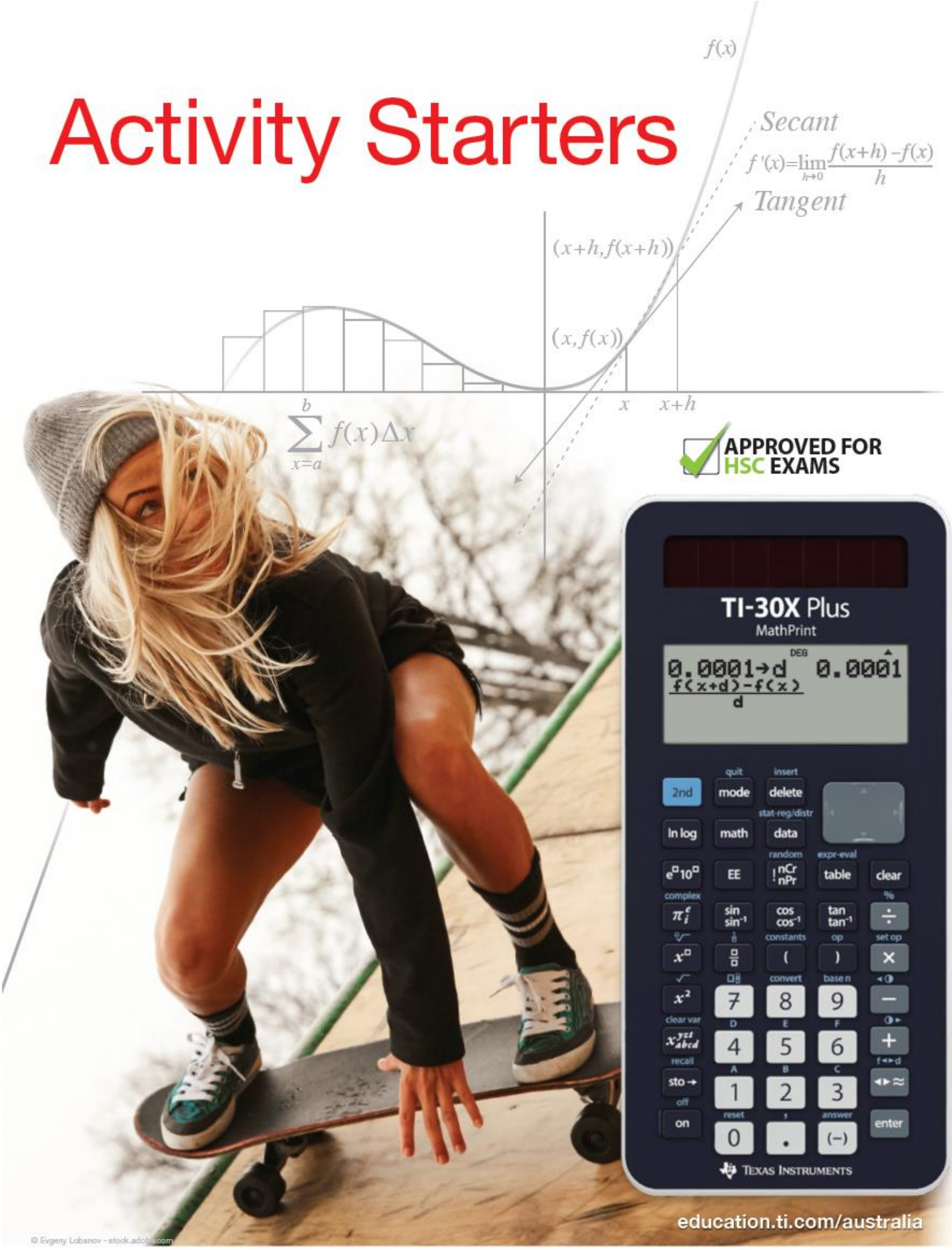
Activity Starters



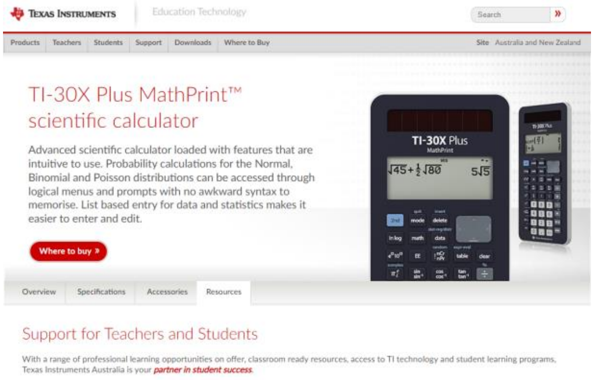
Texas Instruments

Contents

Introduction

This booklet contains extracts from a selection of the activities available from the Texas Instruments Australia website.
Full versions of these activities and many more are available free and include:

- Student worksheets
- Answers
- Teacher notes
- Video tutorials (as applicable)
- PowerPoint Presentations (as applicable)



In addition to the free activities, you will also find: http://education.ti.com/aus/nsw

Student Courses

No prerequisites or sign-ups required, the FREE online course is a great way for student's to develop their TI-30X Plus MathPrint™ technology skills. Students can take the quiz at the end of the course to obtain a personalised certificate of completion.

Student Tutorials

These videos cover the mathematics and associated calculator operations. Students are encouraged to download the associated worksheet in advance and follow along whilst watching the tutorials. Worked solutions, answers and tips are also provided for worksheets.

Webinars

Led by experienced teachers, the webinar program is designed to demonstrate the effective use of TI technology to help build understanding. Live events provide the opportunity to ask questions, receive a copy of the notes and a PD certificate.

Access Free TI-30X Plus MathPrint™ technology

Schools can access TI-30X Plus MathPrint calculators and a TI-SmartView™ emulator software license for every Maths teacher, classroom loan kits and customised professional development.

e-Newsletter Subscription

Keep up to date with our latest news, workshops, activities and opportunities through our e-Newsletter, delivered direct to your inbox.

The extracts included in this booklet include some basic calculator instructions, teacher notes and sample questions, indicative of the content associated with the complete activity. For senior mathematics content, head to the Senior Curriculum Inspirations section of the Texas Instruments website, activities are sorted by subject and content. For junior and middle school classes, check out the resources tab for the TI-30XPlus MathPrint calculator; there you will find booklets that you can download for each year level.

Calculator Setup & Settings

Before you drive a new car, you need to familiarise yourself with various controls and settings; it's the same with a new calculator. The top of the calculator screen provides reminders and information about current settings, but how do you adjust them? If the screen is not dark enough, how do you adjust the contrast, or perhaps you want to set everything back to factory default. This page provides information about the most common adjustments.

Settings & Screen Prompts

Screen prompts and reminders are only visible when the corresponding mode or function is active. The screen shown opposite includes all the indicators, most of them should be familiar. Extras include:

L1, L2 and L3: These represent the calculator's lists.

Number base: H = Hex, B = Bin & O = Oct

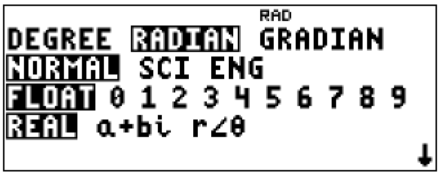
Hour Glass: Calculator is busy

Up/Down/Left/Right: Arrows indicate that more content is visible beyond the current screen, use the navigation keys accordingly.

quit
mode

Mode Settings

Press the mode key to see or change the calculator settings. Notice the arrow in the bottom right corner of the screen, this indicates that more options are available.



Place the cursor over the required setting and press:

enter

To exit the mode settings press:

2nd | mode (quit)

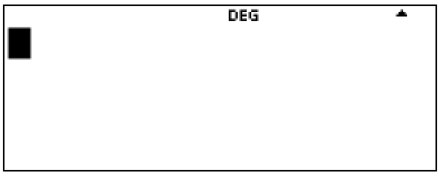
Clear Home Screen

Press the clear button:

clear

Previous calculations still exist; however they will not be displayed. To exit press:

2nd | mode (quit)



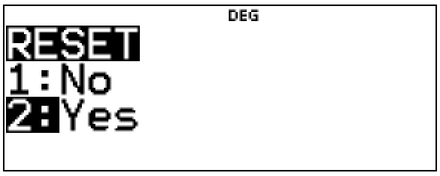
Notice the up arrow (top right of screen) indicates more content is available.

