Home Run Derby: Swingin' for the Bleachers

Materials

- *TI-Nspire™ CX or TI-Nspire™ CX CAS handheld with Softball.tns file OR*

<u>*Optional Materials:*</u>

- *TI-Nspire™ CX or TI-Nspire™ CX CAS handheld*
- Smartphone or Video Camera with tripod
- LoggerPro™ Software
- Meter Stick
- Softball bat and ball
- Two Orange Cones

Introduction to the Investigation

Mathematics can be used to model (simulate) reality so that we can make predictions or determine likely outcomes. In particular, we can use quadratic functions to model trajectories of ballistic objects. In this investigation, we will play a game of Home Run Derby and use mathematics to try and determine a winner. In Home Run Derby, contestants take turns trying to hit home runs and the person who hits the most wins. Here we will modify the game just a bit and instead use mathematical modeling to determine who hits the ball the farthest.

As you may have experienced in a mathematics or physics class, we often use a theoretical function involving the acceleration of gravity to describe the height or path of a projectile that assumes no air resistance; however, unless we are on the moon, this seldom fits reality. Instead, we will be collecting actual data of the flight path of a softball and use them to predict how far the ball traveled in the air before hitting the ground. In this investigation your instructor may choose to go outside and, using a camera or smartphone, record video of students hitting a softball or s/he may choose to use the provided file *softball.tns* where a photo of the ball's path has already been created.

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Collecting Your Own Video (If using the file Softball.tns, skip to the section titled, Analyzing the Photo Image, p. 5)

Go outside to a baseball, softball, or other flat field. Every student will get three attempts to hit a softball "up the middle" as far as s/he can. We will record video of the hits and pick the best one that the contestant wishes to have analyzed. In order to make fair comparisons, we want the trajectories to be in roughly the same plane of flight. To be able to accomplish this, we will use hits that follow a path between two orange cones placed about 15 feet on each side of the pitcher. This will keep us from considering hits that are at too great of an angle toward left or right field. Note that any angle deviating from a "straight up the middle" path will make comparisons of flight paths difficult.

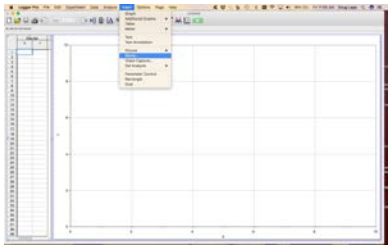
Place a camera or smartphone at a distance from home plate perpendicular to the path the ball will travel (see image above). A tripod will be helpful here to maintain consistent camera angle and distance. You will need to have it far enough away from the batter to capture several video frames of the ball's position as it ascends, but not so far away that it is difficult to make out the image of the ball in each frame. A fluorescent colored softball may make it easier to see the ball in the video. Also, have a student stand behind the hitter at a safe distance holding a meter stick perpendicular to the ground. This will be used to determine the scale of the video once it is imported to the software.

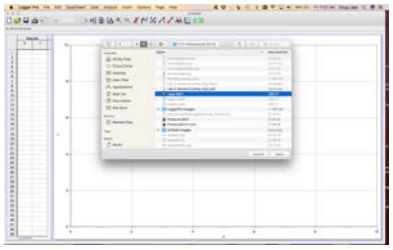
Each student who wishes to participate in the contest gets three hits (contact with the ball) to try and get the ball airborne between the cones. Record each of the attempts. When all video attempts are recorded, go back into the classroom to analyze the video.

Each student will choose which of his/her three video attempts will be used for comparison. Once a representative video is chosen for each participant, in a poll, have the class members decide who they think are the top three hitters. These students' video will be used to predict the distance of the hit.

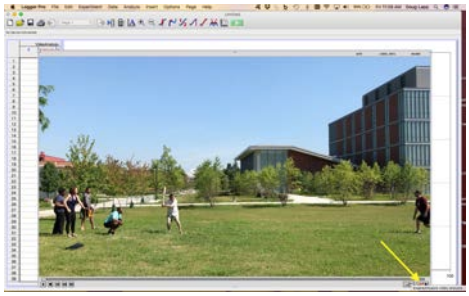
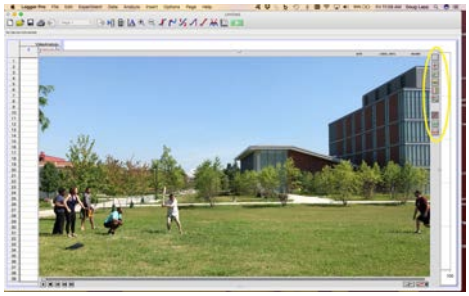
<u>Using LoggerPro™</u> (This can be done ahead of time by the teacher to save time)

Open the software LoggerPro™. To import the video, select Movie… from the Insert menu and then locate the video on your computer and click on it.

