TEXAS INSTRUMENTS

VCE Specialist Series:

Cross product, lines and planes

Question: 1.

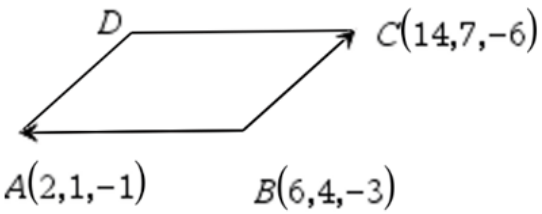
Find a unit vector perpendicular to both $4\underset{\sim}{i}+3\underset{\sim}{j}+2\underset{\sim}{k}$ and $8\underset{\sim}{i}+3\underset{\sim}{j}+3\underset{\sim}{k}$.

Question: 2.

Find the area of triangle OAB, where O is the origin, A has position vector $\underset{\sim}{i}-\underset{\sim}{j}$ and B has position vector $3\underset{\sim}{i}+4\underset{\sim}{j}-6\underset{\sim}{k}$.

Question: 3.

Find the area of the parallelogram ABCD where A, B, D have position vectors $2\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}$, $6\underset{\sim}{i}+4\underset{\sim}{j}-3\underset{\sim}{k}$ and $14\underset{\sim}{i}+7\underset{\sim}{j}-6\underset{\sim}{k}$.



Question: 4.

Find the equation of the line of intersection of two planes $2x-y+z=0$ and $x-3y+z=2$. Give your answer in the parametric form.

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Question: 5.

A plane passes through the points $A(2,2,-1), B(3,2,-1), C(4,3,5)$.

Find the cartesian equation of the plane.

Question: 6.

Show that the lines $\underset{\sim}{r}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$ and $\underset{\sim}{r}_2 = \begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$ are skew.

Question: 7.

The line l has equation $\dfrac{x-1}{2} = \dfrac{y-1}{-2} = \dfrac{z+3}{-1}$ and the point A has coordinates $(1,2,-1)$.

Find the shortest distance between A and l.

Question: 8.

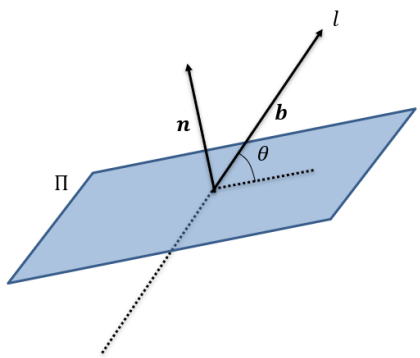
Find the acute angle between the line l with equation

$\underset{\sim}{r} = 2\underset{\sim}{i} + \underset{\sim}{j} - 5\underset{\sim}{k} + \lambda(3\underset{\sim}{i} + 4\underset{\sim}{j} + 2\underset{\sim}{k})$

and the plane with equation

$\underset{\sim}{r} \cdot (3\underset{\sim}{i} + 4\underset{\sim}{j} + 2\underset{\sim}{k}) = 2.$

Give your answer in degrees to one decimal place.



Question: 9.

Find the point of intersection of the line l and the plane $\prod$ where:

$l : \underset{\sim}{r} = -\underset{\sim}{i} + \underset{\sim}{j} - 5\underset{\sim}{k} + \lambda(\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k})$

$\prod : \underset{\sim}{r} \cdot \left(\underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}\right).$

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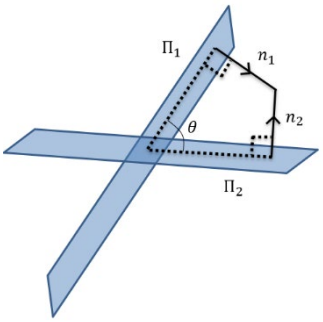
Question: 10

Find the acute angle between the planes $4x+4y-7z=13$ and $7x-4y+4z=6$.

Give your answer to the nearest degree.