RESET

There are two options to reset the calculator. Option (1) Press:

2nd | 0 (reset) … 2 (B)

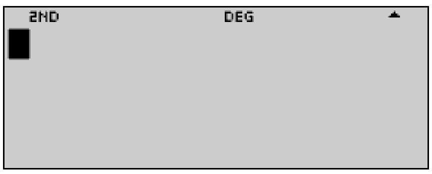
The second option is to use the physical reset button on the back of the calculator.



Contrast

To lighten or darken the screen press:

2nd | – … OR … 2nd | +



Calculator Navigation

The TI-30XPlus MathPrint calculator includes some advanced features such as menus, templates and navigation tools. A brief list of navigation tips is included here. For more detailed information, check out the video tutorials.

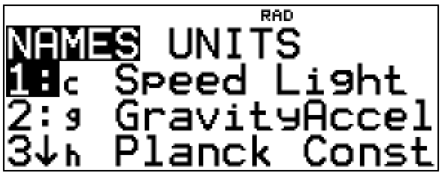
Home Screen

The calculator has multiple menus, wizards, lists and more. To exit out of an environment and return to the home screen there are several options:

Escape or Quit: 2nd mode (quit)

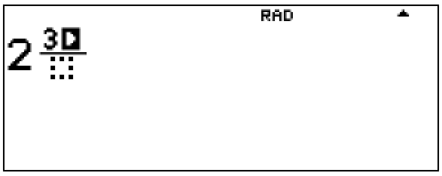
Use item number: 1 (A)

Select item and press: enter



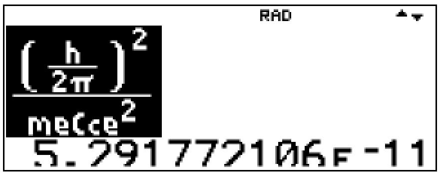
Navigation Pad / Arrow Keys

Move the cursor by tapping the corresponding arrow. In the example shown here numerical values have been entered for the whole number and numerator in a mixed fraction. The next item is the denominator, the arrow indicates that pressing the right arrow will navigate to the next space in the template, the down arrow will also navigate to the denominator.



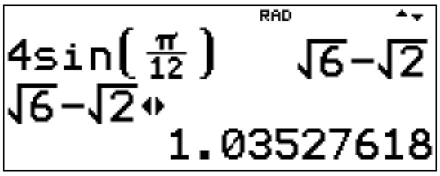
Copy & Paste

Use the navigation keys to select an item, then press Enter to paste the selection to the current calculation.



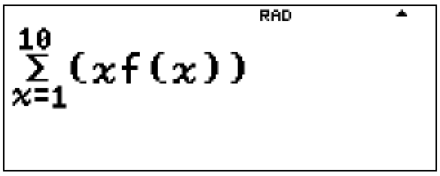
Answer Toggle

Use this key to swap or toggle between exact or fraction form and decimal representation.



Templates

Templates provide an intuitive format, enter the corresponding values and formulas while using the arrow keys to navigate around the template.

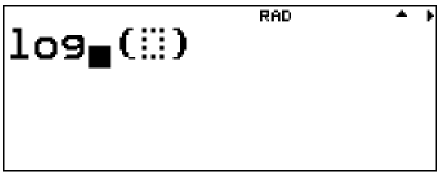


Multi-Tap Keys

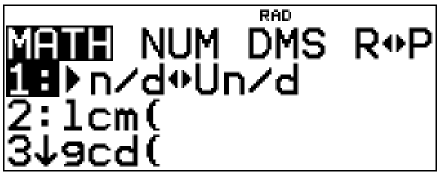
Tap these keys multiple times in succession to see the range of functionality offered. Multi-tap keys include:

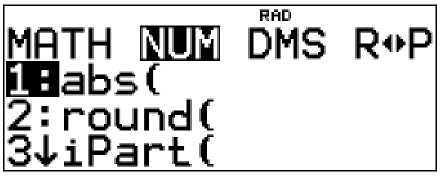
ln log | $e^{\square}\ 10^{\square}$ | ! nCr nPr (random) | $\pi\ ^e_i$ (complex) | $x\ ^{yzt}_{abcd}$ (clear var) | sin $\sin^{-1}$ | cos $\cos^{-1}$ | tan $\tan^{-1}$

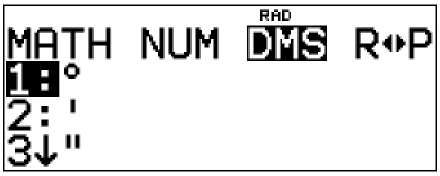
To use a multi-tap key for successive entries such as πe, select the first item then use the right arrow to proceed to the next entry, this breaks the cycle of the multi-tap key.

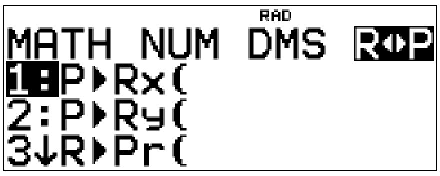


Menus







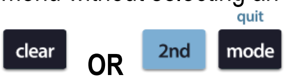


Menus exist in several locations. The **math** menu contains a total of twenty four (24) commands sorted into four categories:

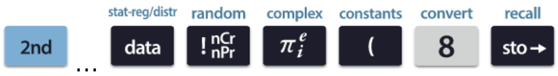
- Maths (MATH),
- Number (NUM),
- Degrees Minutes & Seconds (DMS)
- Rectangular/Polar coordinates (R◀▶P)

Use the navigation pad to navigate through the menu. If you know the menu item, you can enter the item number. Pressing the up-arrow at the top of a menu list will automatically scroll up.

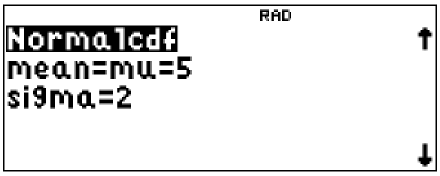
To exit a menu without selecting an item, press:

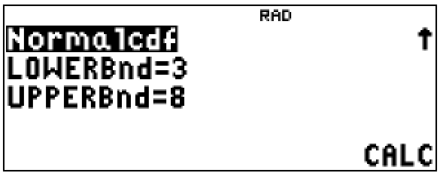


Some menus are associated with the second function:



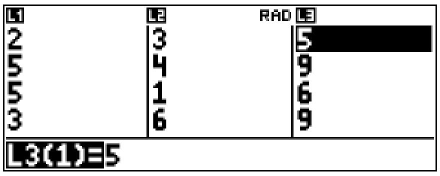
Wizards





In addition to menus and templates, some commands include a wizard. The purpose of the wizard is to reduce the need for specific calculator syntax. The example shown opposite if for the Normal Cumulative Density Function. Prompts exists for each parameter: mean, standard deviation and the upper and lower bound values.

Lists



The lists be populated one entry at a time or automatically using sequences or formulas. The lists opposite show a section of 50 random dice rolls in List 1 (L1) and another 50 in List 2 (L2). The sum of the two dice has be automatically calculated in List 3 (L3). To access the lists press:

If the lists contain unwanted data, press the data key again and select option 4 to **Clear All** lists.

Factors that Count

The aim of this investigation is for students to determine a rule that identifies the quantity of factors for any given number using prime factorisation. The investigation requires students to:

- Engage in systematic exploration;
- Use tables to sort and summarise findings;
- Make and test conjectures;
- Use appropriate mathematical terminology and notation.

The investigation helps students appreciate the purpose of writing numbers in factorised form. Students are also introduced to the concept of algebra, as a tool that can be used to generalise. The investigation has students 'doing' mathematics, a vastly different experience than completing multiple skill and drill style questions.

Calculator Instructions

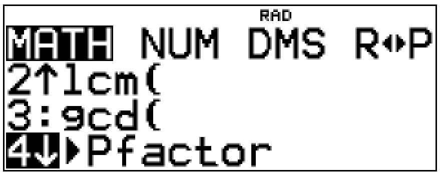
The TI-30XPlus MathPrint calculator has a Prime Factorisation command: **Pfactor** that can be used to determine the prime factorisation of any number less than 1,000,000, numbers higher than this quantity will generate a domain error.