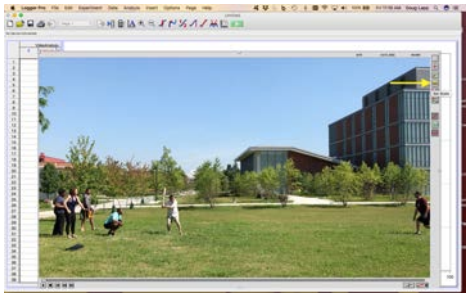
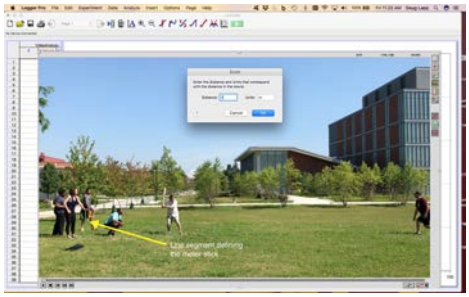




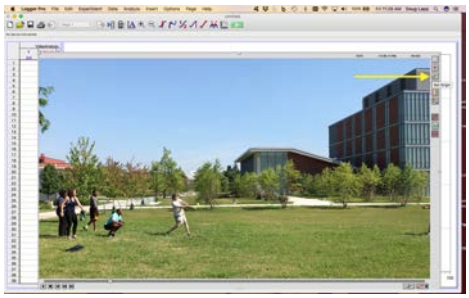
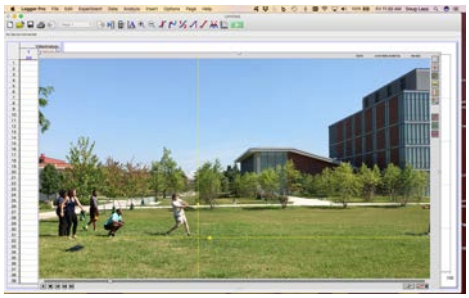
Now enable the video analysis menu by clicking on the button in the lower right corner of the window. This will make another menu of buttons appear on the upper right side of the window.

To set the scale for the video, select the **Set Scale** button (looks like a ruler) from the side menu and then click and draw a line segment that goes from the top to the bottom of the meter stick being held by the student in the video. The dialogue box will then ask you to define this distance (define it as 1 meter).

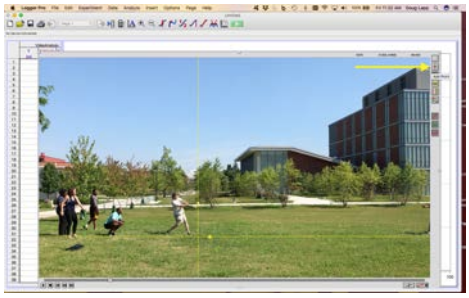
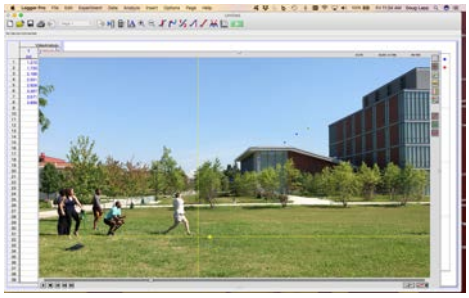



We now need to set an axis system for the video. To do this, select the **Set Origin** tool in the upper right corner of the window and drag the axis system until the horizontal axis lines up with the feet of the batter and bottom of the meter stick and the vertical axis lines up with the image of the ball just as it is being struck by the bat. Note you will need to have already advanced the video to this point in time.

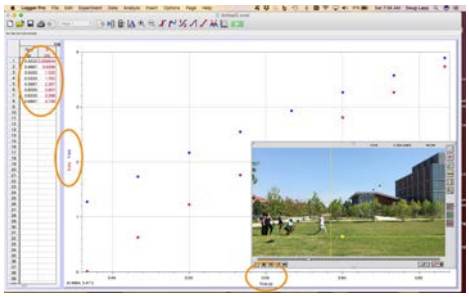
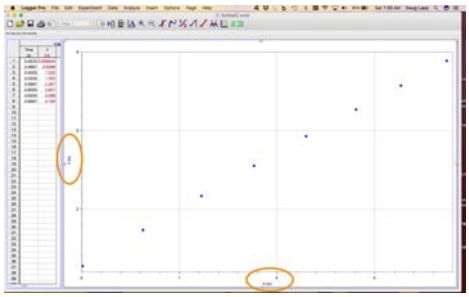



