Bobo the Forgetful Elf

Bobo the Forgetful Elf makes trikes and wagons at the North Pole.
After a day's work, Bobo's supervisor asks him how many of each kind he made so he can be paid. Sadly, he forgets.

The toys have already been sent to the warehouse. The supervisor sees that there are 31 fewer wheels in Bobo's shop than were the previous day.

How many three wheeled trikes and four wheeled wagons did Bobo make?
Explain the reasoning for your answer.

Bobo the Forgetful Elf

Teacher's Notes

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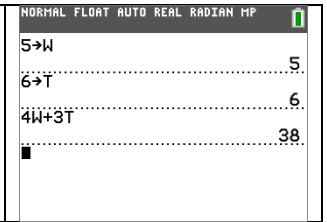
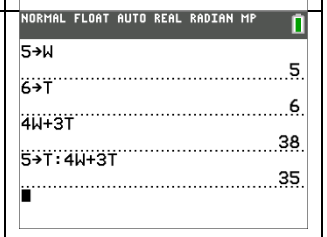
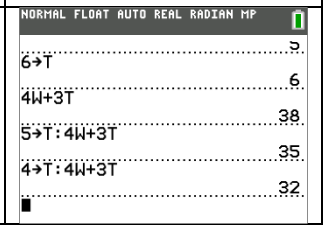
After giving students time to play with the problem, the following might be discussed:

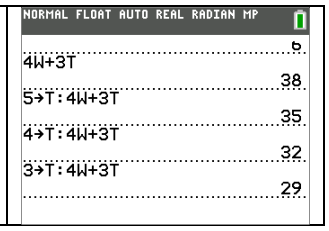
- Bobo's forgetfulness is not complete amnesia. While he may not remember exact details or write down any notes, he does remember he spent a day making trikes and wagons. For example, Bobo would be able to tell you, if asked, whether or not he made all trikes and no wagons, or vice versa. But now Bobo is so embarrassed he has retired to his room and will not open the door.
- Pose the question "Would either of these cases (0 wagons, all trikes or 0 trikes, all wagons) be even possible? Could we know without asking Bobo?"
  This question will lead to a discussion about divisibility and prime numbers.
  Since 31 is prime, the answer is no to both cases.
- One of the Bobo's coworkers feels strongly that 5 wagons were made.
  What number of wheels were needed to build 5 wagons and 6 trikes?
  What number of wheels were needed to build 5 wagons and 5 trikes?
  What number of wheels were needed to build 5 wagons and 4 trikes?
  What number of wheels were needed to build 5 wagons and 3 trikes?
  Ask students to organize their work, perhaps having them work in groups on the board.
- Using boxes completed with numbers might highlight the concept of *variable*.

$$4\cdot\boxed{5}+3\cdot\boxed{6}=38 \qquad 4\cdot\boxed{5}+3\cdot\boxed{6}=38$$
$$4\cdot\boxed{5}+3\cdot\boxed{5}=35 \qquad 4\cdot\boxed{5}+3\cdot\boxed{5}=35$$
$$4\cdot\boxed{5}+3\cdot\boxed{4}=32 \qquad 4\cdot\boxed{5}+3\cdot\boxed{4}=32$$
$$4\cdot\boxed{5}+3\cdot\boxed{3}=29 \quad \Longrightarrow \quad 4\cdot\boxed{5}+3\cdot\boxed{3}=29$$
$$4\cdot W + 3\cdot T = \text{Total \# Wheels}$$

- From students' work,
  guide the class to build the expression $4W+3T$,
  where W gives the number of wagons and T gives the number of trikes.
- Shown below is a sequence of screens to help collect the work done by students to build a pattern. This could be done on the teacher's display calculator.

The screen to the right gives the number of wheels needed to build 5 wagons and 6 trikes.	
Press ▲ four times to select 6→T, press enter to paste it on the entry line, then press 2nd ◀ to move to the beginning of the line to change 6→T to 5→T. Press 2nd [:] to separate a new entry on the same line, then scroll the history by pressing ▲ twice, selecting 4W+3T. Press enter to paste it on the entry line, and press enter to evaluate the expression, giving you 35.	
For the third screen, press ▲ twice to select 5→T:4W+3T, press enter to paste it on the entry line, then press 2nd ◀ to move to the beginning of the line to change 5→T to 4→T, and press enter to evaluate the expression, giving you 32.	

For the fourth screen, repeat as before, changing 4→T to 3→T.
Press [enter] to evaluate the expression, giving you 29.



