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Structuring Lessons to Develop Mathematical Thinkers

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Structuring Lessons to Develop Mathematical Thinkers

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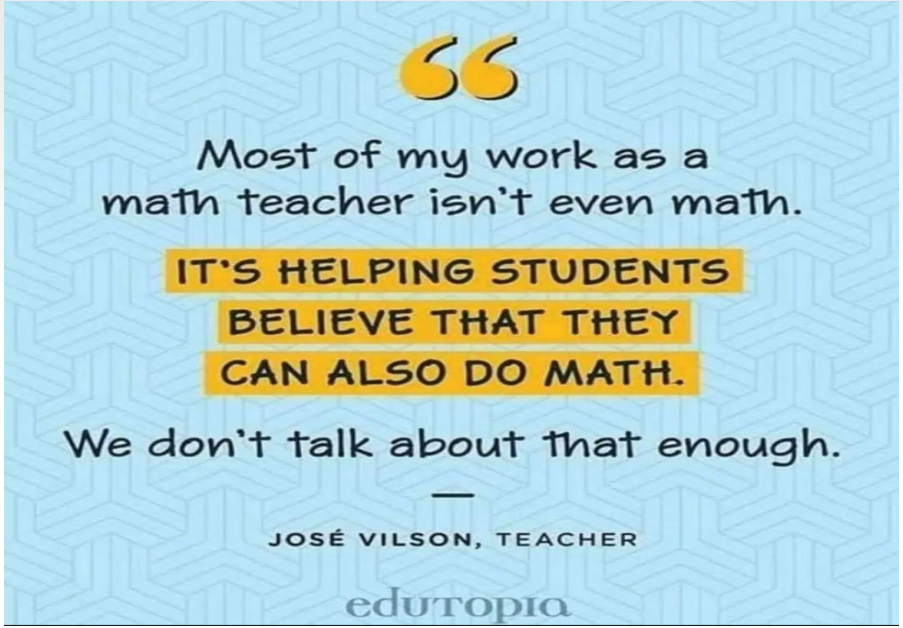
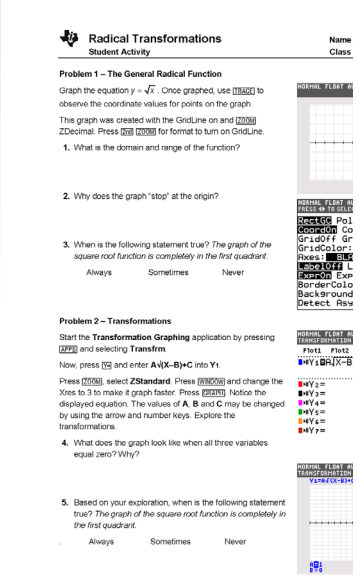
LEARNING OUTCOMES

- To build a lesson designed so that students see themselves as capable doers of mathematics
- Select appropriate tasks to encourage curiosity, discussion, collaboration, and written and verbal reflection
- Use technology as a tool for investigation and interpretation
- Extend the content/topic as needed to meet the needs of the students


Most of my work as a
math teacher isn't even math.
IT'S HELPING STUDENTS
BELIEVE THAT THEY
CAN ALSO DO MATH.
We don't talk about that enough.
—
JOSÉ VILSON, TEACHER
edutopia

Structuring Lessons to Develop Mathematical Thinkers



Radical Transformations
Student Activity

Name
Class

Problem 1 – The General Radical Function

Graph the equation $y = \sqrt{x}$. Once graphed, use TRACE to observe the coordinate values for points on the graph.

This graph was created with the GridLine on and ZOOM ZDecimal. Press 2nd ZOOM for format to turn on GridLine.

1. What is the domain and range of the function?

2. Why does the graph "stop" at the origin?

3. When is the following statement true? *The graph of the square root function is completely in the first quadrant.*

   Always Sometimes Never

Problem 2 – Transformations

Start the **Transformation Graphing** application by pressing APPS and selecting **Transfrm**.