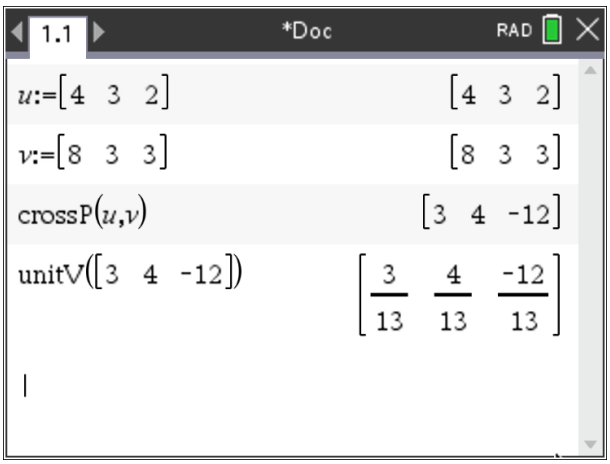
Question 11

Find the point of intersection of the following two lines

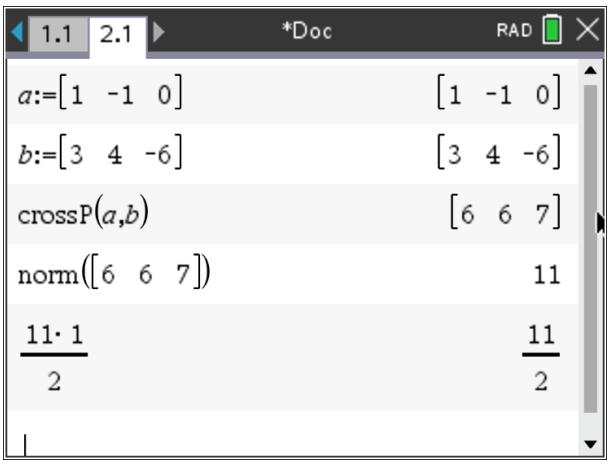
$$\underset{\sim}{r}_1 = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}3\\4\\5\end{pmatrix} \text{ and } \underset{\sim}{r}_2 = \begin{pmatrix}1\\4\\4\end{pmatrix} + \mu\begin{pmatrix}2\\2\\3\end{pmatrix}$$

Answers

Question 1



Question 2



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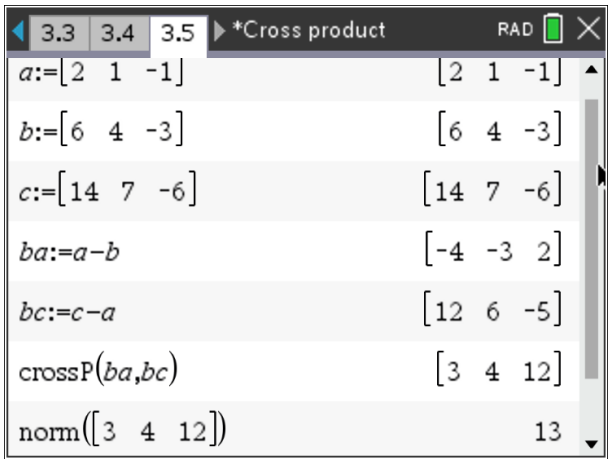
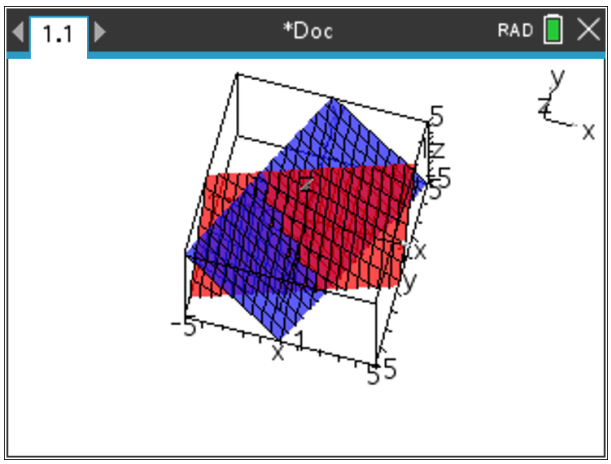
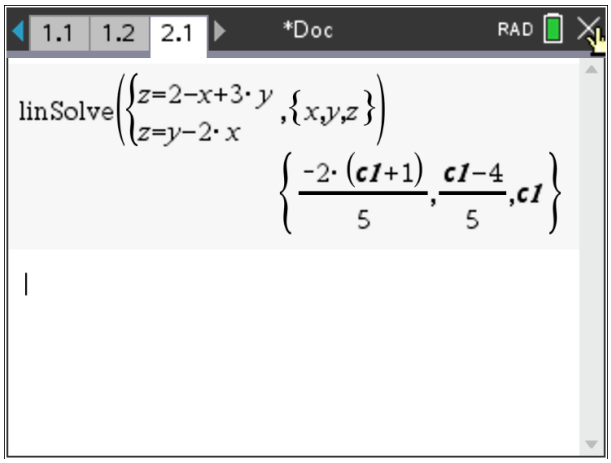
Question 3