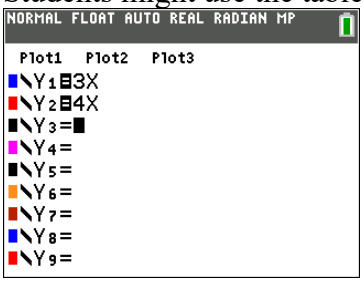
Ask students to discuss what pattern they see and use it to write the next three entries and answers without performing any calculations.

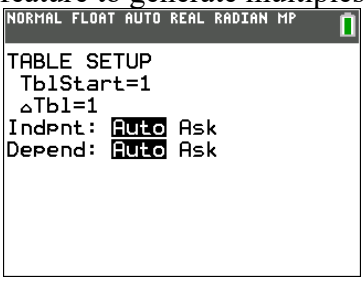
- After students share that each of the total number of wheels decrease by 3, ask them why this would be the case. Build a consensus that one less trike means three less wheels.
- Another pattern students may notice is that the total of toys (wagons plus trikes) decreases by 1.
  Why? The next row represents one less trike.

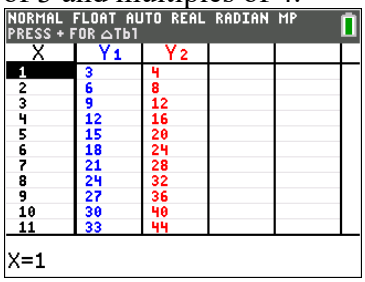
$4 \cdot W + 3 \cdot T =$ Total # of Wheels	$W + T =$ Total # Toys
$4 \cdot \boxed{5} + 3 \cdot \boxed{6} = 38$	$\boxed{5} + \boxed{6} = 11$
$4 \cdot \boxed{5} + 3 \cdot \boxed{5} = 35$	$\boxed{5} + \boxed{5} = 10$
$4 \cdot \boxed{5} + 3 \cdot \boxed{4} = 32$	$\boxed{5} + \boxed{4} = 9$
$4 \cdot \boxed{5} + 3 \cdot \boxed{3} = 29$	$\boxed{5} + \boxed{3} = 8$
$4 \cdot \boxed{5} + 3 \cdot \boxed{2} = 26$	$\boxed{5} + \boxed{2} = 7$
$4 \cdot \boxed{5} + 3 \cdot \boxed{1} = 23$	$\boxed{5} + \boxed{1} = 6$
$4 \cdot \boxed{5} + 3 \cdot \boxed{0} = 20$	$\boxed{5} + \boxed{0} = 5$

- This problem provides opportunities for multiple entry points. Return students to groups to strategize and solve the problem. For those who are struggle, here are some possible strategies.
  - Continue the above "Try and Revise" strategy and split the work.
  - With sketches of trikes and wagons use exchanges. (See the last page.)
  - List multiples of 3 and multiples of 4 to see if any combinations make 31.
  - Subtract all possible multiples of 4 from 31 to see if any are divisible by 3.
  - For additional strategies, see *Comparing Quantities* unit from the "Mathematics in Context" series (Wisconsin Center for Education Research and the Freudenthal Institute, Encyclopedia Brittanica Educational Corporation, 1996)

Students might use the table feature to generate multiples of 3 and multiples of 4.

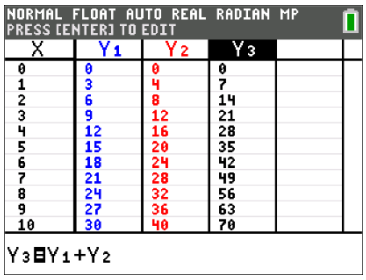






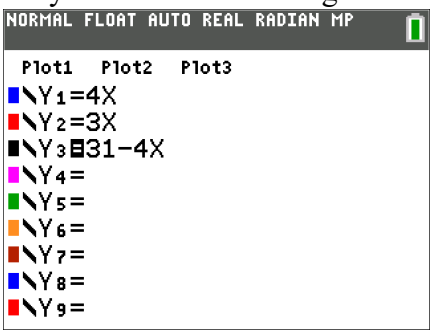
X	Y1	Y2
1	3	4
2	6	8
3	9	12
4	12	16
5	15	20
6	18	24
7	21	28
8	24	32
9	27	36
10	30	40
11	33	44

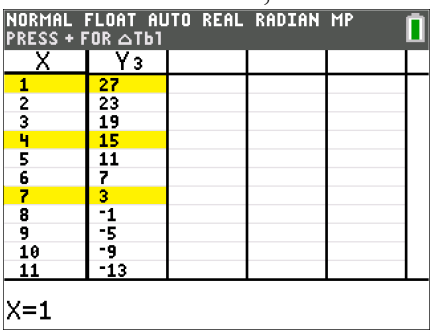
The multiples of 3 are shown in Y1. The multiples of 4 are shown in Y2.
Ask students what X, Y1 and Y2 each represent in terms of Bobo's problem situation.
Answer: X is the number built, Y1 is the number of wheels used for trikes, Y2 is the number of wheels used for wagons. You might ask students (or a student might ask you), let's add the two columns to see if the total of wheels from trikes and the total of wheels from wagons is 31 for some value of X.
Ask students what they notice about Y3 = 3X + 4X.
Answer: they are multiples of 7.
Why would this be true? Answer: combining like terms
What does this table tell us about the solution?
Answer: Bobo made a different number of trikes than wagons.
Otherwise, the total number of wheels would have been a multiple of 7.



