

35 YEARS OF
IMPAC(T³TM)



Teachers Teaching
with Technology™

International Conference

AP[®] Calculus: From Those in the Know

Moderator:

Tom Dick, Ph.D.

T³ National Instructor

Panelists:

Julie Clark, Ph.D.

*Chief Reader
AP[®] Calculus*

Stephanie Ogden

*AP[®] Calculus Director
The College Board*

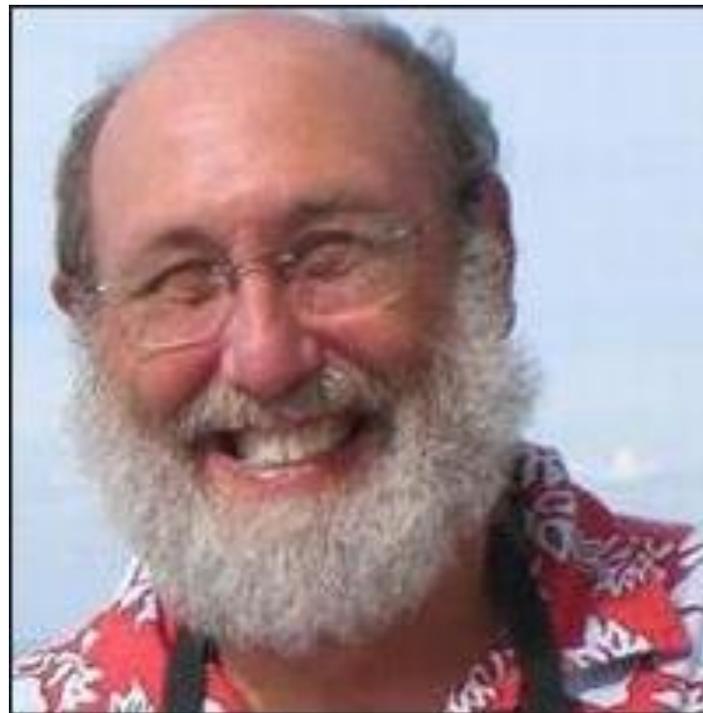
Benjamin Hedrick

*Assessment Specialist
Educational Testing Service*

Tom Dick

Moderator

- Professor Emeritus
- Former Math Department Chair
- Oregon State University
- T³ National Instructor
- Former Chair of AP Calculus Development Committee



Julie Clark

Panelist

- Associate Professor of Mathematics, Statistics, & Computer Science – Hollins University, Roanoke, VA
- B.S. Davidson College
- M.S., Ph.D. University of Virginia
- AP Calculus Chief Reader & Member AP Calculus Development Committee, 2019-2024
- “Mother” of Math Pups (Pythagoras, Trig, Tukey, Merkle, Bonferroni, Bernoulli, and Sqrt)



Stephanie Ogden

Panelist

- College Board Lead for AP Calculus & AP Statistics
- Taught Secondary and Higher Education for more than 30 years
- Principal Investigator for Knox County Schools' First to the Top Grant
- Established Tennessee STEM Innovation Network and the L & N STEM Academy
- T³ Regional Instructor



Benjamin Hedrick

Panelist

- Senior Manager, ETS STEM Assessments
- Former College Board Course Lead for AP Calculus and AP Statistics
- B.S. Mathematics and Statistics – Pennsylvania State University
- Masters Degree – Duke University
- Ph.D. – Stanford University
- Former Chair of AP Calculus Development Committee
- Taught calculus at all levels as well as in Japan & Finland



Agenda

- » Julie Clark, Ph.D. - Presentation
- » Stephanie Ogden, Ph.D. - Presentation
- » Benjamin Hedrick - Presentation

AP[®] Calculus From Those in the Know Panel Discussion

Julie Clark, Hollins University (VA) – AP Calculus Chief Reader



Virtual Conference

#T3IC



AP Calculus Development Committee 2022-2023



AP Calculus Development Committee 2022-2023

- ▶ Co-Chair: Frances Rouse, UMS-Wright Preparatory School, Mobile, AL
- ▶ Co-Chair: Mark Kannowski, DePauw University, Greencastle, IN
- ▶ CB Advisor: Erik Ahlquist, St. Louis Park High School, St. Louis Park, MN
- ▶ Mary Beisiegel, Oregon State University, Corvallis, OR
- ▶ Anisah Nu'Man, Spelman College, Atlanta, GA
- ▶ Brian Shay, Canyon Crest Academy, San Diego, CA
- ▶ Jerome White, The Willow School, New Orleans, LA
- ▶ Chief Reader: Julie Clark, Hollins University, Roanoke, VA
- ▶ College Board: Stephanie Ogden

AP Calculus Development Committee Incoming Members

- ▶ CRD: Sharon Taylor, Georgia Southern University, Statesboro, GA
- ▶ Co-Chair: Alex McAllister, Centre College, Danville, KY
- ▶ Karen Sleno, Flushing High School, Flushing, MI

Development Committee Responsibilities

- ▶ Plan, develop, and approve each exam.
- ▶ Participate in the Reading.
- ▶ Consider statistical analysis associated with the exam.
- ▶ Review, revise, and update the Course & Exam Descriptions.
- ▶ Advise the Chief Reader on Reading issues.
- ▶ Participate in Outreach Events.
- ▶ Regular face-to-face meetings once-twice a year
- ▶ Virtual meetings many times a year

Typical AP Calculus Exams

“Forms”

- US Main exam
- International Exam
- US Alternative Exam (late administration) & other confidential forms.

“Parts”

- Section I: 45 Multiple Choice Questions
- Section II: 6 Free Response Questions
- Calculator and Non-calculator parts
- AB and BC exams (3 common FR questions)

The Reading: Personnel 2023

- ▶ CR: Julie Clark
- ▶ 3 ACRs: Steve Kokoska, Alex McAlister, Sharon Taylor
- ▶ 8 Els: Virge Cornelius, Stephen Davis, Tom Dick, Mark Kiraly, Kristi Meyer, Tom Polaski, Frances Rouse, Karen Sleno
- ▶ 36 QLs
- ▶ 63 Early Table Leaders (ETLs)
- ▶ 79 TLs (21 in Kansas City, 58 at home)
- ▶ 1170 RDs (385 in Kansas City, 785 at home)
- ▶ 1 CRD: Mike Boardman
- ▶ Total: 1361

AP Calculus Reading Facts and Figures



Pre-Reading Participants

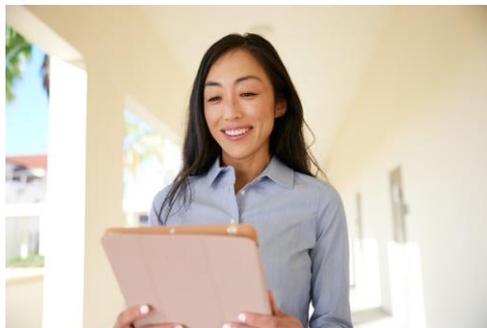
Students and Exams

410,522 exams submitted for scoring

2,463,132 total student responses scored

Why should I consider becoming an AP reader?

AP readers often refer to the AP Reading as one of the best professional experiences they have ever had.