Now, press Y= and enter **A√(X−B)+C** into **Y1**.

Press ZOOM, select **ZStandard**. Press WINDOW and change the Xres to 3 to make it graph faster. Press GRAPH. Notice the displayed equation. The values of **A**, **B** and **C** may be changed by using the arrow and number keys. Explore the transformations.

4. What does the graph look like when all three variables equal zero? Why?

5. Based on your exploration, when is the following statement true? *The graph of the square root function is completely in the first quadrant.*

   Always Sometimes Never

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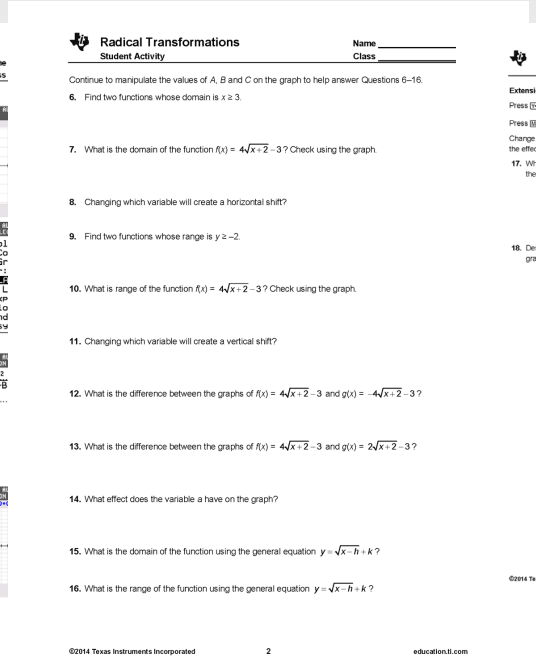

Radical Transformations
Student Activity

Name _____
Class _____

Continue to manipulate the values of *A*, *B* and *C* on the graph to help answer Questions 6–16.

6. Find two functions whose domain is $x \geq 3$.

7. What is the domain of the function $f(x) = 4\sqrt{x+2} - 3$? Check using the graph.

8. Changing which variable will create a horizontal shift?

9. Find two functions whose range is $y \geq -2$.

10. What is range of the function $f(x) = 4\sqrt{x+2} - 3$? Check using the graph.

11. Changing which variable will create a vertical shift?

12. What is the difference between the graphs of $f(x) = 4\sqrt{x+2} - 3$ and $g(x) = -4\sqrt{x+2} - 3$?

13. What is the difference between the graphs of $f(x) = 4\sqrt{x+2} - 3$ and $g(x) = 2\sqrt{x+2} - 3$?

14. What effect does the variable *a* have on the graph?

15. What is the domain of the function using the general equation $y = \sqrt{x-h} + k$?

16. What is the range of the function using the general equation $y = \sqrt{x-h} + k$?

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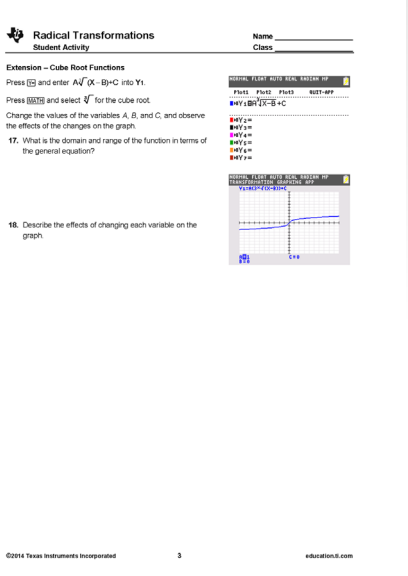

Radical Transformations
Student Activity

Name _____
Class _____

Extension – Cube Root Functions

Press Y= and enter $A\sqrt[3]{(X-B)}+C$ into **Y1**.

Press MATH and select $\sqrt[3]{\ }$ for the cube root.

