematical expressions, such as $2 \pi$ or $3 / 4$. Use the menu to switch between rectangular ( $x$, $y$ ) and polar $(r, \theta)$ notation.
3. Press ENTER twice to close the dialog box. Sketchpad constructs and displays a point at the specified coordinates. When you alter the coordinate system (by dragging the origin or changing its unit length), the point remains at the specified coordinates.


Change the coordinates of an already plotted point by selecting the plotted point and choosing [F6 Graph: (TI-89: 2nd [F6]) 6 Edit Plotted Point.

To plot a point at a coordinate position that you've computed with measurements or the calculator, follow these steps:

1. Select two measurements or calculations, the first defining an $x$ coordinate, the second defining a $y$-coordinate.
2. Press F66 Graph: (TI-89: 2nd [F6]) 6 Plot As (x, y). Sketchpad constructs a point whose rectangular coordinates are equal to the two selected measurements. Changing the values of the measurements (by altering the geometric properties they measure) changes the coordinates of the point.

Or, on a polar grid:

1. Select two measurements or calculations, the first defining a radial distance $r$, the second defining an angle $\theta$.
2. Press F66 Graph: (TI-89: 2nd [F6]) 6 Plot As (r, $\theta$ ). Sketchpad constructs a point whose polar coordinates are equal to the two selected measurements. Changing the values of the measurements (by altering the geometric properties they measure) changes the coordinates of the point.

Your choice of equationtype determines the function's independent and dependent variables and how the graph will appear when plotted. You may choose a new equation-type at any point while entering or editing your function, and Sketchpad will update the independent and dependent variables.

Choose Plot Function to graph any function on the active coordinate system.

When you choose Plot Function, the function calculator appears, allowing you to enter the function you wish to plot. The function calculator works in the same manner as the regular Sketchpad calculator, with the following additions:

1. A new menu, F5 Graph, appears in the calculator's menu bar. Choose the equation-type you wish to graph from this menu. Each equation-type has an independent variable (on the right) and a dependent variable (on the left). Choices are $y=f(x), x=f(y), r=f(\theta)$, and $\theta=f(r)$.
2. At the left edge of the calculator entry line, Sketchpad displays the dependent variable of your equation. (You can't edit this in the calculator entry line, but you can change it with the
F5 Graph menu.)
3. In addition to measurements, the F1 Values menu includes your function's independent variable for entry into your equation. (You can also type the independent variable from the keypad.) Change independent variables with the F5 Graph menu.

Example Graph: $\boldsymbol{y}=\mathbf{2 x + 1}$

## Display

1. Press [F6 Graph: (TI-89: 2nd [F6]) 7 Plot Function. Sketchpad displays the function calculator, with $y=f(x)$ as the default equation to graph.
2. Press 2 and then the multiplication key. Press F1 Values: $1 \mathbf{x}$ to insert $x$ into $y=2 * x+1=$ your expression (or press $\mathbf{x}$ on the keypad). Then press + and $\mathbf{1}$.
3. Press ENTER to close the function calculator. Sketchpad displays both the function $(y=2 x+1)$ and the constructed function plot (a line).


See Performance Tips (page 86) for additional notes about using function plots.

## About Function Plots

Function plots in Sketchpad behave like loci-they are plotted by evaluating the function at a number of samples. You can determine the number of samples in a function plot: for a specific plot, use Graph Properties (see page 48); or for all plots, use Plot Preferences (see page 52). If the number of samples is too low, the plot may not be visually accurate (depending on the complexity of the function). There is a tradeoff between speed and accuracy-the higher you set the number of samples, the better the representation of the plot on-screen, but the slower it will be when dragging objects.

As with loci, function plots may have points constructed on them, but cannot be intersected, transformed, or measured. You can alter the interval over which Sketchpad evaluates the function by dragging the arrowheads that appear at the plot's endpoints with the Arrow tool or by specifying the domain of the independent variable numerically (using Graph Properties, see page 48 ).