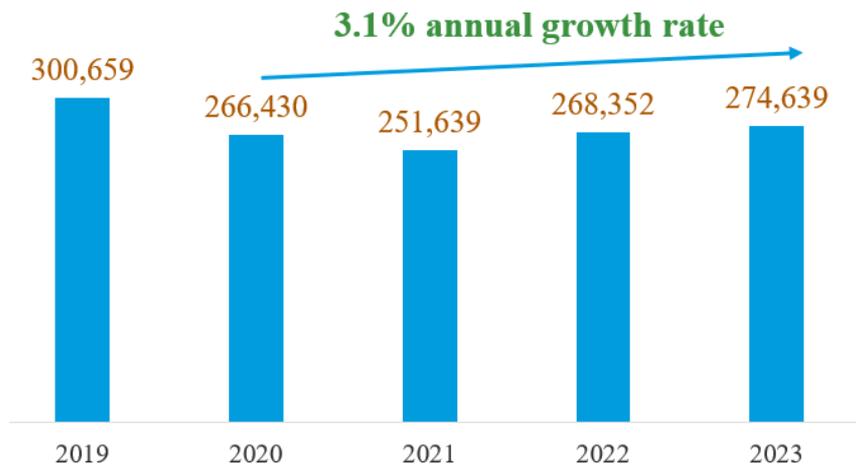


- **AP readers enjoy the experience.**
- **Experiencing the AP Reading leads to positive changes in the classroom.**
- **AP readers learn to apply rubrics with fidelity.**
- **AP readers gain exposure to the full universe of student responses—gaining valuable insight into the quality and depth of student responses from the entire pool of AP Exam takers.**
- **AP readers are compensated for their effort**
- **AP readers can earn Continuing Education Units (CEUs) and Professional Development Hours**

To learn more and apply, visit: collegeboard.org/apreading

AB Exam Volumes

AP Calculus AB Volume, 2019 – 2023

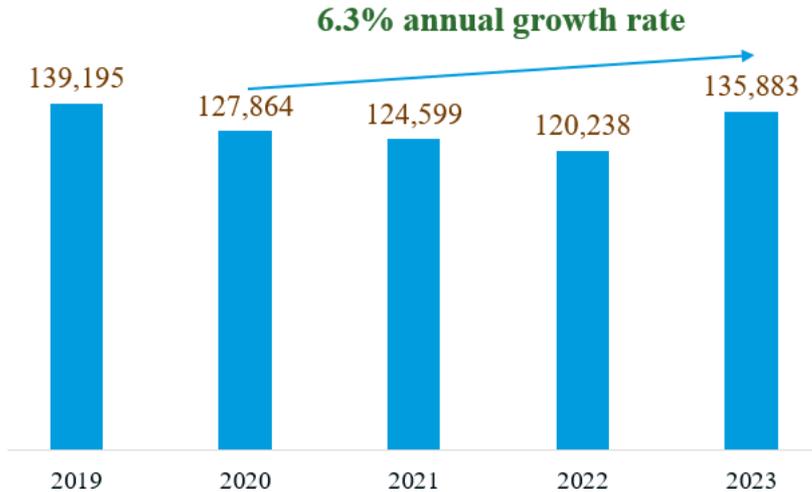


Observations

- 2.6 % decrease in 2019, before the pandemic
- 11.4% decrease in 2020
- 3.1% annual growth rate since 2020:
 - 5.6% decrease in 2021 (decreasing at a decreasing rate)
 - 6.6% increase in 2022
 - 2.2% increase in 2023

BC Exam Volumes

AP Calculus BC Volume, 2019 – 2023

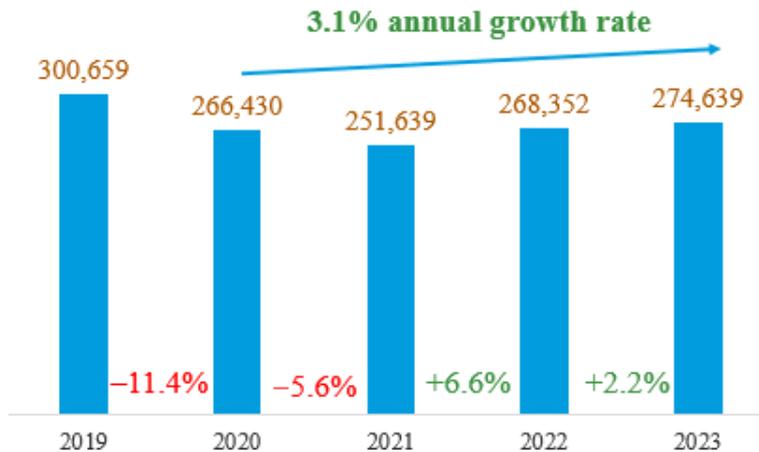


Observations

- Relatively stable in 2019
- 8.1% decrease in 2020
- 6.3% annual growth rate since 2020:
 - 2.6% decrease in 2021
 - 3.4% drop in 2022 (recall AB saw 6.65% growth in 2022)
 - 13% growth in 2023

Interactions

AP Calculus AB Volume, 2019 – 2023



AP Calculus BC Volume, 2019 – 2023



2022: BC students opting for the AB exam?

2023: BC boom, almost to pre-pandemic level

AP Calculus AB Exam Score Distribution

(All versions of 2023 exam)

58%

of students scored 3 or higher

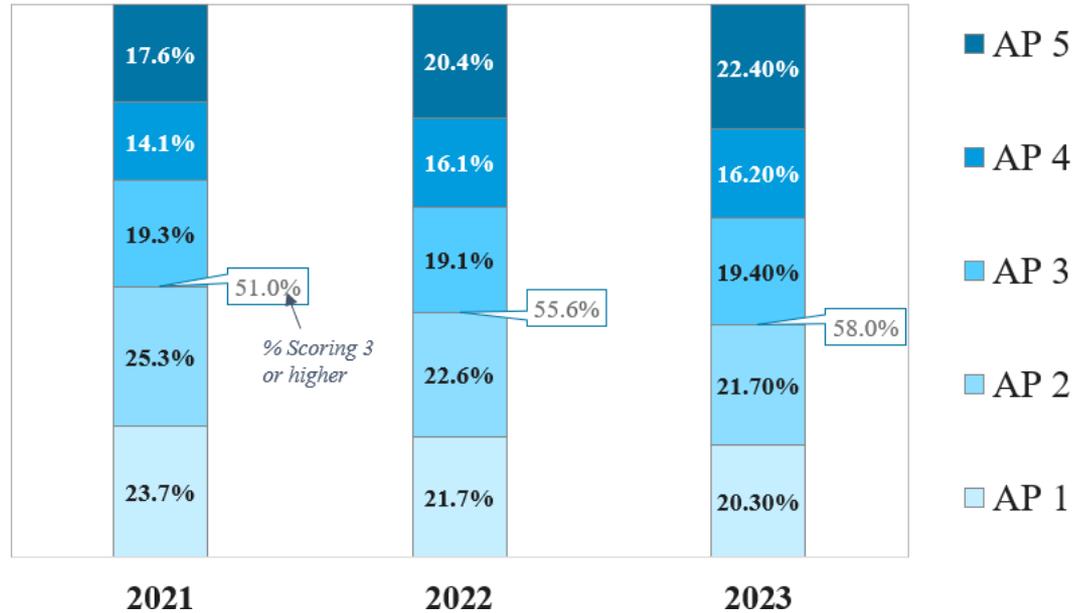
AP Exam Score	Approximate % Students EARNING this AP Score	Approximate % Students BELOW this AP Score
AP 5	22.4%	77.6%
AP 4	16.2%	61.4%
AP 3	19.4%	42.0%
AP 2	21.7%	20.3%
AP 1	20.3%	NA

AP Calculus AB Exam Score Distribution

Observations

- In 2019, 58.4% of students scored 3 or higher.
- Performance dipped (2022) and began to recover in 2021 & 2022.
- The 2023 cohort of AB students scored comparably to the 2019 cohort.