❶ To find the prime factorisation of 36 press:

3 6 math

Scroll down to option **4 ▸Pfactor**, then press:

enter … to paste the command on the home screen.



❷ The command is pasted on the home screen, press:

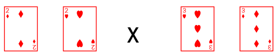
enter

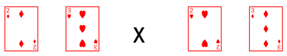
The prime factorisation of 36 is displayed: $2^2 \times 3^2$.

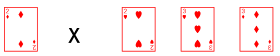
The factors for any number can be generated by manipulating the prime factors, an important concept for students to understand. Playing cards make a great manipulative and allow students to work cooperatively in pairs. For each starting number, students gather the appropriate cards, move them around, group them and record their findings. Manipulating the prime factorisation in this way is the catalyst that helps students make the quantum leap towards establishing a general formula.

Example:

The prime factorisation of 36 is given by: $2^2 \times 3^2$, students will need two (2's) and two (3's). Students group the cards and write the corresponding expressions for each factor pair. (See below)

 = (2 x 2) x (3 x 3) = 4 x 9

 = (2 x 3) x (2 x 3) = 6 x 6

 = 2 x (2 x 3 x 3) = 2 x 18

 = 3 x (2 x 2 x 3) = 3 x 12

The arrangements of the prime factors, in addition to 1 and 36, reveals all the factors of 36:

1, 2, 3, 4, 6, 9, 12, 18, 36 Qty: 9 Factors.

Use the calculator to determine the prime factorisation of 1225. Gather the necessary cards to represent this prime factorisation and consider the outcome. How is this similar to 36? How is it different? What would be your prediction for the quantity of factors for 225 or 104,329? The number 1715 could be represented with four playing cards, but it has a different quantity of factors. Why? The factors that count activity provides a layer of scaffolding by seeding the exploration with carefully selected numbers to start students on their journey.

Sample Findings:

Number	Prime Factorisation	Bases	Exponents	Qty Factors
196	$2^2 7^2$	2, 7	2, 2	9
100	$2^2 5^2$	2, 5	2, 2	9
1183	$7^1 13^2$	7, 13	1, 2	6
175	$5^2 7^1$	5, 7	2, 1	6
250	$2^1 5^3$	2, 5	1, 3	8
56	$2^3 7^1$	3, 7	3, 1	8
80	$2^4 5^1$	2, 5	4, 1	10
7203	$3^1 7^4$	3, 7	1, 4	10
693	$3^2 7^1 11^1$	3, 7, 11	2, 1, 1	12
140	$2^2 5^1 7^1$	2, 5, 7	2, 1, 1	12
350	$2^1 5^2 7^1$	2, 5, 7	1, 2, 1	12

Students identify: when the exponents are the same, the quantity of factors is the same.

Students identify this by informally considering the ‘arrangements’ of the cards and also by tabulating results.

Once students focus on the exponents, the problem comes down to finding a connection between the last two columns.

Struggling to find the answer?

Try adding ‘one’ to each exponent.

Extension – Factor Sums

The prime factorisation of a number is also a great way of determining the sum of the factors for any number.

Example:

$200 = 2^3 \times 5^2$.

Factors {1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200}.

Sum the factors: 465

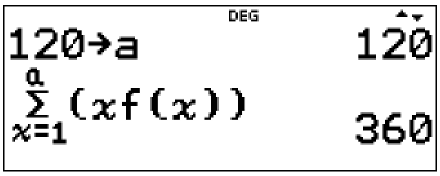
Calculate: (1 + 2 + 4 + 8) x (1 + 5 + 25)

The calculate expression is derived directly from the prime factorisation, in other words, it is not necessary to determine each and every factor in order to calculate the sum. How and why does this work? The reason why it works provides a delightful introduction to ‘expanding’.

Advanced Calculator Operations

Suppose you want to use an algorithm to sum all the factors for some number: a.
This could be done using the following:

$$\sum_{x=1}^{a} xf(x)$$



The function $f(x)$ returns a one (1) if x is a factor of a, otherwise it returns a zero (0). The function uses two calculator commands: *fPart* and *iPart* where *fPart* returns the fraction part and *iPart* the integer part of a number or expression. What is the definition for $f(x)$?

This single calculation is relatively time consuming! To locate and sum the factors of 120 takes approximately 9 seconds. For 360 it takes approximately 27 seconds! **Do not try large numbers!**

Euclidean Algorithm: Highest Common Factor

The TI-30XPlus MathPrint has a command to determine the highest common factor or greatest common divisor (GCD) and also the lowest common multiple (LCM), however algorithms are an important part of mathematics, so teaching students how these algorithms work can help them understand how some digital technologies work.

The Euclidean Algorithm for finding GCD(A,B) is as follows:

If A = 0 then GCD(A,B)=B, since the GCD(0,B)=B, [Stop]

If B = 0 then GCD(A,B)=A, since the GCD(A,0)=A, [Stop]

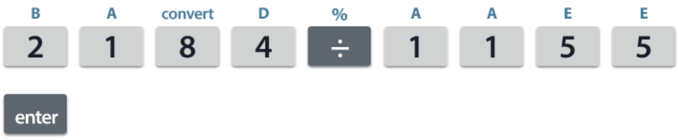
Write A in quotient and remainder form (A = B·Q + R)

Find GCD(B,R) using the Euclidean Algorithm since GCD(A,B) = GCD(B,R)

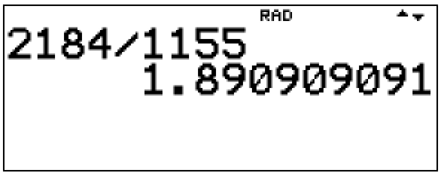
Putting the Algorithm to work:

What is the highest common factor of 2184 and 1155?

1 Start by dividing the larger number by the smaller.

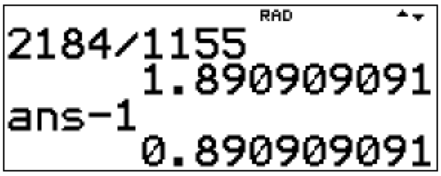


We're not interested in the quotient, just the remainder. The problem is the remainder has been expressed as a decimal.



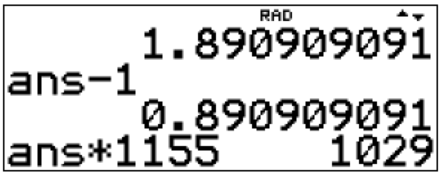
2 One way to calculate the remainder is to subtract the quotient (1) from the answer. Type:

Notice that the calculator automatically pastes the previous result and expresses it as "ans".



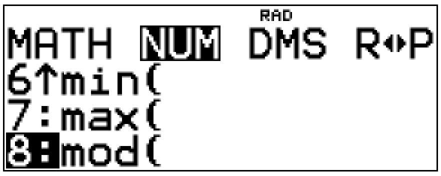
3 This answer can be multiplied by the original divisor (1155):

So 2184 ÷ 1155 = 1 and 1029 remainder.

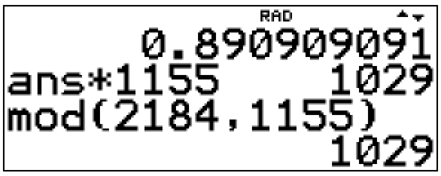


4 There is an easier way to calculate the remainder on the calculator, it's referred to as 'modular' arithmetic. Press:

Notice that the up-arrow cycles over to the bottom of the number menu.



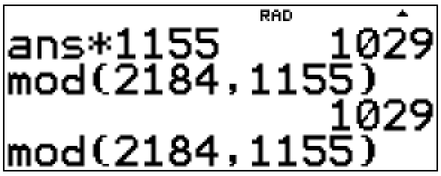
5 The syntax for the modular command is mod(quotient,divisor):



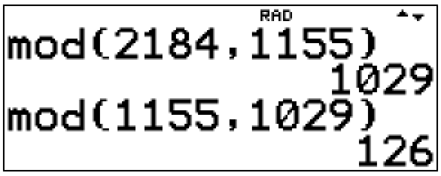
6 Modular arithmetic is now carried out on the smaller of the previous two values: (2184, 1155) and combined with the remainder as:

Mod(1155, 1029)

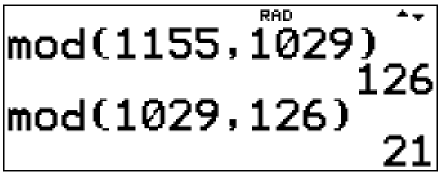
This time, copy and paste the previous entry:



7 Navigate back to 2184 and type over the top of the entries:



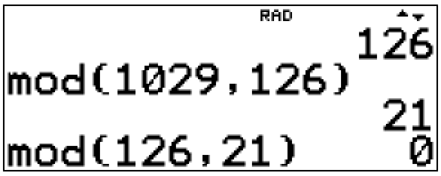
8 The delete key is used here to reduce the quantity of digits in the modular arithmetic command.