Recall that Sketchpad evaluates certain trigonometric functions in the current units set in Preferences. Thus, $\mathrm{y}=\sin (x)$ will display differently if your angle units are set to radians than if they are set to degrees. (The sine function repeats after 360 units when $x$ is evaluated in degrees, but in only $2 \pi$ units when $x$ is evaluated in radians.) You can change your units at any time with Preferences (see Units Preferences, page 52).

Finally, you can alter the equation of the function being plotted by selecting the function equation (not the function plot) and pressing F66 Graph: (TI-89: 2nd [F6]) 7 Edit Function.

## Tips and Techniques



Once you understand how Sketchpad works and what you can do with it, there are a variety of techniques you can use to work more efficiently. This chapter describes many of those techniques.

A family of oscillating spirals. This sketch shows the family of curves $\theta=80 r+25 \sin 400 r+k$, where $\theta$ is interpreted in degrees. The graph of $\theta=80 r$ is a simple spiral, and adding $25 \sin 400 r$ to the right side causes the spiral to oscillate as it moves away from the origin. Adding the varying parameter $k$ rotates the entire oscillating spiral about the origin. In this sketch, $k$ is the angle formed by a point on a circle, the circle's center, and a second point on the circle. Tracing the function plot as one of the points on the circle animates around the circle
 graphs a family of curves related by rotation.

Dilating a polar graph. This sketch is based
 on the graph of $r=\sin 6 \theta+2$, with $\theta$ interpreted in radians. Dilating a point on this graph away from the origin, and then connecting it to its dilated image with a segment, creates one spoke of the inner "flower." Dilating this segment away from the origin creates a second collinear spoke (on the outer flower). The locus of the spokes and their endpoints, as the original point travels along the curve, completes the figure. For complicated graphs like these, Sketchpad requires a high number of plot samples and ample time to calculate the graph.

## Shortcuts

| Toolbox Shortcuts | TI-89 keystrokes | TI-92 Plus <br> /Voyage ${ }^{\text {TM }} 200$ PLT keystrokes | Effect |
| :---: | :---: | :---: | :---: |
|  | $\square 1$ | $\bullet$ F1 | activates the current Selection Arrow tool immediately (typing $\bullet 1$ or $\bullet$ F1 again cycles through Selection Arrow tools) |
|  | $\square 2$ | - F2 | activates the Point tool |
|  | $\square 3$ | -F3 | activates the Compass tool immediately |
|  | $\bullet 4$ | -F4 | activates the current Straightedge tool immediately (typing $\bullet 4$ or $\bullet$ F4 again cycles through Straightedge tools) |
|  | $\checkmark 5$ | -F5 | activates the Text tool immediately |
|  | $\checkmark 6$ | - F6 | activates the Custom Tools tool immediately |
|  | - | -F8 | hides the toolbox if it is showing or shows the toolbox if it is hidden (see Show/Hide Toolbox, page 57, for more information) |

## Menu Command Shortcuts

| TI-89 keystrokes | TI-92 Plus <br> /Voyage 200 PLT keystrokes | Effect |
| :---: | :---: | :---: |
| T Z | $\square \mathrm{z}$ | F1 Edit: 1 Undo command |
| 1 x | $\square x$ | F1 Edit: 2 Redo command |
| $\bullet \square$ | $\bullet \square$ | F1 Edit: 3 Delete command |
| $\square \mathrm{A}$ | $\square \mathrm{A}$ | F1 Edit: 5 Select All command |
| $\dagger \mathrm{P}$ | $\bullet \square$ | F1 Edit: 6 Select Parents command |
| T C | $\square$ | F1 Edit: 7 Select Children command |
| 10 | -0 | F1 Edit: 8 Properties command |
| T N | $\square \mathrm{N}$ | F1 Edit: 9 Sketch: 1 New command |
| $\square \mathrm{H}$ | $\square \mathrm{H}$ | F2 Display: 1 Hide Objects command |
| 1 J | $\square J$ | F2 Display: 3 Show/Hide Labels command |
| - STO | - STO | F2 Display: 6 Trace command |
| - CLEAR | CLEAR | F2) Display: 7 Clear All Traces command |
| $\pm$ U | $\bullet \cup$ | F2 Display: 8 Animate command |