Approximate % of students earning each AP Exam Score



AP Calculus BC Exam Score Distribution

(All versions of 2023 exam)

78.3%
of students scored 3 or higher

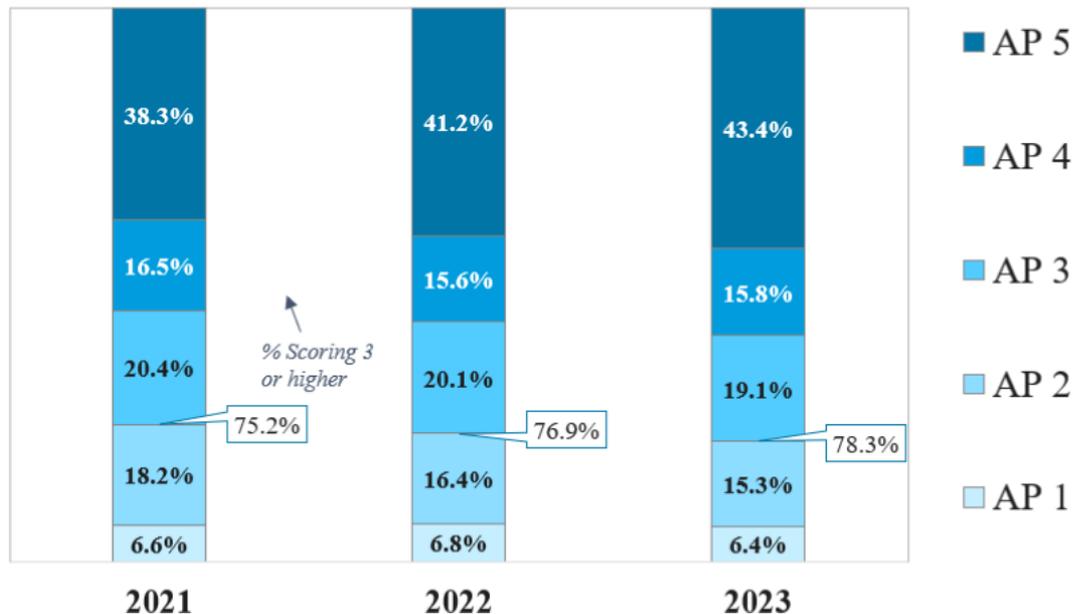
AP Exam Score	Approximate % Students EARNING this AP Score	Approximate % Students BELOW this AP Score
AP 5	43.4%	56.6%
AP 4	15.8%	40.8%
AP 3	19.1%	21.7%
AP 2	15.3%	6.4%
AP 1	6.4%	NA

AP Calculus BC Exam Score Distribution

Observations

- In 2019, 81.0% of students scored 3 or higher.
- Recall that in 2022, more BC students than usual appeared to take the AB exam.
- Even though the size of the BC cohort grew by 13% in 2023, the scores improved.

Approximate % of students earning each AP Exam Score



AP Calculus AB/BC Mean Student FRQ Scores

(Operational exam administration, common questions)

Question	Mean Score
Question AB1/BC1 Modeling Rates with Riemann sum and MVT, Calculator required	AB: $3.81 / 9 \approx 42.3\%$ BC: $5.29 / 9 \approx 58.8\%$ Combined: $4.27 / 9$ points $\approx 47.5\%$
Question AB3/BC3 Modeling with Differential Equation, Calculator not allowed	AB: $2.08 / 9 \approx 23.1\%$ BC: $3.78 / 9 \approx 42\%$ Combined: $2.61 / 9$ points $\approx 29\%$
Question AB4/BC4 Graphical Analysis of Functions with L'Hospital's Rule, Calculator not allowed	AB: $2.72 / 9 \approx 30.2\%$ BC: $4.07 / 9 \approx 45.2\%$ Combined: $3.14 / 9$ points $\approx 34.9\%$

AP Calculus AB/BC Mean Student FRQ Scores

(Operational exam administration, AB-only questions)

Question	Mean Score
Question AB2 Acceleration, Distance and Total Distance, Calculator required	4.63 / 9 points \approx 51.5%
Question AB5 Tabular and FTC, Calculator not allowed	4.48 / 9 points \approx 49.8%
Question AB6 Implicit Differentiation and Related Rates, Calculator not allowed	2.79 / 9 points \approx 31%

AP Calculus AB/BC Mean Student FRQ Scores

(Operational exam administration, BC-only questions)

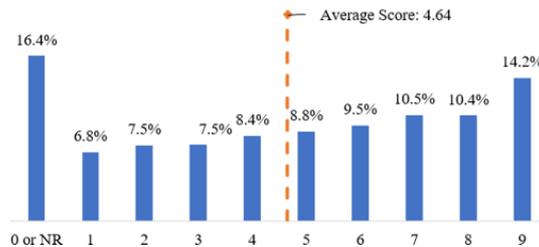
Question	Mean Score
Question BC2 Parametric Particle Motion, Calculator required	5.26 / 9 points \approx 58.4%
Question BC5 Area-Volume with Improper Integral & Integration by Parts, Calculator not allowed	4.75 / 9 points \approx 52.8%
Question BC6 Taylor Polynomials and Lagrange Error Bound, Calculator not allowed	4.25 / 9 points \approx 47.3%