The ANS key could be used to populate the second term in the modular arithmetic command, however, in this particular situation, the concealment of the value in ANS makes it harder for students to follow.

9 Euclid's algorithm has been repeated one more time (opposite) and a result of zero obtained, at which point the algorithm declares: "STOP"

The zero tells us that 21 is a factor of 126, and therefore a factor of both 2184 and 1155; it is the highest common factor.



The Highest Common Factor (Greatest Common Divisor GCD) command is available directly from the MATH menu, test the result from Euclid's algorithm to confirm it produces the same result.

Once students know how to generate the Greatest Common Divisor using the algorithm, they should express each of the numbers as a product of their prime factors.

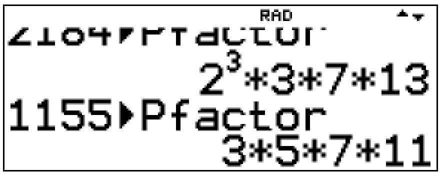
Prime factorisations:

$2184 = 2^3 \times \mathbf{3} \times \mathbf{7} \times 13$

$1155 = 3 \times \mathbf{5} \times \mathbf{7} \times 11$

$21 = \mathbf{3} \times \mathbf{7}$

From here we see that the common factors are: 3 and 7, so the highest common factor of 2184 and 1155 is 21 =3 x 7



Try exploring other numbers.

Why does the algorithm work?

How could you apply the algorithm to determine the highest common factor of: 4620, 2730 and 7735?

Use the prime factorisation to explore the lowest common multiple of two numbers.

Exploring Recursion: Babylonian Square-roots & More!

Some calculator functions are treated as 'black box' operations, the square-root operation is perhaps the first such function that students encounter. The square-root of a number is generally introduced as the opposite or inverse of squaring a number. The square-root of a number is easy to calculate for perfect squares such as: 1, 4, 9, 16 … but how are other numbers evaluated? Algorithms to determine the square-root of a number have existed for more than 4,000 years, indeed, they pre-date the use of zero!

Suppose you want to calculate the square-root of 200. A common response from students is 20, double the square-root of 100. This is obviously not true, but let's use it to create a better estimate.

Student's estimate: 20

Check: $200 \div 20$

The 'check' is designed to see whether our guess or estimate is correct by dividing our original number by the estimate for the square-root.

If the check is not equal to the original estimate, then we can improve our estimate by averaging these two quantities.

Revised Estimate: $\dfrac{20+(200\div 20)}{2}=15$ [Algorithm]

The new estimate (15) represents an improvement on our original estimate (20). This estimate can be revised again using the algorithm:

Revised Estimate: $\dfrac{15+(200\div 15)}{2}\approx 14.167$ [Algorithm]

Use [f◄►d ◄►≈] to toggle between factional and decimal answers.

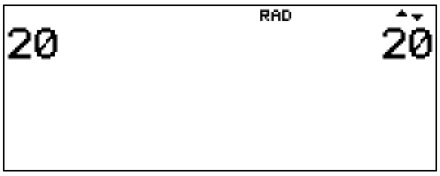
Despite our original estimate being poor, two applications of the algorithm produced a remarkably accurate estimate! This process can be made efficient by using either a memory location or the answer (**ans**) feature.

Babylonian Calculator Instructions

1. Let's explore $\sqrt{200}$ using the '**ans**' feature. Enter the original estimate:

   [2] [0] [enter]

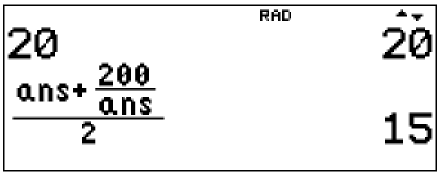
   This process makes **ans** = 20.



2. Use the fraction tool to write the expression shown opposite.

   [fraction] ... to access '**ans**' press: [2nd] [(−)]

   Once this calculation has been executed, the new '**ans**' is 15. This value will be used in the next calculation.



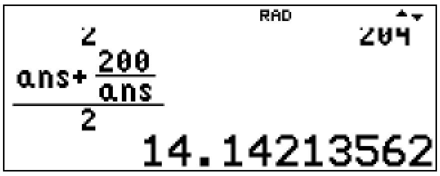
3. To repeat the algorithm simply press:

   [enter]

   Note: Some answers will be provided in fraction form.

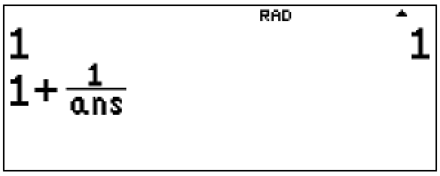
   Repeat the algorithm a few more times and notice what happens to the answer. After just four applications of the algorithm, the estimate produces the result:

   $14.14213562 \approx \sqrt{200}$ This result is accurate to 8 decimal places!

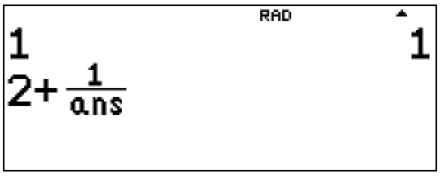


More Recursion

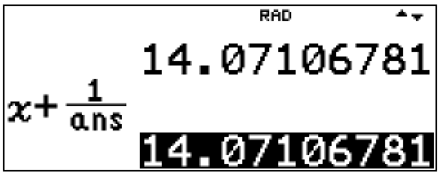
Try this interesting recursion (shown opposite).
What will be the output for the first step?
Does this recursive process tend towards a specific number?



Here is a slight variation to the previous recursion. (opposite)
What will be the output of the first step?
Does this recursive process tend toward a specific number?



Try this problem in reverse. An integer value was stored in x and the recursion process repeated many times.
The result: $7+5\sqrt{2}$
What value was stored in x?



The numerator and denominator in the fractions generated in the first exploration should have looked familiar. Try the recursion again and watch closely. What is happening? Why are we seeing this pattern?

Successive terms in the second investigation may not have looked familiar; they represent a more general form of the same recursive sequence. Try searching the internet for “metallic ratios”.

To find the value of x (above) in the third investigation, you could use trial and error since x is an integer value, or you could use algebra. Will numbers generated in this way ever produce an integer result?

Extension

1. The TI-30XPlus MathPrint has the capability of storing a function.

   Start by storing $\frac{1+\sqrt{5}}{2}$ in memory location a and $\frac{1-\sqrt{5}}{2}$ in b.

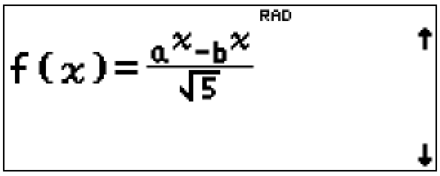
   Use the store key: recall sto→ and multi-tap memory keys: clear var x^{yzt}_{abc}

2. To define a function, use the table key and select **Add/Edit function**.

   expr-eval table

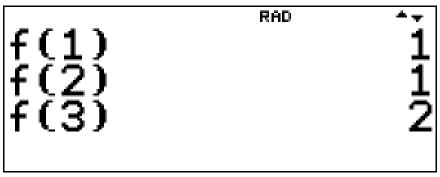
3. Define f(x) as shown opposite. **Once it has been defined press**:

   enter then return to the home screen 2nd quit mode



4. Use the table key to access f(x) [Option 2] and try entering integer values for x. What is this function doing?

   Try calculating: $\frac{f(25)}{f(24)}$. What does it equal? (Approximately)