X	Y1	Y2	Y3
0	0	0	0
1	3	4	7
2	6	8	14
3	9	12	21
4	12	16	28
5	15	20	35
6	18	24	42
7	21	28	49
8	24	32	56
9	27	36	63
10	30	40	70

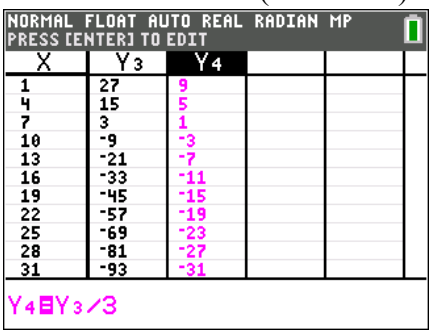
Students could explore subtracting the multiples of 4 from 31 as shown in Y3.
Below, the value of X is now the number of wagons made and Y3 is the number of wheels available to make trikes. Students might scan Y3 for multiples of 3. What patterns do they notice?
Answer: for every increase in 1 of X, the values of Y3 decrease by 4.
Why? One additional wagon consumes 4 more wheels, or 4 less wheels available for trikes.



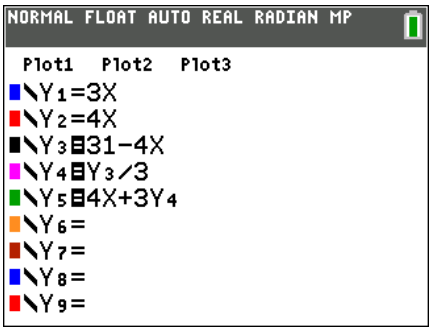


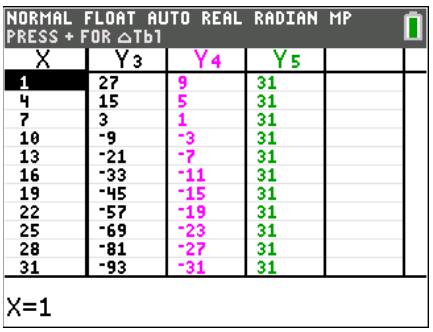
When X= 1, 4, and 7 we have the outputs of Y3 divisible by 3.

Create Y4 = Y3/3 using a table with step size of 3.
Notice in each row that Y4 decreases by 4 (the number of wheels of 1 less wagon) when X increases by 3.
On the last page, using sketches, students can show that an increase of 3 wagons (12 wheels) means a decrease of 4 trikes (12 wheels) if the number of wheels are unchanged.

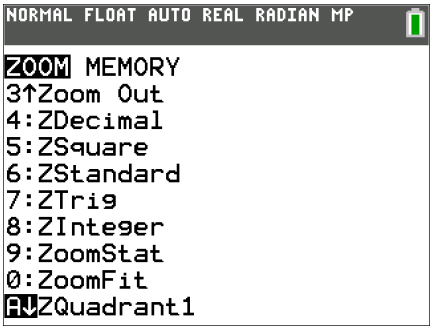


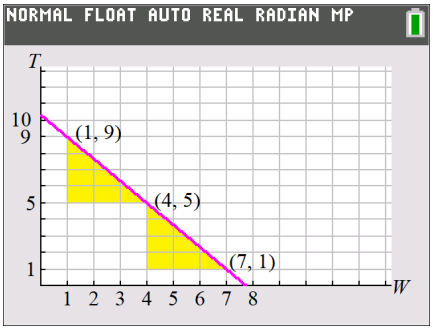
Create the expression 4W+3T in Y5, where X= W and Y4 = T.





Based on the level of your class, graph $y = \frac{31-4x}{3}$ with a gridline and show the three possibilities (1, 9), (4, 5), and (7, 1).