AB Results 2023

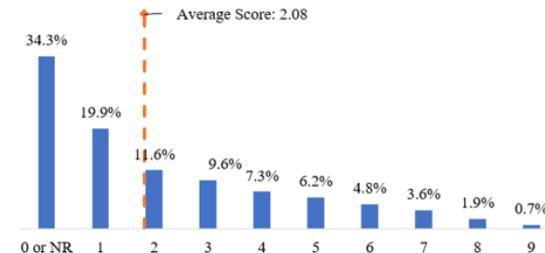
AB1



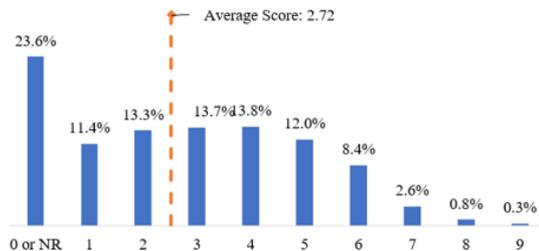
AB2



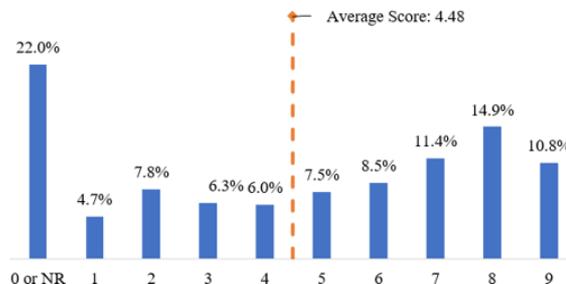
AB3



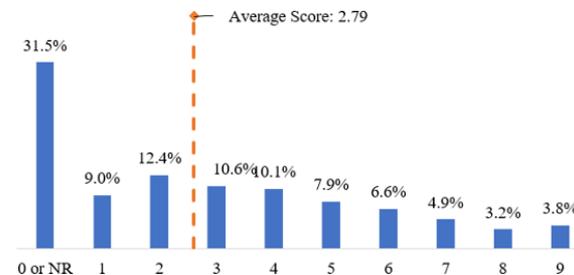
AB4



AB5



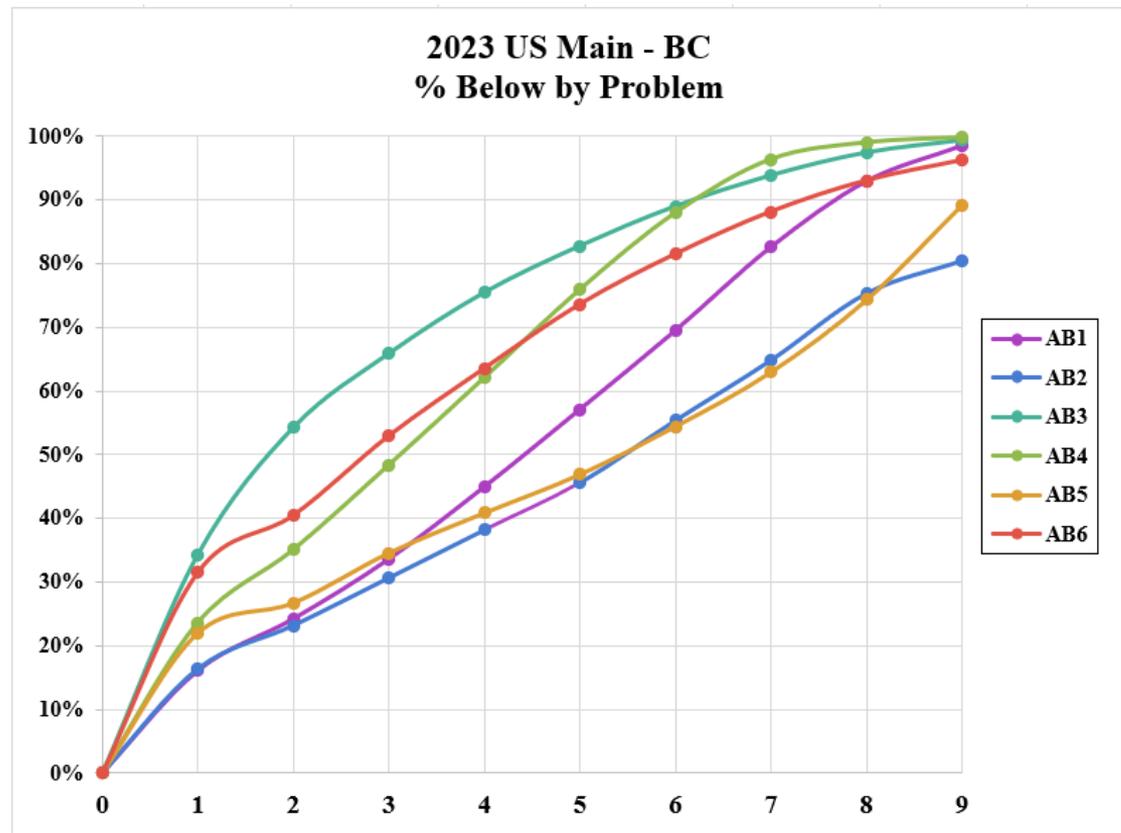
AB6



Results 2023 (AB FRQs)

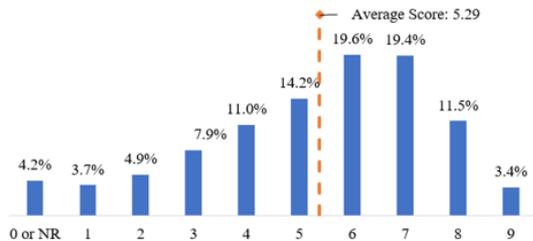
2023 US Main <i>N</i> = 125,991				
Question	Mean	St. Dev.	% 9s	% 0s
AB-1	3.81	2.57	1.5	14.6
AB-2	4.64	3.14	14.2	13.2
AB-3	2.08	2.28	0.7	31.3
AB-4	2.72	2.17	0.3	21.2
AB-5	4.48	3.26	10.8	18.0
AB-6	2.79	2.69	3.8	24.8