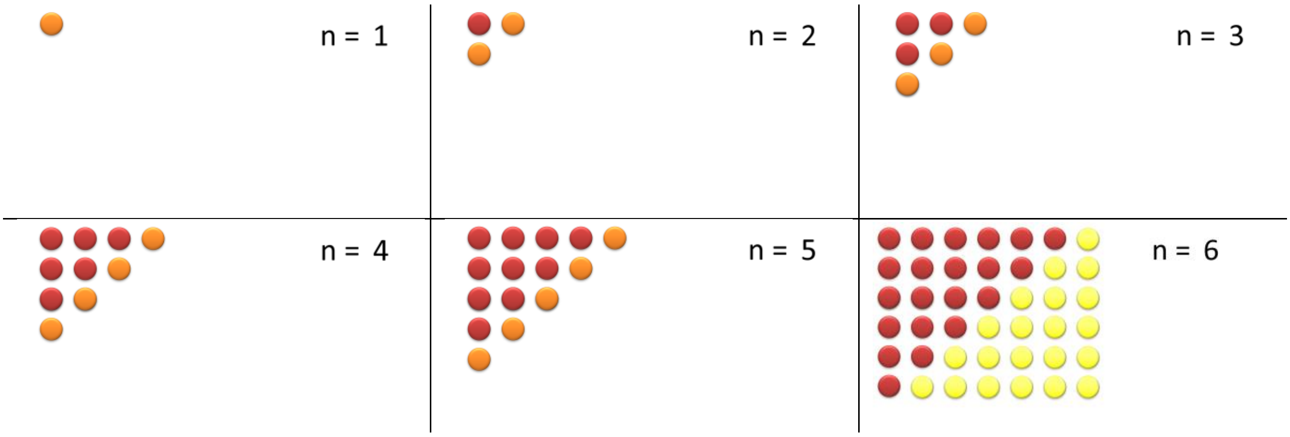


Triangular Numbers

What is the sum of the first *n* whole numbers? There are several ways this problem can be explored. This activity includes powerful visuals via a PowerPoint slide show, students see why they are called triangular numbers! The activity can be done in middle school (quadratics) or senior mathematics classes as a delightful way to introduce proof by induction.

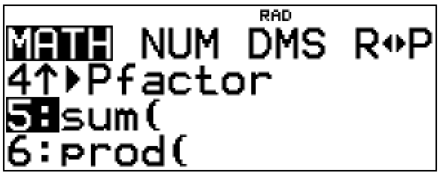
Visual Representation

The PowerPoint slide show contains a series of animations to help students develop a formula.

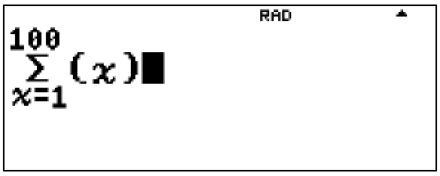


Calculator Instructions

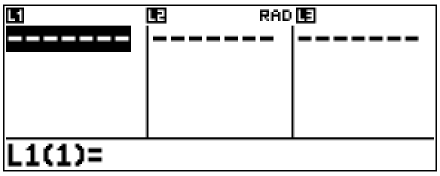
1 The calculator can determine the sum of the first 100 numbers using the summation tool. Press:



2 The summation template is intuitive, use the arrow keys to navigate. The screen opposite shows how to calculate the sum of the first 100 numbers.
Try other sums such as the sum of the first 50 whole numbers.
What about the sum of the whole numbers from 50 to 100?

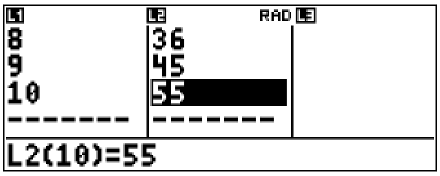


3 The calculator lists can also be used by students to explore these sums. Press:



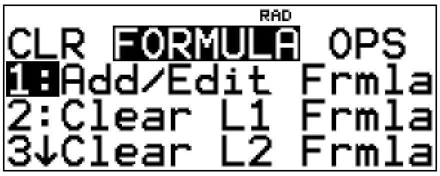
If lists need to be cleared, press the Data key again and select option 4.

4 Enter the values from 1 through to 10 in list 1 (L1).
Enter the progressive sums in list 2 (L2).
While this can be done automatically, for the purposes of this task, enter the values manually using mental arithmetic.



5 Navigate across to L3 and press:

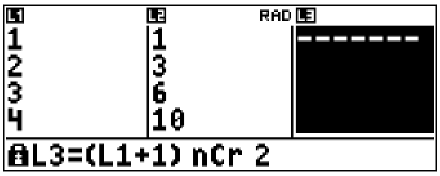
… select **Add / Edit formula**



6 Type in the formula: **(L1 + 1) nCr 2** (shown opposite)

To access the reference to List 1 (L1), press:

The combinatorics command can be accessed via the multi-tap key:



Once the formula has been entered in list 3 (L3), enter additional numbers in list 1, notice that the values in list 3 are automatically generated courtesy of the formula.

Why does the formula in List 3 work? A section of Pascal's triangle is shown here displaying the triangular numbers. A brief study of the corresponding diagonal soon shows why this works. The formula for the triangular numbers (sum of the first n whole numbers) can be derived from the combinatorics expression:

$$^{n}C_{r}=\frac{n!}{(n-r)!r!}$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

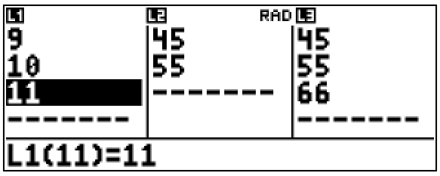
The triangular numbers occur in the 'second' diagonal:

$$^{n}C_{2}=\frac{n!}{(n-2)!2!}=\frac{n(n-1)(n-2)(n-3)...}{2(n-2)(n-3)...}=\frac{n(n-1)}{2}$$

the **first** triangular number occurs at $n + 1$.

7 Another way to determine the equation for the sum of the first n whole numbers is to use the data!

Navigate to List 1 and make sure it is the same length as List 2, note that you can press the 2nd function key followed by the down arrow to jump straight to the bottom of a list. Delete values that do not align to List 2.



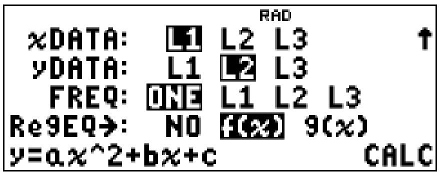
8 Quadratic regression can be used to determine the equation. Press:

Select Quadratic Regression.

9 Make the selections opposite, store the equation in: $f(x)$.

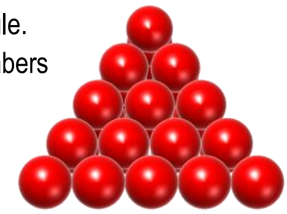
Select the CALC (calculate) option to see the coefficients: a, b and c for the quadratic equation.

With the equation stored in $f(x)$, students can perform calculations such as $f(10)$ to see the sum of the first 10 whole numbers.



The tetrahedral numbers also appear in Pascal's triangle. Tetrahedral numbers are the sum of the triangular numbers and can therefore be generated using:

$$\sum_{x=1}^{n}\left(\frac{x(x-1)}{2}\right)$$



1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

The 7th triangular number appears in the next row of Pascal's triangle. Use the summation tool to determine this number. Why does this work?

Probability Simulation: Maximum Dice

The lists on the TI-30XPlus MathPrint can be used to store random numbers and therefore perform probability simulations. Consider a simple board game where players roll two dice. and move forward an amount equal to the highest number rolled. For example, if a player rolls a 3 with dice one and a 5 with dice two, the player moves forward 5 spaces.

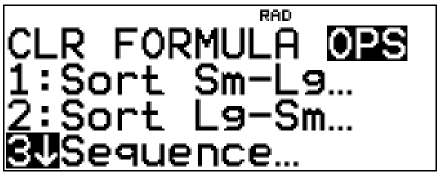
What would be:

- The most common number of squares moved forward?
- The average number of squares moved forward?

❶ The calculator's lists can be used simulate up to 50 rolls at once.

The first step is to generate a 'sequence' of random numbers:

… select **Sequence**

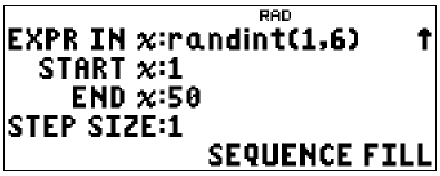


❷ The first dice rolls go into List 1 (L1). Select List 1 and press **ENTER** to continue.