Menu Command
Shortcuts
(continued)

| Other Keyboard Shortcuts (continued) | TI-89 keystrokes | TI-92 Plus <br> Voyage <br> keys 200 PLT | Effect |
| :---: | :---: | :---: | :---: |
|  | ENTER | ENTER | when pointing a drawing tool at a path object, constructs a point on the object |
|  | ENTER | ENTER | when pointing a drawing tool at an intersection of two objects, constructs the point of intersection |
|  | 2nd (+ + ) | 2nd (+ + ) | when positioning the second point with a Straightedge tool, constrains the straightedge's slope to $15^{\circ}$ increments (in other words, after creating the first control point of a straight object, press and hold 2nd while using $(\cdot)$ to position the second control point) |
| ESC Key | ESC is a general-purpose shortcut for "escaping" from the current Sketchpad activity in both the TI-89 and the TI-92 Plus/Voyage ${ }^{\text {TM }} 200$ <br> PLT. The action taken by ESC depends on your current activity, but if you press ESC repeatedly, eventually you will return to a sketch in which the Translate Arrow is the active tool, no objects are selected, no animation is active, and no traces are visible. <br> Specifically, pressing ESC: |  |  |
|  |  |  |  |
|  | Specifically, pressing ESC: <br> 1. Closes a pulled-down menu without choosing any command in it, if a menu is showing. |  |  |
|  | 2. Chooses the Translate Arrow if it is not the active tool, stopping the active drawing tool if you are in the midst of using a drawing tool to construct objects. (The objects you are in the process of constructing will disappear.) |  |  |
|  | 3. Deselects any selected objects. |  |  |
|  | 4. Stops the active animation, if no objects are selected and an animation is active. |  |  |
|  | 5. Clears all visible traces if traces are visible, the Translate Arrow is chosen, no animation is active, and no objects are selected. |  |  |

## Construction and Display Hints

This section describes useful techniques for editing constructions, making particular types of figures, and modifying the appearance of your sketch.

## Editing Constructions

## Fixing a Distance

## Fixing an Angle

In general, objects in Sketchpad always behave according to the rules by which they were constructed. A point constructed as the midpoint of a segment will always appear at the midpoint of that segment. Thus, if you create a construction based on a point on a segment and later decide that you needed that point to be defined on a circle, you need to change the construction itself.

A simple way to edit an incorrect construction is to delete the incorrect portion (see Delete, page 43) and reconstruct it from scratch. For example, select a point on a segment, delete it, and construct a new point on a circle instead. However, when you delete an object, all objects that depend on it are deleted as well, so this method is not helpful if the (incorrectly defined) point on the segment defines many other objects in your sketch. In this case, Split and Merge (see page 43) can be used to redefine your constructions. Specifically, Split can redefine a dependent point to be independent, and Merge can redefine an independent point to be dependent. Thus, you could Split the point on a segment off the segment, and then select the (split) point and the circle and Merge them, in essence changing a point on a segment (in two steps) to be a point on a circle.

Occasionally you may wish to create a segment of fixed length-for instance, a segment that is exactly 3 cm long. While you could create a segment with the Segment tool, measure its length, and drag the segment's endpoints until the measurement was exactly 3 cm , this segment would not be constructed to be exactly 3 cm long. That is, nothing in your definition of the segment indicates that it must remain 3 cm in length, and when you drag it, its length will change.
To fix a distance in Sketchpad, use the Transform menu to translate a point by a fixed distance (see Translate, page 67). For instance, select a point $A$ and construct its translated image (by 3 cm at $0^{\circ}$ ) $A^{\prime}$. Then construct a segment between $A$ and $A^{\prime}$. This horizontal segment will always be exactly 3 cm in length, because you've constructed its endpoints to be exactly that far apart. If you want your segment to have an arbitrary slope-rather than being fixed as horizontal-instead construct a circle centered at $A$ passing through $A^{\prime}$. Then use the Segment tool to draw an arbitrary radius of this circle. (A radius of this circle will always be-by definition- 3 cm in length, and you can drag it to have any slope.)