Change the values of the variables *A*, *B*, and *C*, and observe the effects of the changes on the graph.

17. What is the domain and range of the function in terms of the general equation?

18. Describe the effects of changing each variable on the graph.

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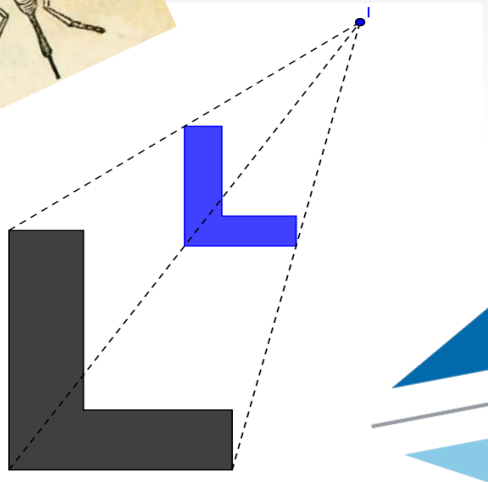
Invite students to be doers

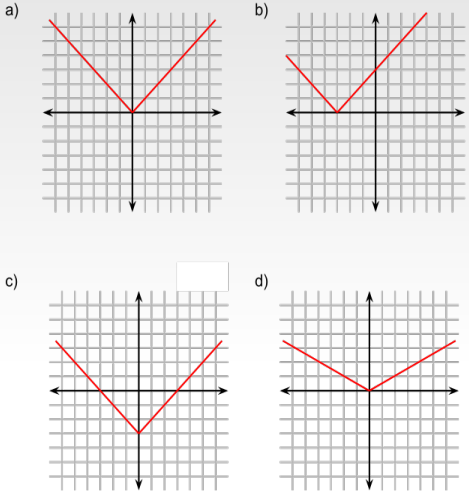
"If we want students to build this **authentic** understanding of the discipline of mathematics, they need to engage in these wonderful verbs (***notice, wonder, imagine, ask, investigate, figure, reason, connect, prove***) as they learn new mathematical content throughout the year." – Tracy Zager

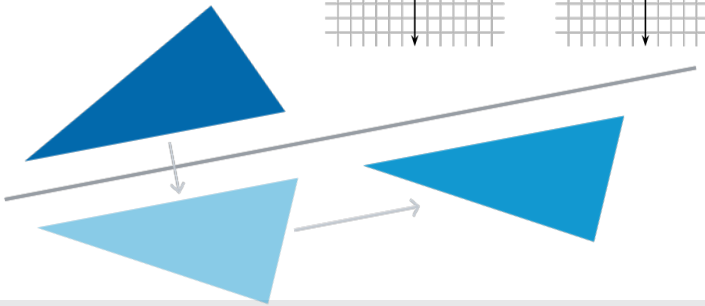
BECOMING THE MATHEMATICS TEACHER YOU WISH YOU'D HAD –

TRACY ZAGER

Are you curious?