❸ To insert the random integer command (randint) press:

… select **randint(**

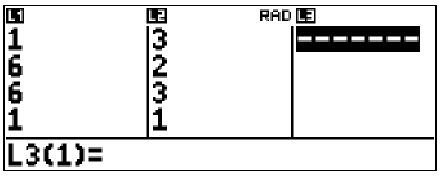
Syntax: randint(lower# , upper#).



Lower = **1** (smallest random integer) Upper = **6** (largest random integer).

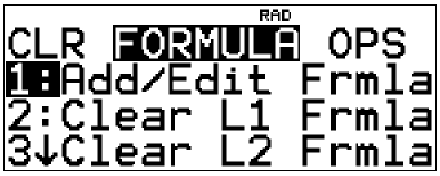
To generate 50 dice rolls set: **Start** = **1** & **End** = **50** then select **Sequence Fill**.

❹ Repeat the process for List 2 (L2)



❺ Use list 3 (L3) to automatically identify the largest value for each dice pair. Navigate to List 3 and press:

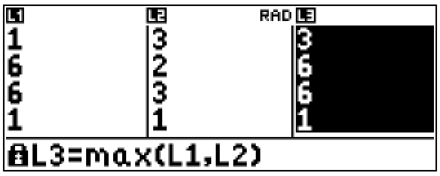
… navigate to **Formula** and select **Add/Edit**



❻ The formula is: max(L1,L2).

The maximum command is located in the **MATH** > **NUM** menu.

List 1 (L1) and List 2 (L2) can be found in the **Data** menu.



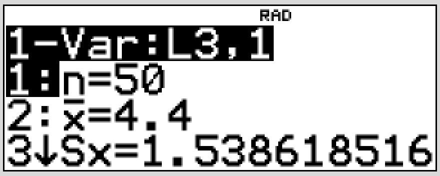
Most electronic devices generate pseudo random numbers. There are advantages and disadvantages to this nuance. If it is desirable for all students to generate the same data, students can use the same seed value. To randomise across the class, students can use the last four digits of their mobile phone number as the seed value. To seed the random number generator, enter your seed value then press:

Analysing the data

Scroll through List 3 and use the data to populate the frequency table: (Sample data shown below using 1234 seed)

MAXIMUM #	**1**	**2**	**3**	**4**	**5**	**6**
FREQUENCY	2	6	7	5	15	15
EST.PROB	0.04	0.12	0.14	0.10	0.30	0.30
$x.P(x)$	0.04	0.24	0.42	0.40	1.50	1.80
					$\sum x.P(x)$	4.4

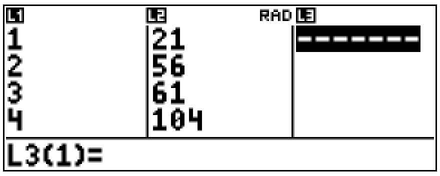
The calculator retains the list formula so it is quick and easy to generate another sample!
Based on the above data students can also calculate the mean using the calculator's statistical tools.



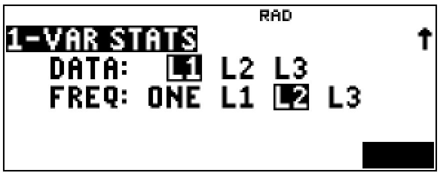
Students can collect multiple samples from other students.

The data shown opposite represents the combination of samples from a total of 10 students, bringing the total simulation to 500 data points.

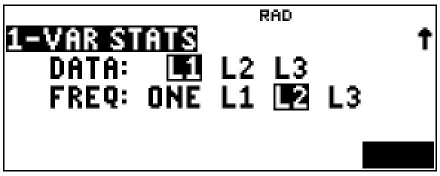
Off screen results include: 5 (107) and 6 (151)



This time the data has frequency (stored in List 2).



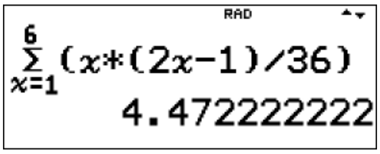
The average number of spaces moved when choosing the maximum of two dice is approximately: 4.36



These estimates can be compared with theoretical probabilities.

DICE 1 / DICE 2	**1**	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	2	2	3	4	5	6
3	3	3	3	4	5	6
4	4	4	4	4	5	6
5	5	5	5	5	5	6
6	6	6	6	6	6	6

MAXIMUM #	**1**	**2**	**3**	**4**	**5**	**6**
FREQUENCY	1	3	5	7	9	11
					$\sum x.P(x)$	4.472



Probability Distributions: Binomial PDF & CDF

The distributions menu contains two options for the binomial distribution:

- Binomialpdf
- Binomialcdf

Probabilities can also be computed using the formula:

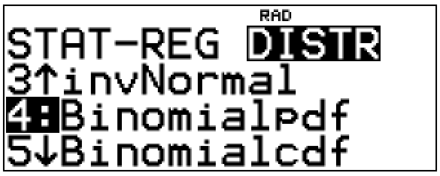
$$P(x) = {}^{n}C_{x}(p)^{x}(1-p)^{n-x}$$

Scan the QR code to watch a video on the binomial distribution.

1. To access the binomial distribution press:

2. Use the navigation pad (right arrow) to go across to the distribution menu (**DISTR**) then down to Binomialpdf or Binomialcdf.

Select **Binomialpdf** [Press **ENTER** or **4**]



3. Using the Binomialpdf option, you can calculate the probability for:

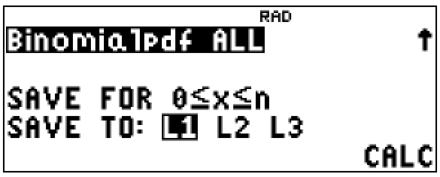
- **SINGLE** – nominated value of x.
- **LIST** – multiple selected events specified by list (1,2 or 3)
- **ALL** – all possible outcomes, results stored to a list.

Navigate across to select **ALL** and press:

4. The video tutorial (QR code) refers to the probability of various (natural) hair colours. In the video it states: “the probability of a person having brown hair is 78%”. Let’s consider a group of 10 students. We can calculate the probability of 0, 1, 2 … students having brown hair.

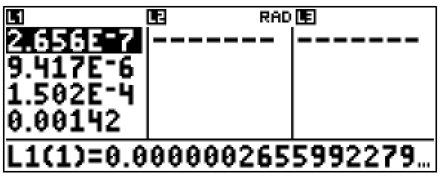
Enter 10 for the quantity of trials (students) and 0.78 for the probability of success then press **ENTER** or the **down arrow** to navigate to the next screen.

5. Choose the list where the results will be stored then press **ENTER** on the **CALC** option.



6. All the probabilities are stored in a list.

Now try clearing the list and specify selected event in List 1. Repeat the binomialpdf command and choose the LIST option.



Sample Problem

Mobile phone numbers in Australia consist of 10 digits. The first two digits are 04, this leaves 8 other digits, assuming the remaining digits are random and independent, calculate each of the following probabilities for the remaining 8 digits:

i) Contains exactly 1 five.

ii) Contains no more than 2 zero's.

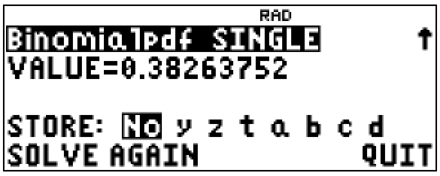
iii) The digits add up to an even number.

Sample Problem - Solutions

The probability of a specific digit is 1/10 since there are 10 different digits: 0, 1, 2, … 9 of equal probability.

i) Use binomialpdf and select SINGLE.

n(trials) = 8, p(success) = 0.1 x = 1. Pr(x = 1) = 0.3826



ii) No more than 2 zeros, there can be 0, 1 or 2 zeros.

n(trials) = 8, p(success) = 0.1, $x \le 2$ or x = {0, 1, 2}. Consider the following approaches:

- Use binomialcdf with x = 2 [Most efficient approach]
- Store 0, 1 & 2 in LIST1 then use binomialpdf and select the LIST option. To reduce the likelihood of a transcription error use the Sum List option in the OPS menu. [Press DATA key twice]
- Use binomialpdf and compute three individual events (0,1 & 2), add the results together.