Question 4





$$x = \frac{-2t-2}{5},\ y = \frac{t-4}{5},\ z = t.$$

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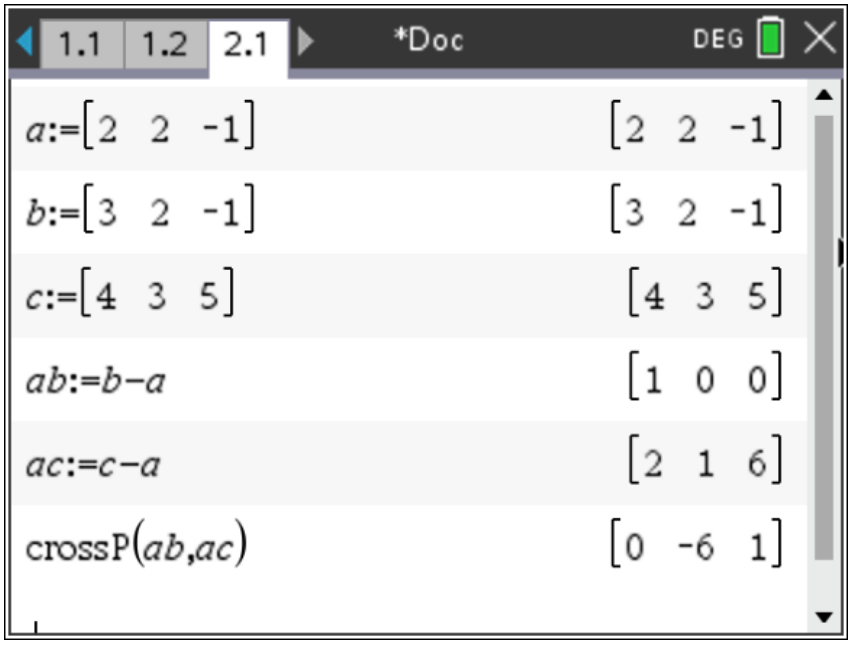
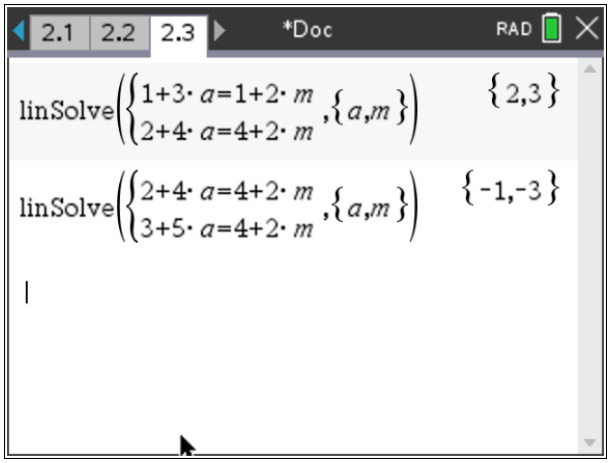
Question 5



$$-6y+z=-13$$

Question 6



We arrive at the contradiction when solving the equations in pairs.