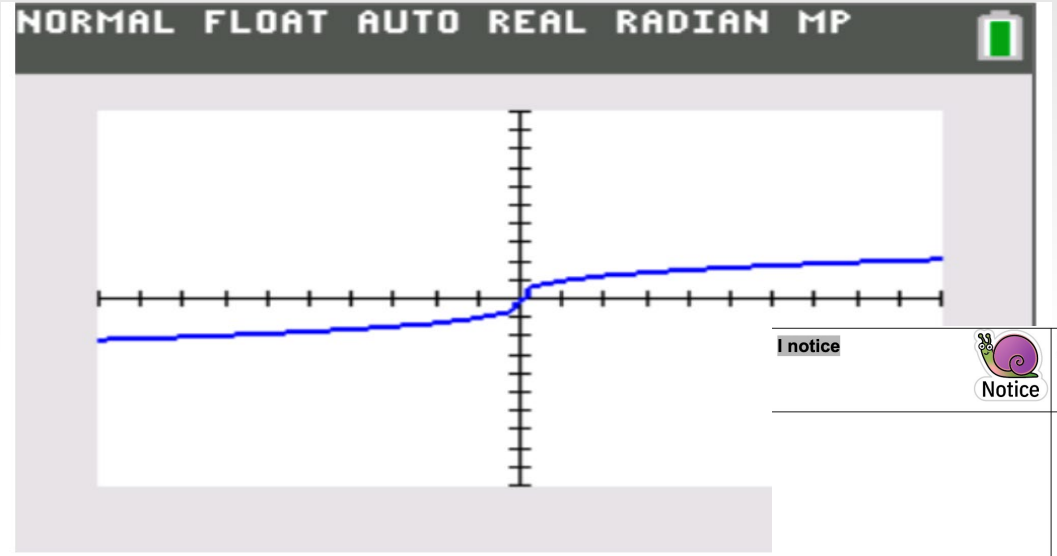
Mathematical Instructional Routines

Instructional routines are **specific** and **repeatable** designs for learning that support both the teacher and students in the classroom… enabling all students to engage more fully in learning opportunities while building crucial mathematical thinking habits. (Kelemanik, Lucenta, & Creighton, 2016).

Think, Write, PAIR, SHARE

What are some differences between a linear and nonlinear function?

Notice and Wonder



I notice	I wonder

Connect and Reflect

How do we use these routines effectively?

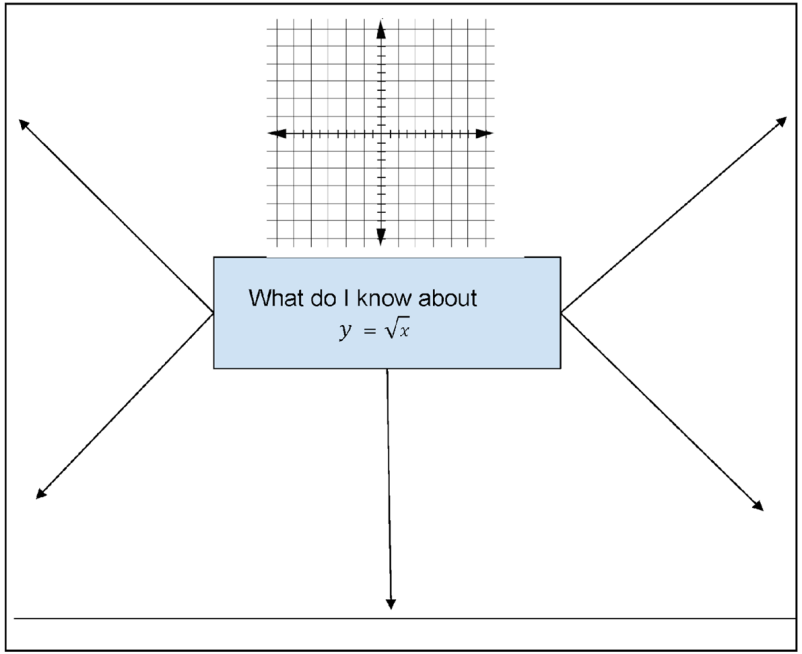
Do these routines:

- encourage discussion and provide the opportunity for reflection?
- deepen understanding?
- encourage and respect ALL voices are heard?