AB Results 2023

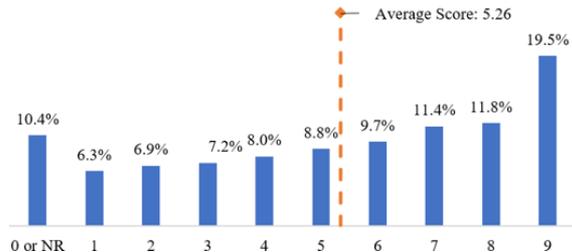


BC Results 2023

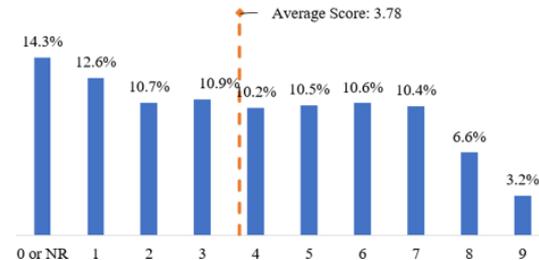
BC1



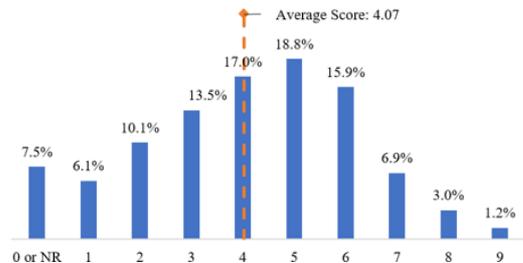
BC2



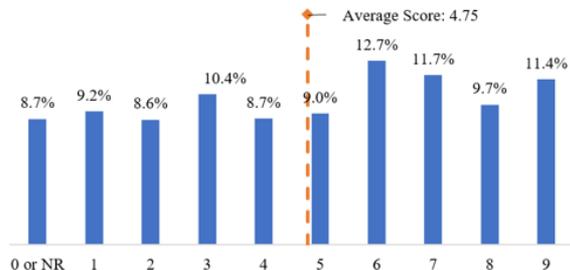
BC3



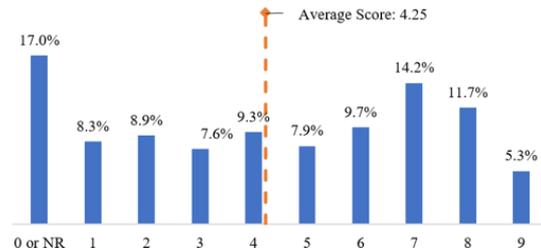
BC4



BC5



BC6

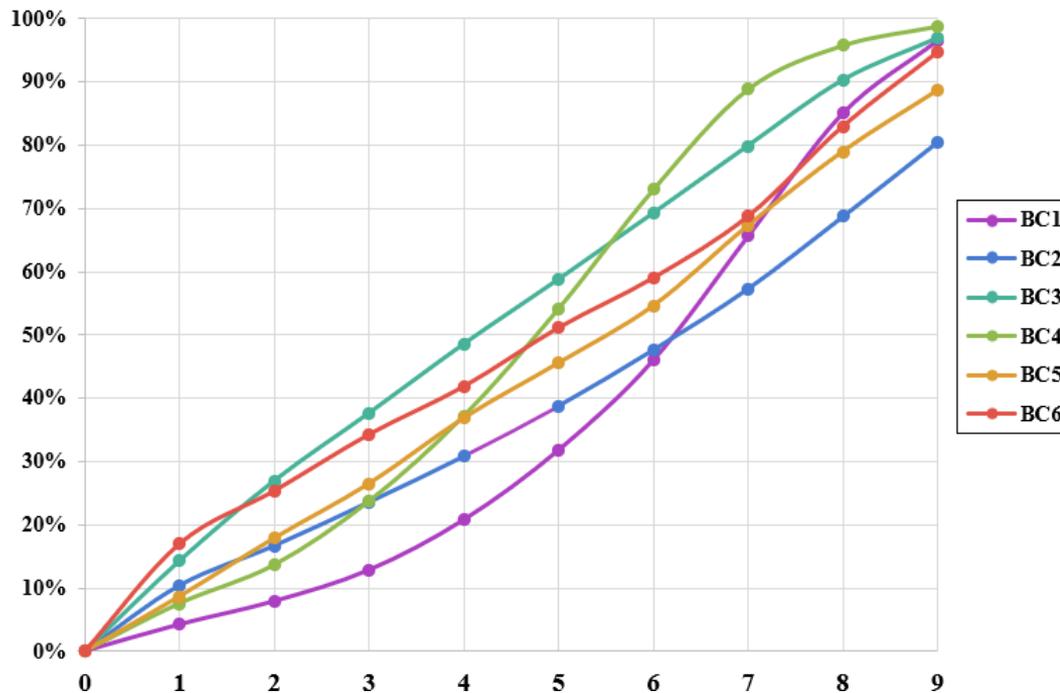


Results 2023 (BC FRQs)

2023 US Main <i>N</i> = 57,222				
Question	Mean	St. Dev.	% 9s	% 0s
BC-1	5.29	2.23	3.4	3.9
BC-2	5.26	3.04	19.5	9.3
BC-3	3.78	2.69	3.2	13.5
BC-4	4.07	2.12	1.2	7.0
BC-5	4.75	2.85	11.4	7.1
BC-6	4.25	2.97	5.3	13.1

BC Results 2023

2023 US Main - BC
% Below by Problem



AP Calculus 2023 AB1/BC1: Modeling Rates, Riemann sum, MVT

t (seconds)	0	60	90	120	135	150
$f(t)$ (gallons per second)	0	0.1	0.15	0.1	0.05	0

1. A customer at a gas station is pumping gasoline into a gas tank. The rate of flow of gasoline is modeled by a differentiable function f where $f(t)$ is measured in gallons per second and t is measured in seconds since pumping began. Selected values of $f(t)$ are given in the table.

- (a) Using correct units, interpret the meaning of $\int_{60}^{135} f(t) dt$ in the context of the problem. Use a right Riemann sum with the three subintervals $[60,90]$, $[90,120]$, and $[120,135]$ and to approximate the value of $\int_{60}^{135} f(t) dt$.

AP Calculus 2023 AB1/BC1: Modeling Rates, Riemann sum, MVT

t (seconds)	0	60	90	120	135	150
$f(t)$ (gallons per second)	0	0.1	0.15	0.1	0.05	0

- (b) Must there exist a value of c for $60 < c < 120$, such that $f'(c) = 0$? Justify your answer.
- (c) The rate of flow of gasoline, in gallons per second, can also be modeled by $g(t) = \left(\frac{t}{500}\right) \cos\left(\left(\frac{t}{120}\right)^2\right)$ for $0 \leq t \leq 150$. Using this model, find the average rate of flow of gasoline over the time interval $0 \leq t \leq 150$. Show the setup for your calculations.
- (d) Using the model defined in part (c), find the value of $g'(140)$. Interpret the meaning of your answer in the context of the problem.

Common misconceptions / errors

□ Part (b): Mean Value Theorem (MVT)

- ◆ Failing to state that f is continuous *because* it is differentiable.
- ◆ Attempting to use the Intermediate Value Theorem

□ Part (d): Interpretation

- ◆ $g'(140) = 0.005$
- ◆ The **rate** at which gasoline is flowing into the tank is **decreasing** (or **changing**) **at a rate** of 0.005 **gallons per second per second** at time $t = 140$.

AP Calculus 2023 AB2: Acceleration, Displacement, Distance

2. Stephen swims back and forth along a straight path in a 50-meter-long pool for 90 seconds. Stephen's velocity is modeled by $v(t) = 2.38e^{-0.02t} \sin\left(\frac{\pi}{56}t\right)$, where t is measured in seconds and $v(t)$ is measured in meters per second.
- (a) Find all times t in the interval $0 < t < 90$ at which Stephen changes direction. Give a reason for your answer.
- (b) Find Stephen's acceleration at time $t = 60$ seconds. Show the setup for your calculations, and indicate units of measure. Is Stephen speeding up or slowing down at time $t = 60$ seconds? Give a reason for your answer.
- (c) Find the distance between Stephen's position at time $t = 20$ seconds and his position at time $t = 80$ seconds. Show the setup for your calculations.
- (d) Find the total distance Stephen swims over the time interval $0 \leq t \leq 90$. Show the setup for your calculations.

Common misconceptions / errors

❑ Part (a): When does Stephen's direction change?

◆ Claiming the change occurs because $v(56) = 0$, rather than because his velocity changes sign.

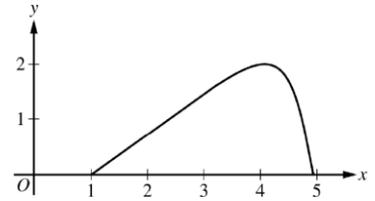
❑ Part (b): Acceleration at $t = 60$; speeding up or slowing down?

◆ No setup for $a(60)$ (requires $v'(60)$)

◆ Believing that the sign of $a(60)$ alone determines whether he is speeding up or slowing down.

❑ Parts (c) and (d): ◆ Confusing total distance with displacement

AP Calculus 2023 BC2: Parametric Particle Motion



For $0 \leq t \leq \pi$, a particle is moving along the curve shown so that its position at time t is $(x(t), y(t))$, where $x(t)$ is not explicitly given and $y(t) = 2\sin t$. It is known that $\frac{dx}{dt} = e^{\cos t}$. At time $t = 0$, the particle is at position $(1, 0)$.

- Find the acceleration vector of the particle at time $t = 1$. Show the setup for your calculation.
- For $0 \leq t \leq \pi$, find the first time t at which the speed of the particle is 1.5. Show the work that leads to your answer.
- Find the slope of the line tangent to the path of the particle at time $t = 1$. Find the x -coordinate of the position of the particle at time $t = 1$. Show the work that leads to your answers.
- Find the total distance traveled by the particle over the time interval $0 \leq t \leq \pi$. Show the setup for your calculations.

Common misconceptions / errors

□ Part (a): acceleration vector

- ◆ Reported the velocity vector
- ◆ Differentiated by hand incorrectly.

□ Part (b): When is particle's speed = 1.5?

- ◆ Found speed at time $t = 1.5$.
- ◆ Used TRACE function on calculator and obtained an inaccurate value

Common misconceptions / errors

□ Part (c): Find particle's x -coordinate at time $t = 1$.