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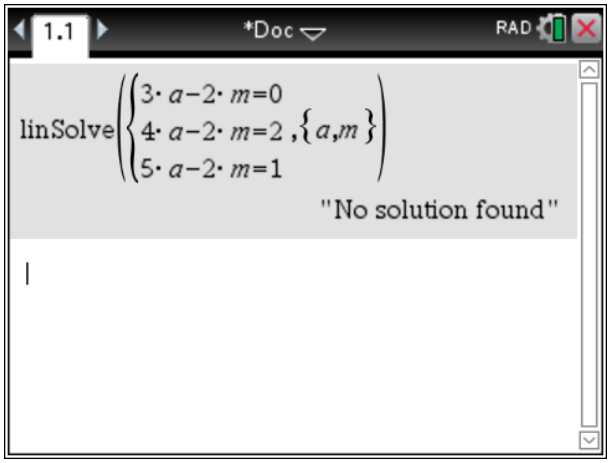
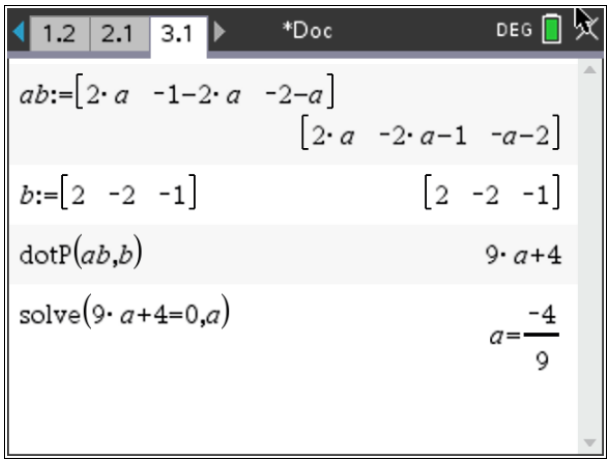
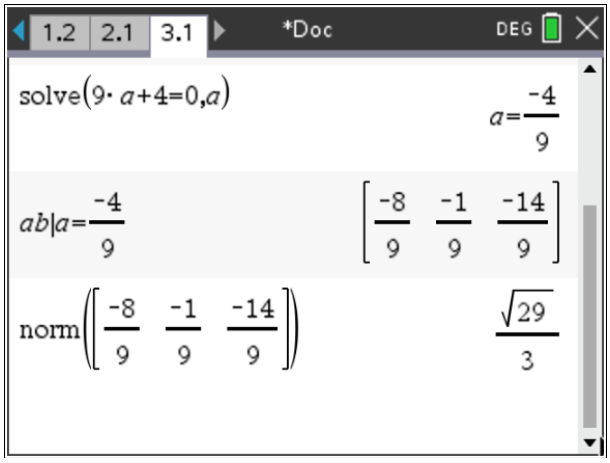
Alternatively:



Question 7





$$\frac{\sqrt{29}}{3}$$

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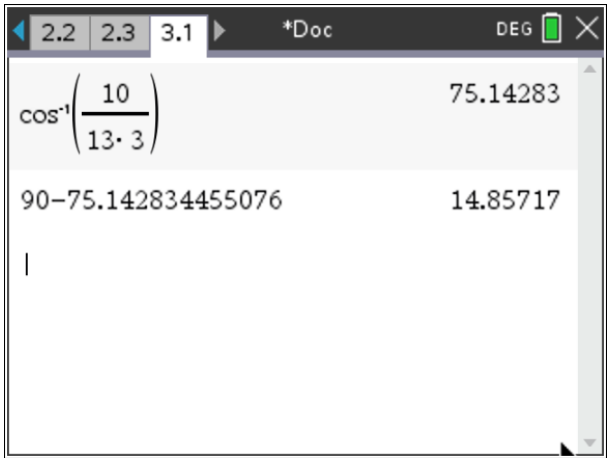
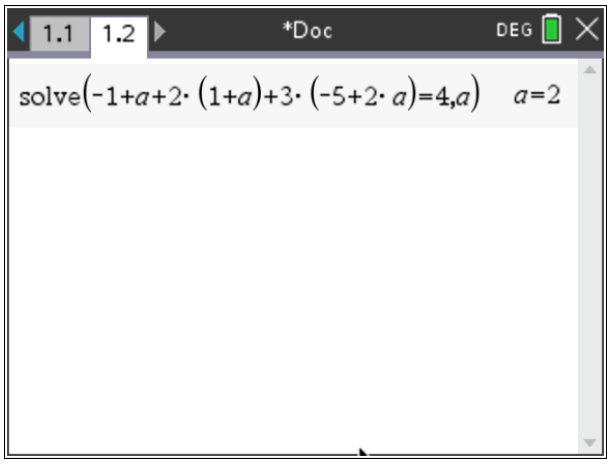
Question 8