Transformation of Functions

Part One: Lets Observe

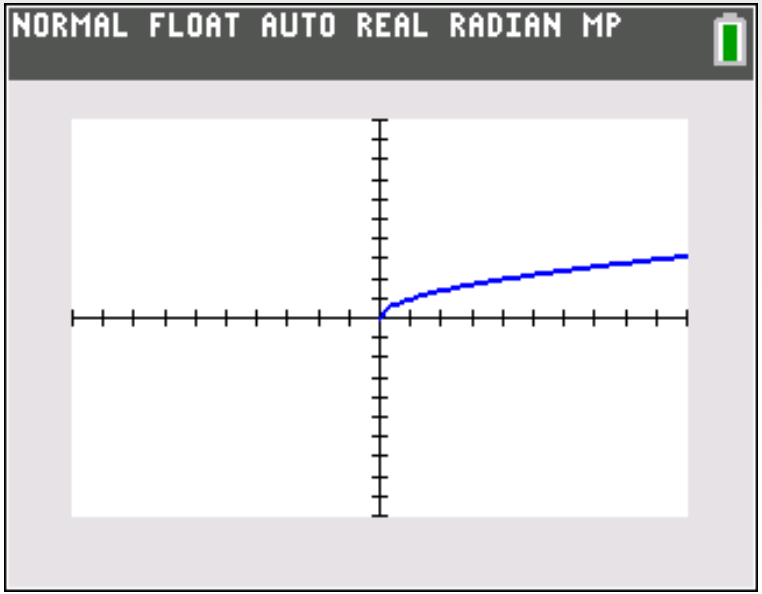
Use your graphing device to graph y = $\sqrt{x}$. What characteristics do you notice about this function, such as **domain, range, intercepts, graph, table of values, symmetry**, etc.?



Lets Connect:

The equation for this function is y = $\sqrt{x}$. Why does the graph for this function **not** appear in quadrants II, III, or IV?

Doing the math: The original task compared to the new one



Part Two: Lets Explore

Use the transformation grap

Press Y= and for **Y_1 enter** v

The values of A, B, and C c

arrows move the cursor fror

of each letter. To change th

alpha, graph, and then dow

Change in the Equation
Let **A**=1, **B**=0, **C**=0

The "A" value changes
Let A values increase, 0<A
Let A values increase, A>1

The "C" value changes	
Let A=1, B=0, C>0 Let C increase	Describe what happens to the graph as C increases. Include a sketch to compare how these graphs compare to the graph of the parent function.
Let A=1, B=0, C<0 Let C decrease	Describe what happens to the graph as B decreases. Include a sketch to compare how these graphs compare to the graph of the parent function.
Reflect: Explain what effect changing the C value has on the graph of the function.	

ct on the Graph
cribe what happens to the graph as B eases. Include a sketch to compare how e graphs compare to the graph of the ent function.
cribe what happens to the graph as B eases. Include a sketch to compare how e graphs compare to the graph of the ent function.
as on the graph of the function.

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#T3IC

Assessment Probes/Consolidation

Part Three: Lets Apply what we have learned

Info Gap - Partner activity

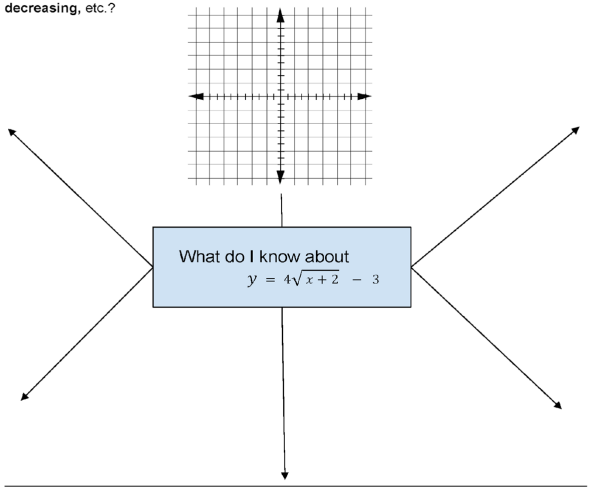
Each partner receives a card. One has a question, the other has some information. The person with the question must ask for specific information from the person with the info. The person with the info must ask why they need this info before giving it to them. When the person with the info has the required information, both solve the problem without seeing the other's cards. After solving, they share their solutions and their work and compare. Use $y = \sqrt{x}$ as the parent function.

Use a graphing calculator to verify your answers.