Answer: $\Pr(X \le 2) = 0.9619$

iii) If the digits add up to an even number they consist of either 0, 2, 4, 6 or 8 odd digits. The probability of any given digit being odd is ½. The most efficient approach in this question is to use a list for the x values.

n(trials) = 8, p(success) = ½, x = L1 (List 1)

Enter the values: 0, 2, 4, 6 & 8 in L1

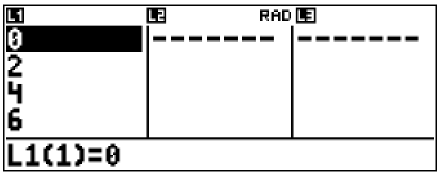
Use binomialpdf and select the LIST option.

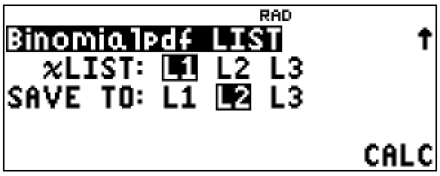
The x values are in list 1 (L1)

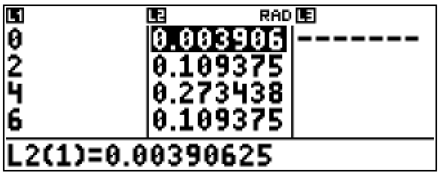
The probabilities will be saved to list 2 (L2)

Use the Sum List option in the DATA > OPS menu.

Ans: ½.   Was this answer 'obvious'?







Discrete Probability Distribution – Mean & Variance

The number of occupants in 6-berth cabins on any given voyage for a particular ship can be summarised by the following probability distribution table:

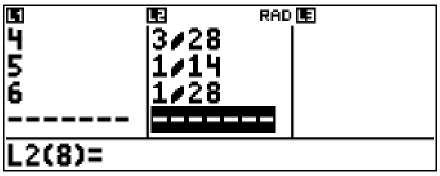
Number of Occupants:	0	1	2	3	4	5	6
Probability:	$\frac{1}{4}$	$\frac{3}{14}$	$\frac{5}{28}$	$\frac{1}{7}$	$\frac{3}{28}$	$\frac{1}{14}$	$\frac{1}{28}$

Calculations involving the mean, variance or standard deviation for this distribution are relatively straight forward, however, the quantity of calculations increases the likelihood of students making a miscalculation. The TI-30XPlus MathPrint includes functionality that can help mitigate such unfortunate events.

1. Start by entering the random variable x in List 1 (L1) and the associated probabilities in List 2 (L2). To access the lists press:

   stat-reg/distr
   data

   To clear the lists, press the Data key again and select option 4.



To keep the probabilities as fractions, use the fraction key.

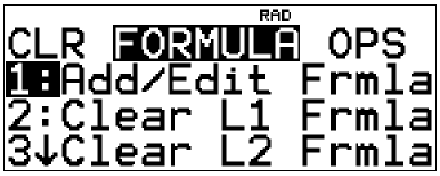
The mean or expected value for the distribution is calculated by:

$$E(x) = \sum x \cdot p(x)$$

Random variable x is stored in List 1 (L1) and the probability in List 2 (L2), calculating the product of L1 and L2 followed by the sum is an efficient way of computing the expected value.

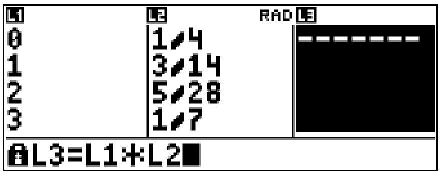
2. Navigate to list 3 and press:

   stat-reg/distr
   data ▸ … select **Add/Edit Formula**



3. To access the list labels press:

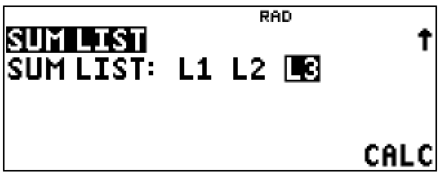
   stat-reg/distr
   data write the formula: L1 x L2 then press: enter



4. To calculate the sum of the values in List 3 press:

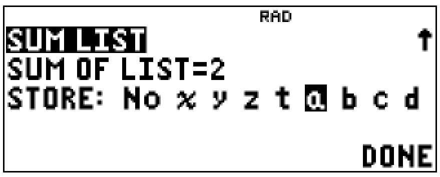
   stat-reg/distr
   data navigate to **OPS** … ▸ ... select **Sum List**

5. Select L3 for the sum followed by **CALC**.



6. The sum of the calculations in List 3 is displayed and can be stored in a variable for later use.

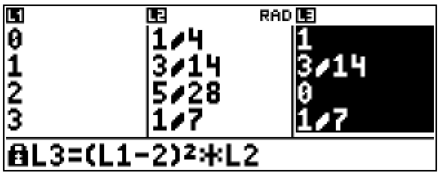
   $E(x) = 2$ (Sum of List = 2)



The variance for the distribution can be calculated by:

$$Var(x)=\sum(x-\mu)^2\cdot p(x) \quad \text{OR} \quad Var(x)=E(x^2)-E(x)^2$$

7. Try calculating the variance for the distribution using the first formula (above). The equivalent calculator formula is shown opposite, where 2 is the mean, determined from the previous calculation. If the mean was stored in 'a', the memory location could be used in place of the 2. **Remember to sum the list!**



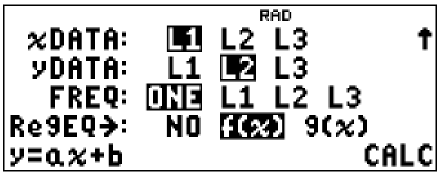
Discrete Probability Distribution - Function

In this particular example the probability distribution can be modelled by a linear function. The equation to the function can be determined using Linear Regression.

1. To determine the equation press:

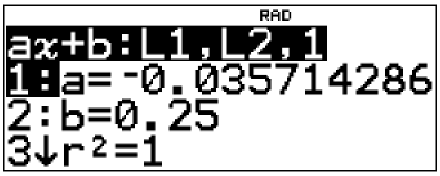
   stat-reg/distr
   2nd data ▾ ... select LinReg ax+b

2. Match the settings shown opposite, store the equation in $f(x)$.



3. The parameters 'a' and 'b' are displayed, so too the perfect correlation.

   The accuracy of the function can be checked by calculating some of the probabilities.

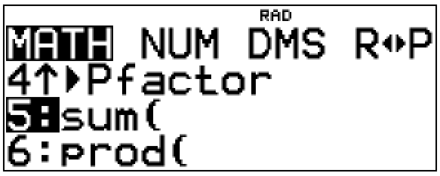


4. Return to the calculator's home screen:

   quit
   2nd mode

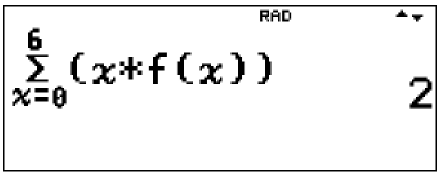
   Use the calculator's summation tool to calculate the mean, press:

   math ▴

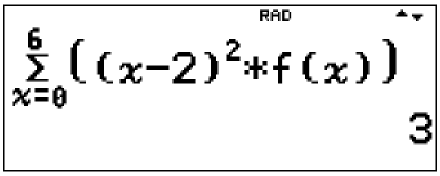


5. Select the summation tool.

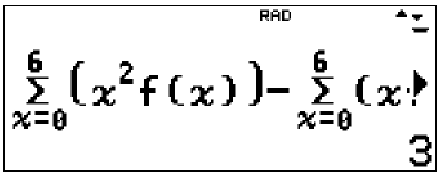
   The sum starts with x = 0 and finishes with x = 6. The function can be recalled using the **table** key.



6. Now try calculating the variance using the formula.



7. This result can be compared with $E(x^2)-E(x)^2$



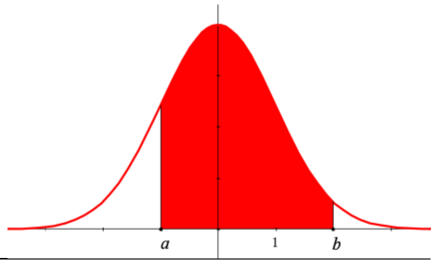
Normal Distribution

The distributions menu contains three options for the normal distribution:

Normalpdf **Normalcdf** **invNormal**

The normal distribution formula:

$$N(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

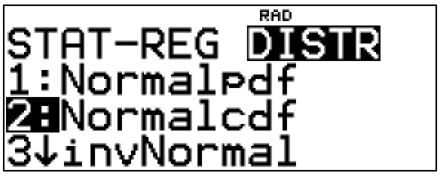


Note: The Normalpdf command uses this formula, however students can define this function in $f(x)$, determine approximate areas and compare the results with those determined by Normalcdf.