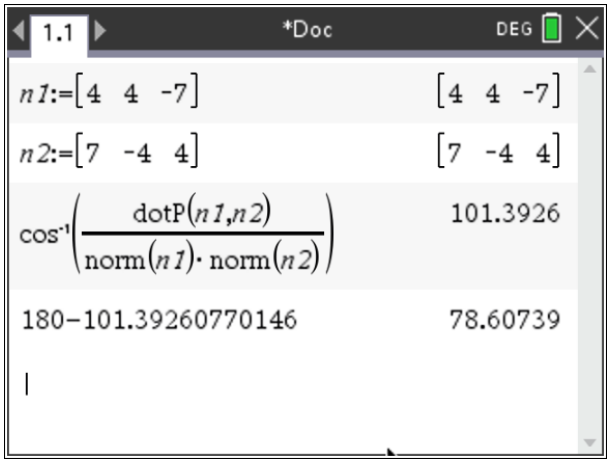
$\theta = 14.9^{\circ}$.

Question 9



$(1, 3, -1)$

Question 10



79°

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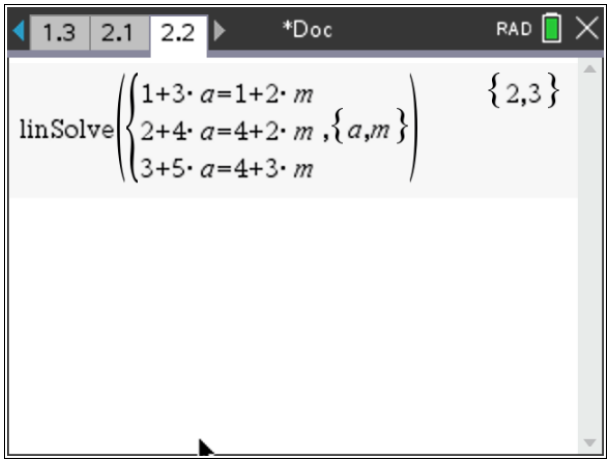
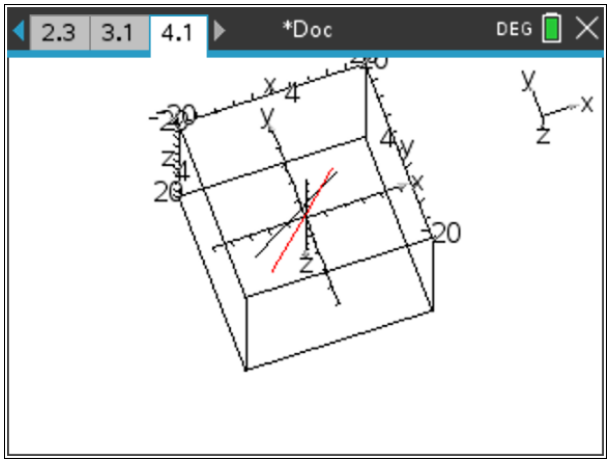
Question 11



$\lambda = 2,\ \mu = 3.$

The two lines intersect in the point $(7,10,13)$.



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