◆ Failing to include a differential, resulting in an ambiguous integrand:

$$\text{e.g., } x(1) = \int_0^1 e^{\cos t} + 1$$

□ Part (d): Total distance traveled

◆ Incorrect setup e.g., $\int_0^\pi \sqrt{1 + \left(\frac{2\cos t}{e^{\cos t}}\right)^2} dt$

◆ Incorrect evaluation e.g., $\int_0^\pi \sqrt{\left(e^{\cos t}\right)^2 + (2\cos t)^2} dt = 13.4467$

Panel Discussion: AP[®] Calculus From Those in the Know

Ben Hedrick

Senior Manager, STEM Assessment, ETS, Princeton, NJ

 CollegeBoard

AP[®]



Virtual Conference

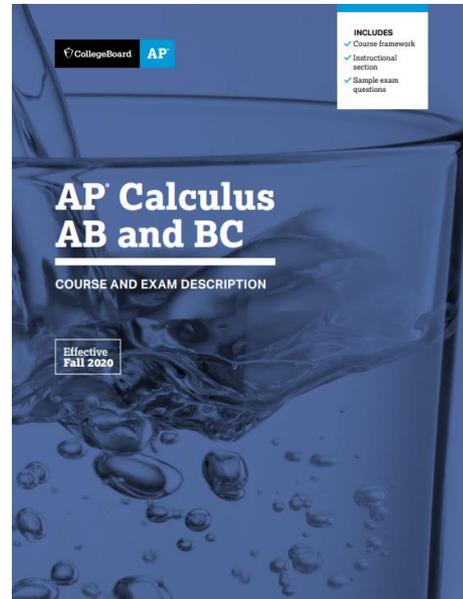
#T3IC



AP Calculus AB and Calculus BC *Course and Exam Description (aka the “CED”)*

AP Central:
apcentral.collegeboard.org

This is the core document for the AP Calculus courses. It clearly lays out the course content and describes the exams and the AP Program in general.



NEW CED, effective 2019-20

- **Curriculum Framework** updated to make things clearer for teachers and students.
- CED organizes the Calculus courses into commonly taught **Units**. Each Unit is broken down into **Topics**.

NEW CED, effective 2019-20

- **Unit 1:** Limits and Continuity
- **Unit 2:** Differentiation: Definition and Fundamental Properties
- **Unit 3:** Differentiation: Composite, Implicit, and Inverse Functions
- **Unit 4:** Contextual Applications of Differentiation
- **Unit 5:** Analytical Applications of Differentiation
- **Unit 6:** Integration and Accumulation of Change
- **Unit 7:** Differential Equations
- **Unit 8:** Applications of Integration
- **Unit 9:** Parametric Equations, Polar Coordinates, and Vector-Valued Functions
(AP Calculus BC only)
- **Unit 10:** Infinite Sequences and Series **(AP Calculus BC only)**

» Mathematical Practices

Practice 1	Practice 2	Practice 3	Practice 4
<i>Implementing Mathematical Processes</i> 1 Determine expressions and values using mathematical procedures and rules.	<i>Connecting Representations</i> 2 Translate mathematical information from a single representation or across multiple representations.	<i>Justification</i> 3 Justify reasoning and solutions.	<i>Communication and Notation</i> 4 Use correct notation, language, and mathematical conventions to communicate results or solutions.

These Practices are categorized into skills, which form the basis of the tasks on the AP Exam.

CED

- » **Big Ideas** weave through the Framework to create unifying themes to help students make connections across topics and develop more meaningful understanding.

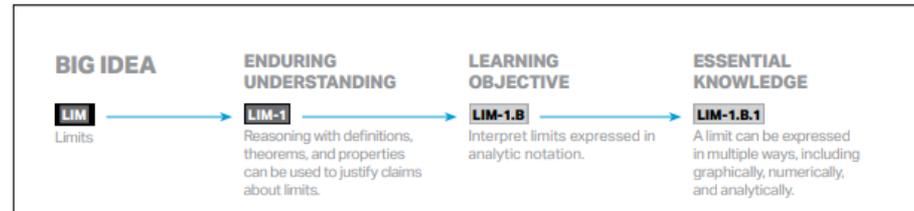
BIG IDEA 1: CHANGE (CHA)

BIG IDEA 2: LIMITS (LIM)

BIG IDEA 3: ANALYSIS OF FUNCTIONS (FUN)

- » **Enduring Understandings**, **Learning Objectives**, and **Essential Knowledge** statements have been clarified and refined.

REQUIRED COURSE CONTENT LABELING SYSTEM



AP Calculus Exams

Section	Question Type	Number of Questions	Exam Weighting	Timing
I	Multiple-choice questions			
	Part A: Graphing calculator not permitted	30	33.3%	60 minutes
	Part B: Graphing calculator required	15	16.7%	45 minutes
II	Free-response questions			
	Part A: Graphing calculator required	2	16.7%	30 minutes
	Part B: Graphing calculator not permitted	4	33.3%	60 minutes

AP Calculus Exams

- » Variety of function types: algebraic, exponential, logarithmic, trigonometric, general (e.g., f and g)
- » Variety of function representations: analytical, graphical, tabular, verbal
- » Variety of content topics across the units
- » Mixture of procedural and conceptual tasks
- » Includes questions in a real-world context
- » Includes questions or parts of questions that require use of the graphing calculator

Exam Weighting for the Multiple-Choice Section of the AP Exam

Units	Exam Weighting (AB)	Exam Weighting (BC)
Unit 1: Limits and Continuity	10–12%	4–7%
Unit 2: Differentiation: Definition and Fundamental Properties	10–12%	4–7%
Unit 3: Differentiation: Composite, Implicit, and Inverse Functions	9–13%	4–7%
Unit 4: Contextual Applications of Differentiation	10–15%	6–9%
Unit 5: Analytical Applications of Differentiation	15–18%	8–11%
Unit 6: Integration and Accumulation of Change	17–20%	17–20%
Unit 7: Differential Equations	6–12%	6–9%
Unit 8: Applications of Integration	10–15%	6–9%
Unit 9: Parametric Equations, Polar Coordinates, and Vector-Valued Functions BC ONLY		11–12%
Unit 10: Infinite Sequences and Series BC ONLY		17–18%

Exam Weighting for the Multiple-Choice Section of the AP Exam

Mathematical Practice	Exam Weighting
Practice 1: Implementing Mathematical Processes	53–66%
Practice 2: Connecting Representations	18–28%
Practice 3: Justification	11–18%

Exam Weighting for the Free-Response Section of the AP Exam

Mathematical Practice	Exam Weighting	
	AB	BC
Practice 1: Implementing Mathematical Processes	37–55%	37–59%
Practice 2: Connecting Representations	9–16%	9–16%
Practice 3: Justification	37–55%	37–59%
Practice 4: Communication and Notation	13–24%	9–20%

Free-Response Instructions

***Emphasize good mathematical communication and writing.
Encourage students to try each part of each question.***

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. You may use the pages in this orange booklet for scratch work, but you must write your answers in the separate Section II: Free Response booklet. **No credit will be given for any work written in this orange booklet.** In the Free Response booklet, write your solution to each part of each question in the space provided for that part. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During Part B, you may continue to work on the questions in Part A without the use of a calculator.

As you begin each part, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2 dx$ may not be written as `fnInt(X^2, X, 1, 5)`.
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

2024 AP Calculus Exam Administration

- Main administration: Paper, In School
 - **Monday May 13, 8 a.m. Local Time**
- Administration 2: Paper, In School
 - **Friday May 24, 8 a.m. Local Time**

Panel Discussion: AP[®] Calculus From Those in the Know

Stephanie Ogden

Director, AP Calculus, College Board

 CollegeBoard

AP[®]



Virtual Conference

#T3IC



What makes a great exam?