$4 \cdot W + 3 \cdot T =$ Total # of Wheels

$4 \cdot \boxed{1} + 3 \cdot \boxed{9} = 31$

$4 \cdot \boxed{4} + 3 \cdot \boxed{5} = 31$

$4 \cdot \boxed{7} + 3 \cdot \boxed{1} = 31$

Connections can be made between the slope −4/3 in terms of the table and the context of the situation:

Connecting to the table:

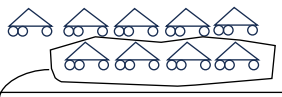
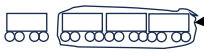
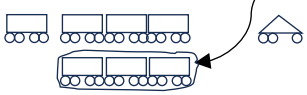
If we increase the number of wagons by 3, we decrease the number of trikes by 4.

Connecting to the context:

Adding 3 wagons means an increase of 12 wheels. Taking away 3 trikes means a decrease of 12 wheels. If we have one solution, say 1 wagon and 9 trikes, then increasing the number of wagons by 3 and decreasing the number of trikes by 3 will keep the same number of wheels, which will also be a possible solution. If an ordered pair is a point on the graph it is a solution to 4W + 3T = 31.
When we move from that ordered pair by going decreasing vertically 4 units (RISE = −4) and increasing horizontally 3 units (RUN = 3), we will stay on the line. This means we have another ordered pair which is a solution to 4W + 3T = 31.

Solution: Although there are three possible solutions, it is most likely that Bobo made 4 wagons and 5 trikes. Had Bobo made only 1 wagon and 9 trikes or had he made only 1 trike and 7 wagons, then he probably would have remembered since these are more unusual combinations. (Bobo doesn't have complete amnesia.)

Students may notice that the total number of toys built (wagons plus trikes) also decreases by 1. Why this may be the case? The next row below is the result of exchanging 4 trikes (12 wheels) into 3 wagons (also 12 wheels). If we change 4 trikes into 3 wagons, we have the same number of wheels, but one less toy. Drawing pictures may help students with this reasoning.

$4 \cdot W + 3 \cdot T =$ Total # of Wheels	$W + T =$ Total # of Toys		
$4 \cdot \boxed{1} + 3 \cdot \boxed{9} = 31$	$\boxed{1} + \boxed{9} = 10$	**1**	 **9**
$4 \cdot \boxed{4} + 3 \cdot \boxed{5} = 31$	$\boxed{4} + \boxed{5} = 9$	**4** 	**5**
$4 \cdot \boxed{7} + 3 \cdot \boxed{1} = 31$	$\boxed{7} + \boxed{1} = 8$	**7** 	**1**

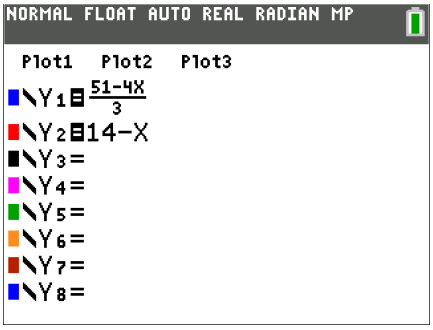
Possible Assessment Questions:

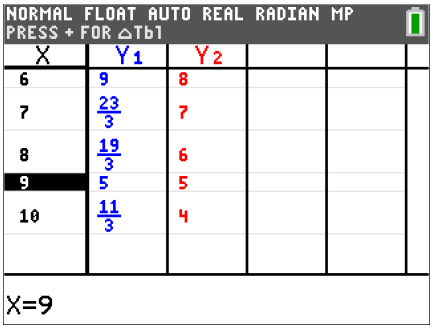
Suppose the next day, instead of 31 wheels used, Bobo used 28 wheels.
What would you think is the likely solution?

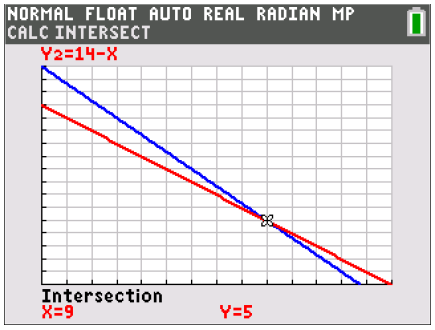
Suppose it were 27 wheels?

Extension to Solving Linear Systems

Return to this activity when you solve systems of linear equations. Present the situation where Bobo uses 51 wheels and makes a total of 14 toys (wagons + trikes). Students can solve 4W+3T = 51 and W + T = 14 with a table, graph, and by substitution or elimination strategies.







Bobo would have made **9 wagons** (9x4 = 36 wheels) and **5 trikes** (5 x 3 = 15 wheels); 36 + 15 = 51.

Acknowledgment: *Bobo the Forgetful Elf* is based on an activity originated by James Hersberger, Purdue University Fort Wayne, hersberg@pfw.edu.