Occasionally you may wish to create an angle of fixed measure-for instance, an angle that measures exactly $45^{\circ}$. While you could create an angle by measuring three points and dragging them until they form an angle of $45^{\circ}$, this angle would not be constructed to be exactly $45^{\circ}$. (That is, dragging it again might change its measure.)

## Stopping Objects from Tracing

To fix an angle in Sketchpad, use F4 Transform: 8 Rotate (see page 68) to construct the rotated image of one point around a marked center point by a fixed angle. Then draw an angle using segments or rays from the vertex to the other two points. Dragging will no longer change the measurement of this angle, because you've constructed it to be fixed in measure.
Sometimes, after tracing several objects during a drag or an animation (see Trace, page 54), you'll wish to stop tracing all traced objects. You can stop any traced object by selecting it and choosing F2 Display: 6 Trace
[Object] to remove the checkmark, but if you have many objects and don't remember which ones are traced, this can be tedious.

You can stop tracing for all trace-activated objects by choosing F1 Edit:
5 Select All followed by F2 Display: 6 Trace [Objects] and finally F2 Display: 6 Trace [Objects] again. The first command selects every object. The first Trace command activates tracing for every (selected) object. Because tracing is now activated for all objects, the second Trace command deactivates tracing for all these objects. The result is that no trace-activated objects remain in your sketch.

## Showing One Object

Sometimes, after hiding many objects, you'll wish to show a hidden object. But the Show All Hidden command shows all hidden objects. If you want to show just one object, showing all of them may be overkill. There are two ways to show just one or a few hidden objects.

The first is to use F1 Edit: 8 Properties to set the visibility of a (hidden) object to show it. While you cannot select a hidden object to display its Properties directly, you can select a related object (a parent or a child of the hidden object) and then navigate to the hidden object's Properties using the Parents and Children menus in the Object Properties dialog box page. (See Object Properties, page 46, for more information.)

A second and more convenient method for showing one or more hidden objects is also available. Note that Show All Hidden shows and selects all hidden objects. Thus, you can first show all hidden objects; then deselect the objects you wish to show from the group of just-shown, selected objects, by clicking them with the Arrow; and then rehide the remaining selected objects (i.e., the objects you don't wish to show) by choosing F2 Display: 1 Hide [Objects].

## Hiding All Labels

Occasionally you may wish to hide the labels of all or most labeled objects to reduce clutter on your screen. Using the Text tool or selecting the objects individually and using Hide Labels can be tedious if there are many labeled objects in your sketch. A quick way to hide all visible labels is to press F1 Edit: 5 Select All, then F2 Display: 3 Show Labels, followed by F2 Display: 3 Hide Labels. The first command selects every object in the sketch. The second causes all selected objects to display their labels. The third command-because all selected objects are already showing labels-causes all selected objects to hide their labels. The result is that all objects in the sketch hide their displayed labels.

## Performance Tips

> Sketchpad is a rich modeling environment for exploring geometry and many forms of visual mathematics. But mathematics is richer still. Mathematicians routinely ask questions that even supercomputers can't answer in any reasonable amount of time! This section lists a few pointers about optimizing the performance of The Geometer's Sketchpad for TI-89, TI-92 Plus, and Voyage ${ }^{\text {TM }} 200$ PLT.

## Selection

## Loci and Function Plots

Take advantage of Sketchpad's default selections. Whenever you construct something, Sketchpad leaves the result selected. In many construction sequences, you'll use that result in your next step. You can frequently reduce the amount of time you spend moving the cursor about if you anticipate and take advantage of Sketchpad's selections. In addition, many of Sketchpad's commands can operate on multiple selections simultaneously. For instance, you can construct three parallel lines instead of one by selecting a point and three lines, or a line and three points, before constructing parallels.
The example below-the sixth midpoint heptagon of a heptagon-is made up only of segments, but would take even an expert several minutes to construct with the Segment tool.