At this point in the analysis, we want to digitize points along the path of the ball. To do this, select the **Add Points** tool from the video analysis menu in the upper right corner of the window. Once the tool is selected, click on the image of the ball and a point will be added to the screen. You will also note that the video is advanced by one frame

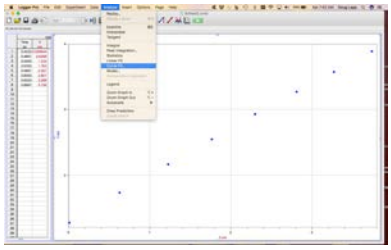
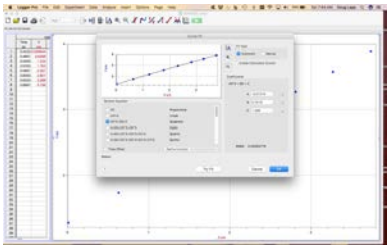
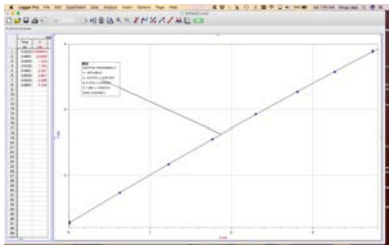
showing the ball in a different location. Click on the new image of the ball and continue this process until you have several data points of the ball's location along the flight path.

At this point, we can use LoggerPro's curve fitting tools to fit a quadratic to the data or we can grab a screen image of the video with all of the digitized points displayed and then import this image into the TI-Nspire CX for analysis. To use LoggerPro, shrink down the video by grabbing the corner of it so that the data and graph can be seen behind it. You will see time on the horizontal axis and both the X and Y-coordinates on the vertical axis. To set the graph to show the flight path of the ball, click on the "Time" label on the horizontal axis and select X from the drop down menu to place the X-coordinate on the axis. Then click on the label for the vertical axis and select Y to place the Y-coordinate on the vertical axis.

To fit a curve to the data, select the Curve Fit option from the **Analyze** menu (see images below). Then select the Quadratic option and click **Try Fit**. You will see the parameters A, B, and C change as it fits a curve to the data.

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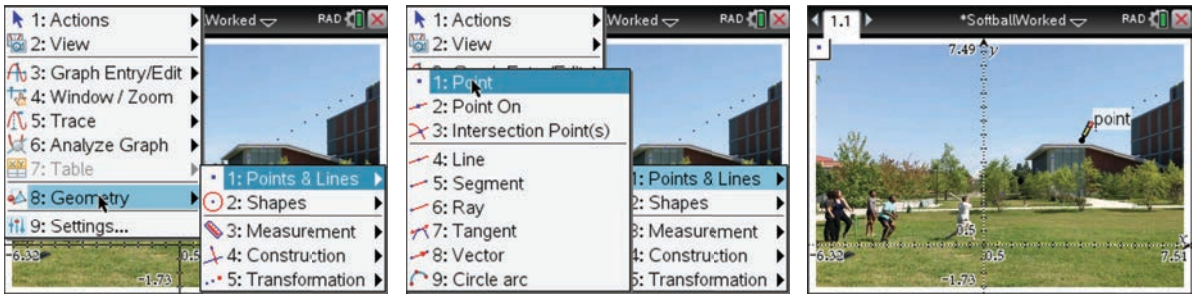
Analyzing the Photo Image

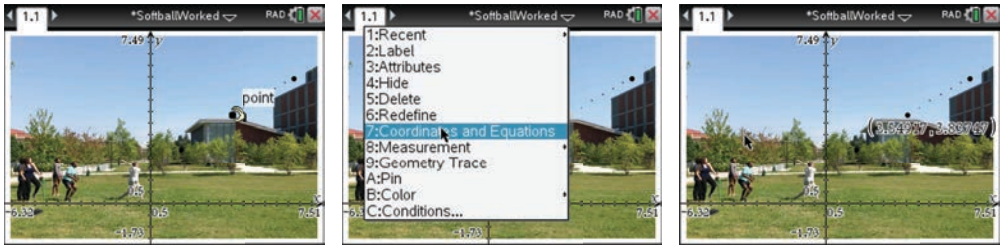
If you use the existing file, *Softball.tns*, the locations of the softball at various frames of the video have already been marked and can be seen in the photo image. Also, the axes and the scale have been adjusted in the TI-Nspire CX CAS so that 1 meter corresponds to the meter stick being held by the student in the photo.

If you wish to use data collected by the class videos, you can use the model found in LoggerPro™ or import a screen image containing multiple locations of the softball and adjust the axes and scale in a manner similar to the *Softball.tns* file provided. Your teacher may have already done this in advance.