Card 1: Info A=2, vertical shift of 3, domain: $x \geq -2$	Card 1: Problem Give an equation for the transformed function.
Card 2: Info Vertex at (1,3) with a = -1.	Card 2: Problem What transformations have occurred?
Card 3: Info A=-1, Passes through (2,1) with a horizontal shift to the left 7 units.	Card 3: Problem What is the vertical shift?

Part Four: We've Got This!(Exit Ticket)

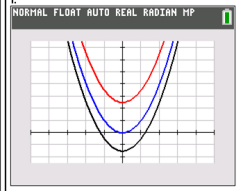
Without using your graphing device, what characteristics do you KNOW about this function, such as **domain, range, intercepts, graph, table of values, symmetry, increasing, decreasing,** etc.?



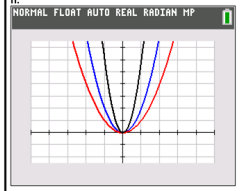
Alternatively

Which One Does Not Belong?

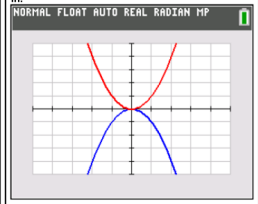
I.



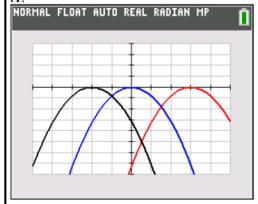
II.



III.



IV.



___ doesn't belong because

___ doesn't belong because

___ doesn't belong because

___ doesn't belong because

Mixed Six

Precalculus – Mixed Six – Transformations with Radical Functions

Calculator Neutral

1. Factual Recall

If f(x) = $\sqrt{x}$, describe how the change to the equation affects the graph of the parent function.

a. f(x) =a$\sqrt{x}$

b. f(x) =$\sqrt{x+b}$

c. f(x) = $\sqrt{x}$ + c

2. Carry out a Procedure

Let f(x) = a$\sqrt{x+b}$ +c
with a=2, b = -5, and c = 4

Determine:

a. f(9)

b. f(0)

c. f(13)

3. Classify a Mathematical Object

The function g is the result of transformations on the function f. Identify the transformations as M(multiplicative), ALR(additive, shifts left or right), AUD(additive, shifts up or down) or R(reflection). If more than 1 transformation, use multiple letters.

M/ALR/AUD/R	f(x) = $\sqrt{x}$
	g(x)=2$\sqrt{x-1}$
	g(x) = $\sqrt{x+3}$ -5
	g(x)= - $\sqrt{x}$ + 4

4. Prove, Show, Justify

Function *h* has the following values at selected values of *x*. Let $g = ah(x-b) + c$, where a, b, and c are positive constants. In the *xy*-plane, the graph of *g* is constructed by applying a vertical dilation(vertical stretch/shrink) by a factor of 4 and a horizontal translation(slide) of -2, and by a vertical translation of 3. Show that g(1) = 31

x	h(x)
0	4
3	7
5	9

5. Extend a Concept

x	-1	0	1	2
g(x)	6	-4	3	7

The table above gives values of a function *g* at selected values of *x*.

Let $f(x) = 3g(2(x+1)) + 4$.

a. Find f(-1)

b. Find a if f(a) = 25

6. Critique a Fallacy

Let g = af(x-b) + c, where *a, b,* and *c* are constants and f(x) = $\sqrt{x}$. In the *xy*-plane, the graph of *g* is constructed by applying a horizontal translation by 2 units left, a vertical dilation by a factor of 3, and a vertical translation of down 6 units.

Martin says that the equation for g(x) is g(x) = 3$\sqrt{x-2}$ - 6 What is wrong with this expression for the function *g*?

Write a correct expression for the function *g*.

Nancy Johnson, Massachusetts

Thinking, Engaging, Discussing, and Persevering

"Success is no accident. It is hard work, perseverance, learning, studying, sacrifice and most of all, love of what you are doing or learning to do."
– Pele

Questions?

One lucky winner receives a TI graphing calculator!

And the winner is …