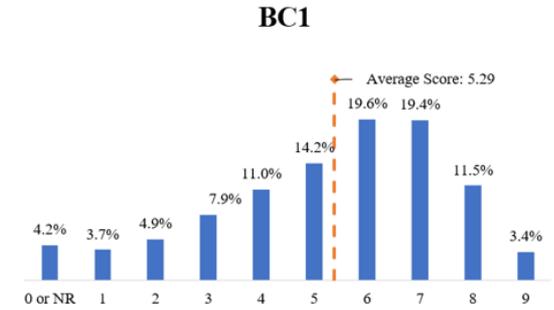
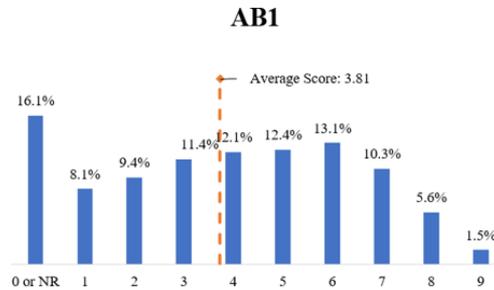
- » The exam assesses the broad scope of the course (and does not assess content or skills that are out of scope)
 - No points are so dense that we cannot tell what they are assessing.
- » The exam generates the data we need to be confident that our scores are fair and accurate.
 - There is a range of question difficulties.
 - All points are used (none are wasted).

What makes a great question?

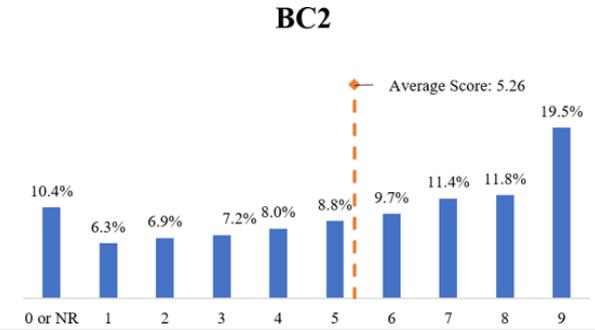
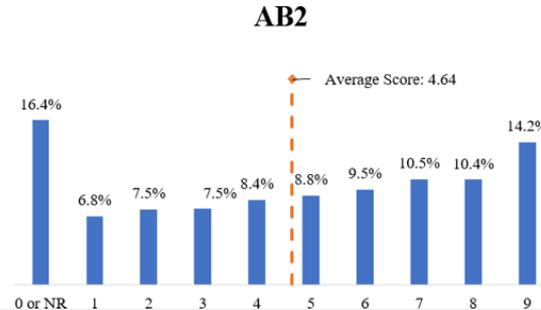
- » All questions:
 - Accessible (fairness)
 - Aligned to the intended difficulty, LO, EK, skill
 - Clear language and stimuli (including “Show the work...” and “...radian mode”)
- » MCQ:
 - One right/best answer
- » FRQ:
 - There are entry points for students at each performance level.
 - All points are used.
 - No points are so dense that we cannot tell what they are assessing.

How'd we do?

- » Accessible (fairness)
- » **Aligned to the intended difficulty, LO, EK, skill**
- » Clear language and stimuli
- » **Entry points** at each performance level.
- » **All points are used.**
- » No overly dense points.



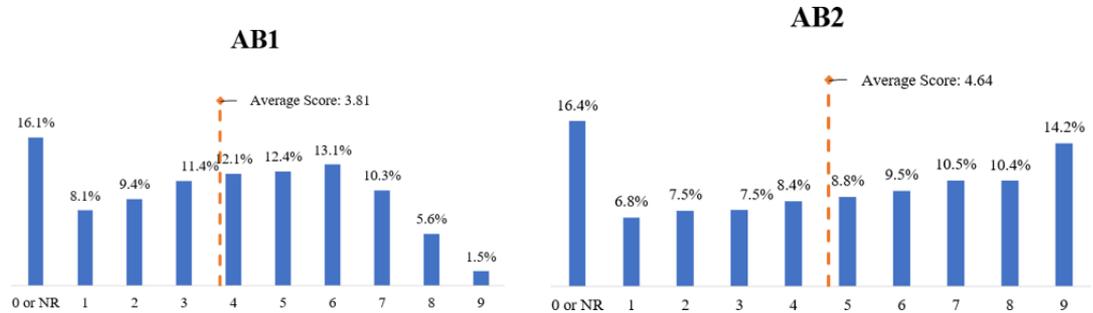
Does this question help to generate the data we need to fairly determine AB subscores?



Advice to share with your students:

» Entry points:

- Try every part.
- Read the question: Show the work or setups and give a reason
- Practice so that you earn every point you thought you earned



Stephen's velocity is modeled by $v(t) = 2.38e^{-0.02t} \sin\left(\frac{\pi}{56}t\right), \dots$

(b) Find Stephen's acceleration at time $t = 60$ seconds. Show the setup for your calculations, and indicate units of measure. Is Stephen speeding up or slowing down at time $t = 60$ seconds? Give a reason for your answer.

Model Solution

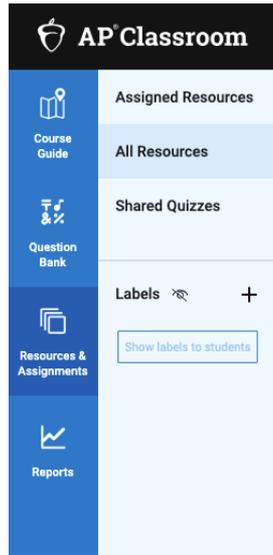
$$v'(60) = a(60) = -0.0360162$$

Stephen's acceleration at time $t = 60$ seconds is -0.036 meter per second per second.

Scoring

$a(60)$ with setup	1 point
Acceleration units	1 point

Practice Resources for all students/all questions



AP Classroom

- Course Guide
- Question Bank
- Resources & Assignments
- Reports

Assigned Resources

All Resources

Shared Quizzes

Labels  +

Show labels to students

RESOURCES & ASSIGNMENTS

All Resources

Progress Checks My Quizzes Practice Exams Videos **Student Practice** 

Filter by

Status

Title

 Unit 1 Practice: Level 1

 Unit 1 Practice: Level 2

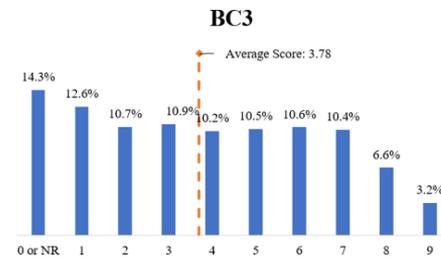
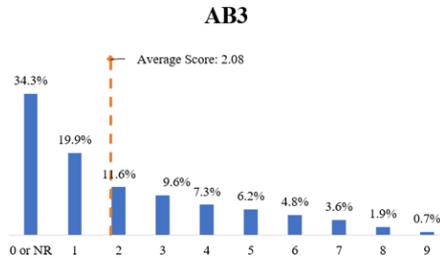
 Unit 1 Practice: Level 3

What Is Student Practice?