For our first analysis, we will assume the ball followed a parabolic path. Pick three points along the path of the ball in the photo and place points over the images of the ball's location. To do this, select menu and then **8:Geometry** → **1:Points & Lines** → **1:Point**. Now click to place points on the image.



Once the points are created, we want to see their coordinates. To do this, hover over the points and press ctrl menu and select **Coordinates and Equations**. Record your coordinates below.



x-coordinate	y-coordinate

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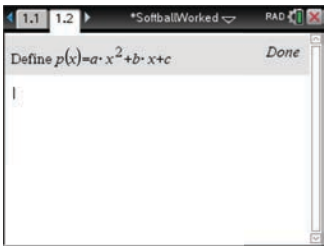
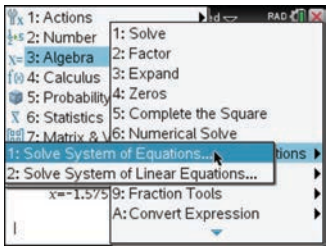
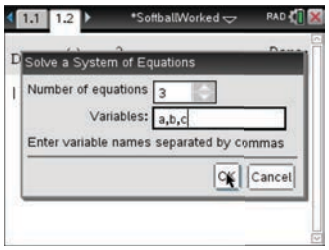
We can now use our computer algebra system to find the parameters needed to fit a parabola to these three data points. We start by defining a generic parabola as $p(x) = ax^2 + bx + c$. Now since each point we have marked along the path corresponds to an x-y pair of coordinates, we can solve a system of equations of the form:

$$p(x_1) = y_1$$
$$p(x_2) = y_2$$
$$p(x_3) = y_3$$

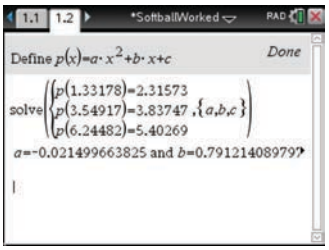
Select your Solve Systems of Equations command by pressing menu followed by **3:Algebra → 7:Solve System of Equations → 1:Solve System of Equations** and then entering the names of the parameters as shown below.







Since you have already defined the function, $p(x)$, now enter the system of equations in the template using the coordinates you found from your points listed in the table above (see example below).



The results will give the values for the parameters *a*, *b*, and *c* that solve the system.

1. Give your quadratic model for the flight path of the softball.

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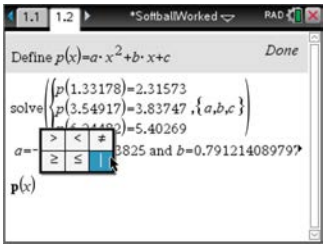
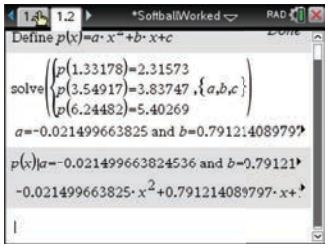
To see the graph of the function on top of the data, enter the name of the function, $p(x)$, on the **Calculator** page followed by the "such that", |, key (ctrl [|≠≥▸]) and copy the previous line listing the parameters. This will make a temporary replacement for *a*, *b*, and *c*. Copying the resulting expression into $f_1(x)$ on the **Graphs** page will display the function on top of the data (see below).





2. We now want to approximate how far the ball traveled before hitting the ground.
   a. In your group, discuss what conditions exist with respect to the y-coordinate of the ball at the moment the ball first hits the ground.

   b. Give an equation with respect to the function, $f_1(x)$, that can be solved to find how far from home plate the ball hit the ground. Explain why solving this equation will give the range of the ball.

   c. If your class collected data from multiple videos, repeat this process for each contestant and determine who hit the ball the farthest. Explain your reasoning.

3. For question 2, we created a model from three data points of the ball's position.
    a. Repeat the same process for finding a model, but this time using just two of your previous points instead of three. Explain your results.

    b. Now place a fourth point on the position of the ball in your **Graphs** page. Using four points, repeat the previous process for finding a new quadratic model and explain your results.

    c. Place another point on the position of the ball in your **Graphs** page (total of 5 points). Using these five points, perform a quadratic regression and give your model. What would be the distance predicted for the hit by your new model?

Extension

Another interesting relationship can be found between time and the y-coordinate. If we fit a quadratic to the relationship between time and the y-coordinate, the coefficient of the squared term is half the acceleration of gravity (assuming no air resistance). Each frame of video was collected in a 1/3 second time interval. Using this information, fit a quadratic model to the relationship between time and the y-coordinate of the ball. What is your predicted value for the acceleration of gravity? Compare/contrast your value to the accepted value of 9.8 m/s^2.

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