The most efficient way to construct it in Sketchpad is to create seven selected points with the point tool. (Hold down 2nd when placing points to keep them selected. On the TI-89, make sure when placing multiple selected points to use the ENTER key instead of the CLEAR key shortcut in order for the 2nd key to have the
 desired effect.) Then repeatedly alternate between the Segments command and the Midpoints command from the F3 Construct menu, without changing your selections between commands. The final figure can be constructed in just a few seconds.

Loci and function plots are the most complex objects that Sketchpad evaluates. In order to plot a locus or a function, Sketchpad must perform thousands of internal calculations for each appearance of the plot. Depending on their complexity, these objects may cause dragging to slow down considerably.
Fortunately, you can control the complexity of Sketchpad's internal calculations. Each (locus or function) plot is composed of a number of samples-positions at which the construction or function is being
evaluated in order to determine the overall shape of the plot. A higher number of samples leads to smoother images, but a lower number of samples leads to faster dragging.
Become familiar with how and when to change the number of samples in a plotted locus or construction-see Locus Properties (page 47), Graph Properties (page 48), and Plot Preferences (page 52). You might wish to keep your Preferences settings to a very low number of samples for new loci and function plots. Then, once you've created a locus or function plot, if its display accuracy is not adequate, increase the number of samples for that plot only, using its Properties dialog box.
After you've manipulated and dragged your construction, if you're interested in a very smooth final image, position the plot as you'd like to see it and then set its Properties to a very high number of samples. If your locus or function is sufficiently complex, and you've requested 800 samples, Sketchpad may take several minutes to evaluate a single image of it. Don't try to drag the plot when its number of samples is so high! Instead, consider using animation in conjunction with tracing to generate a series of (traced) images at high resolution.
Grids

## Delete vs. Undo

While grids are not challenging objects mathematically, they can be slow to display and to drag because the slightest change to a grid requires the entire screen to be redrawn. Unless a coordinate system investigation specifically requires a visible grid, consider hiding the grid before dragging objects. On the small screen of a graphing calculator, hiding the grid may also improve the legibility of your sketch.

Keep in mind that Sketchpad supports unlimited undo and redo of basic operations (see page 42). Though very powerful, unlimited undo has a cost-it uses program memory to remember every basic action you've performed in your sketch. Thus whenever you delete objects in a sketch (see Delete, page 43), Sketchpad deletes them from the screen, but cannot delete them from memory because you might wish to undo the deletion later. Over time, deleted objects take up memory on your graphing calculator that could better be put to other uses. While you should feel free to delete objects whenever you need to, it pays to keep in mind some tips for using the Delete command wisely:

1. If you wish to remove an object you've just created, Undo that object rather than Delete it. If you undo it, Sketchpad can release the memory it uses to remember where that object was created.
2. If you wish to start work in a blank sketch, use F1 Edit: 9 Sketch: 1 New rather than selecting all and then choosing F1 Edit: 3 Delete [Objects]. Moving to a new sketch lets Sketchpad release all the memory it uses to remember objects in your previous sketch.
3. If Sketchpad ever warns you that you are running low on memory, refresh your undo history by saving (see Unlimited Undo and Memory, page 42, for instructions on how to do this). Then delete any unneeded sketches from the VAR-LINK page. To do this, press 2nd [VAR-LINK] to access the VAR-LINK main menu. Use the cursor pad to scroll down to any unneeded file (Sketchpad files are listed as GSP). With the unneeded file selected, press F4 to put a check by its name. In the same way, put checks by the names of all unneeded files. Now press F1 Manage: 1 Delete to delete the file(s). Press ENTER to confirm the deletion. When you're done deleting files, press ESC to return to Sketchpad.

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