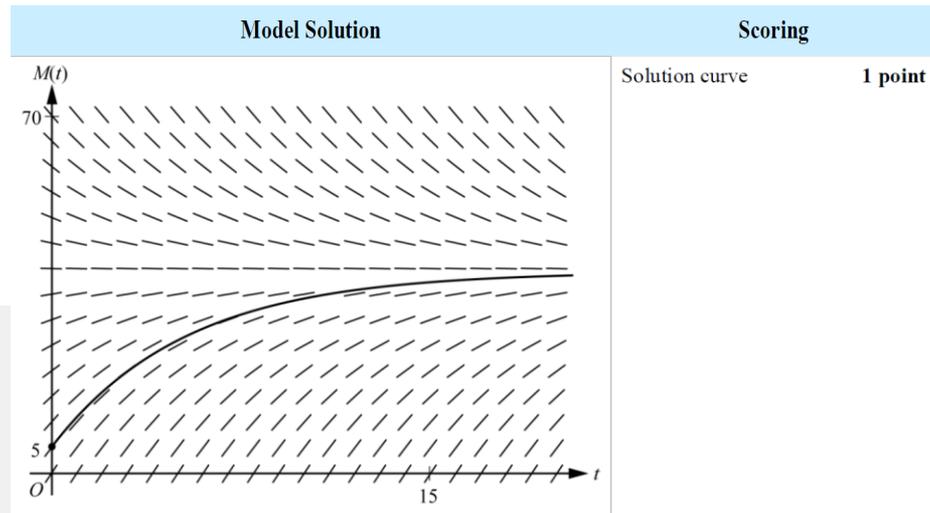
AP Student Practice is a new resource consisting of interactive, scaffolded free-response questions tailored to the content and skills of each unit of the course. This tab lists all your Student Practice assignments for you to view and manage.

[Click here](#) to learn more.

AP Calculus 2023 AB3/BC3: Modeling with Differential Equation



- (a) A slope field for the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ is shown. Sketch the solution curve through the point $(0, 5)$.



All year practice for all students for all questions:

» **Learning how to enter:**

○ **Level 1**

Step-by-step tips

[Video explanation](#)

[Answer](#)

Tip 1 of 4

What is this part of the overall question asking you to find? What type of response are you expected to provide?

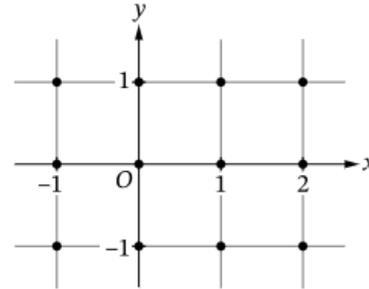
B *I* u x^2 x_2 ↺ ↻ √ ≡ ≡ ≡

slopes at 12 points

4 / 10000 Word Limit

Consider the differential equation $\frac{dy}{dx} = xy^2$.

(a) On the axes provided, sketch a slope field for the given differential equation at the 12 points indicated.



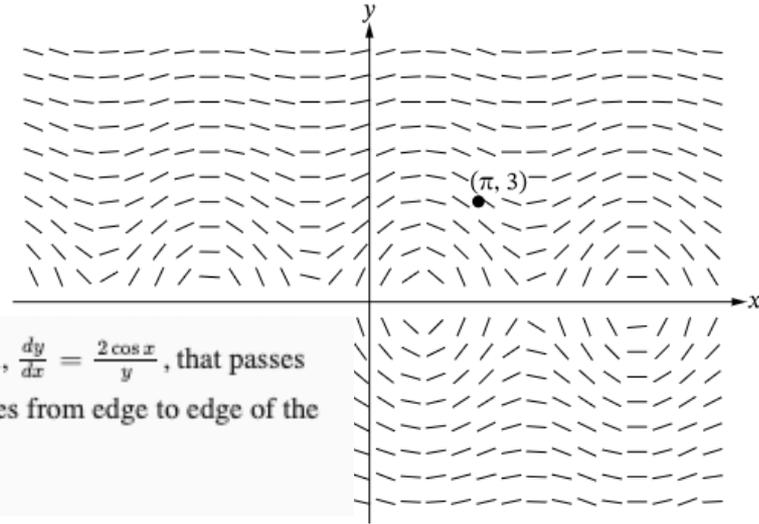
Part (a) is asking you to find the value of the differential equation, $\frac{dy}{dx} = xy^2$, at each of the marked points on the given axes. You're expected to sketch a segment through each point with a slope that matches each corresponding value of the differential equation at that point.

All year practice for all students for all questions:

- » **Learning how to enter:**
 - Level 2, Tip 1

A portion of the slope field for the differential equation $\frac{dy}{dx} = \frac{2 \cos x}{y}$ is given.

(a) Sketch the solution curve through the point $(\pi, 3)$.



Part (a) is asking you to sketch the solution to the differential equation, $\frac{dy}{dx} = \frac{2 \cos x}{y}$, that passes through the point $(\pi, 3)$. Your response will be a smooth curve that goes from edge to edge of the provided graph in the x - y plane.

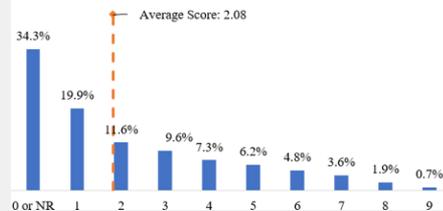
AP Calculus 2023 AB3/BC3: Modeling with Differential Equation

(d) Use separation of variables to find an expression for $M(t)$, the particular solution to the differential

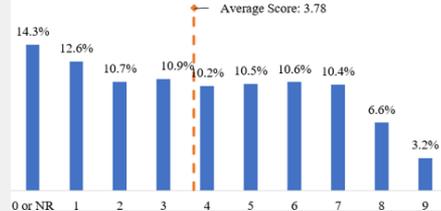
equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ with initial condition $M(0) = 5$.

Model Solution	Scoring
$\frac{dM}{40 - M} = \frac{1}{4} dt$ $\int \frac{dM}{40 - M} = \int \frac{1}{4} dt$	Separates variables 1 point
$-\ln 40 - M = \frac{1}{4}t + C$	Finds antiderivatives 1 point

AB3



BC3



All year practice for all students for all questions:

Level 3: For a specific region, the rate of change of a population of bears, B , is directly proportional to $275-B$, with proportionality constant $k=0.135$. Time, t is measured in years for $t \geq 0$. When $t=0$, the bear population is 90 bears.

(d) Find the particular solution, $y=B(t)$, with initial condition $B(0)=90$.

$\frac{dB}{dt} = 0.135 (275 - B)$	
$\frac{1}{275-B} dB = 0.135 dt$	Separate the variables by dividing each side by $(75 - B)$
$\int \frac{1}{275 - B} dB = \int 0.135 dt$	Integrate both sides
$-\ln 275 - B = 0.135t + C$	Find the antiderivatives using basic rules for integration.
$-\ln 275 - 90 = 0.135 (0) + C$	Substitute the initial condition for t and B .
$-\ln 185 = C$	Solve for C .
$-\ln 275 - B = 0.135t - \ln 185 $	Rewrite the equation using the value for C .
$\ln 275 - B = -0.135t + \ln 185 $	
$e^{\ln 275 - B } = e^{-0.135t + \ln 185 }$	Exponentiate both sides of the equation.
$275 - B = 185e^{-0.135t}$	The initial value works for $275 - B = 185e^{-0.135t}$ and not $275 - B = -185e^{-0.135t}$.
$B = 275 - 185e^{-0.135t}$	Solve for B .

Note: Only the math on the left side is required in your final answer. The text on the right side is provided to help guide you through the mathematical steps taken to arrive at the final answer.

Thank you!

Julie Clark jclark@hollins.edu

Ben Hedrick bhedrick@ets.org

Stephanie Ogden sogden@collegeboard.org



Virtual Conference

#T3IC