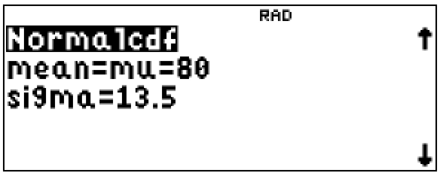
Scan the QR code to watch a video on the normal distribution.

1. To access the normal distribution commands, press:

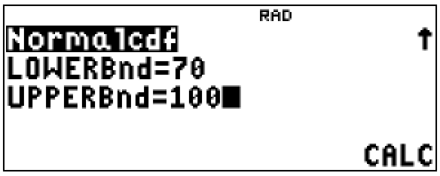
2. Use the navigation pad to move across to the DISTR (distribution) menu. Select option 2: Normalcdf



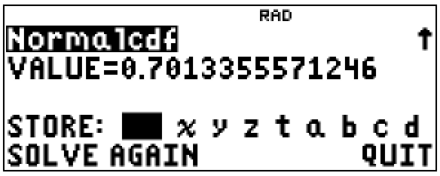
3. Suppose the resting heart rate of sedentary adults is 80 beats per minute with a standard deviation of 13.5. Enter these values for the mean and standard deviation as shown. Use the arrow keys or ENTER to progress through the settings.



4. Suppose we want to calculate the probability that a sedentary adult has a heart rate between 70 and 100 beats per minute. Enter these values as the lower and upper bounds accordingly, then press **ENTER** to calculate.



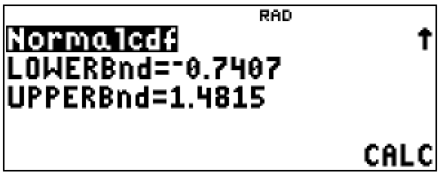
5. The result can be stored in one of the memory locations, this is very useful for conditional probability type questions. Other options from here include Solve Again, the original mean, standard deviation, lower and upper bound values are retained making this an efficient way to do a similar calculation. The final option is to quit from this menu.



6. Students can compare their calculations for the specific mean, standard deviation, lower and upper bound parameters with the equivalent
z-scores for N(0, 1)

$$z=\frac{100-80}{13.5}\approx 1.4815 \quad \&$$

$$z=\frac{70-80}{13.5}\approx -0.7407$$



Sample Problems

Question 1:

Birth weights are normally distributed with a mean of 3000g and a standard deviation of 500g.

i) A Sydney hospital records 1000 births, how many of these would you expect to weigh more than 3400g?

ii) How likely is it that a new born baby will weigh between 2600g and 3600g?

iii) Another hospital typical records 3650 births per year. Babies weighing less than 2200g require access to specialist equipment for 5 days after their birth. Based on this information estimate the minimum quantity of these specialist facilities the hospital should expect, including any assumptions.

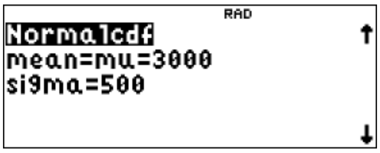
Question 2:

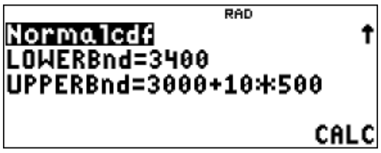
The intelligence quota (IQ) scores for a large population are normally distributed with a mean of 100 and standard deviation of 15. Use the equation for the normal distribution and the trapezoidal rule to estimate the likelihood of an IQ between 95 and 105. [Use intervals of 1 IQ]

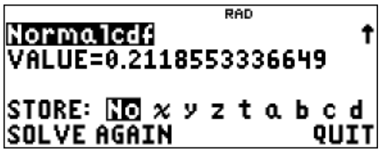
Sample Problem – Solutions

Question 1 (Answers)

i) $\mu = 3000g$, $\sigma = 500g$. Require: $\Pr(X > 3400g)$



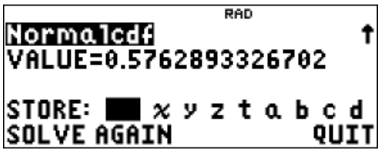




Note: The upper bound used is 10 standard deviations above the mean, sufficiently distant from the mean.

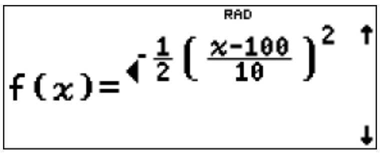
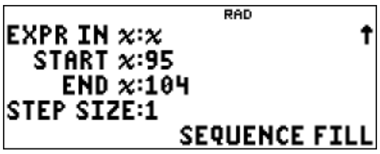
ii) Pr(2600 ≤ x ≤ 3400) = 0.5763

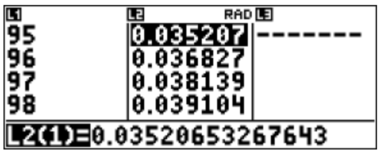
Note: This value could also be deduced from the previous answer. As the normal distribution is symmetrical, the region above 3400 will be the same as the region below 2600 (both lie 400g either side of the mean); ∴ 1 – 2 x 0.211855 = 0.57623

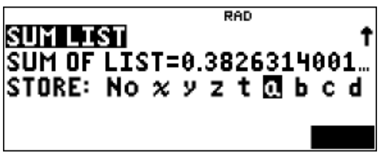


iii) The quantity of babies per year is approximately equivalent to 10 babies per day. The likelihood of a baby weighing less than 2200g (calculator) ≈ 0.0548. A baby requiring the special facility will use it for 5 days, during which time approximately 50 babies are likely to be born. 0.0548 x 50 = 2.74. We 'could' conclude that the hospital would require 3 such facilities. (No rounding at this point!) However, this is the minimum the hospital should expect. For each day that fewer than 10 babies are born, there will be other days where more than 10 are born, and therefore, potentially more babies requiring the facility. What can be stated with a reasonable level of certainty, is that such a hospital with fewer than 3 of these facilities is likely to encounter difficulties.

Question 2 (Answers)





The graph is symmetrical so this result will be the same as the trapezoidal rule and compares favourably with the actual result: 0.3829

Differentiation from First Principles

The calculator can store two different functions: $f(x)$ and $g(x)$. In this section $f(x)$ will be used to define the function and $g(x)$ the approximate gradient using first principles. The calculator does not have a memory location 'h', so 'd' is used instead. Using a memory location is efficient and has the added bonus that the expression appears more symbolic.

Formula:

$$f'(x) = \lim_{d \to 0} \frac{f(x+d) - f(x)}{d}$$

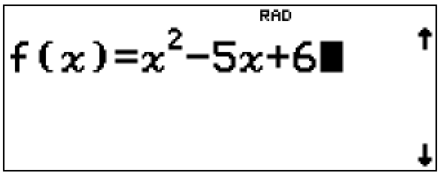
Scan the QR code to watch the video or follow the instructions below.

1 Start with a simple function: $f(x) = x^2 - 5x + 6$

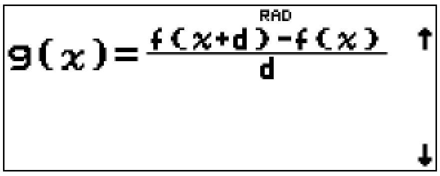
Define the function:

expr-eval
table enter

Use the memory key to enter the 'x' and **ENTER** when finished.

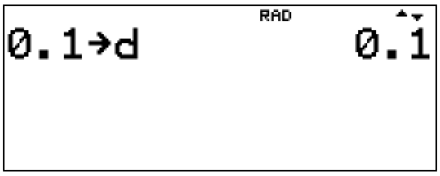


2 Define $g(x)$ using function notation. The calculator will naturally use the most up to date function defined in $f(x)$ and value in d. The **TABLE** key can be used to access $f(x)$, the fraction key to create the vinculum and the multi-tap variable key to access d. Once $g(x)$ is defined, press **ENTER** and then **QUIT** to return to the calculator's home screen.



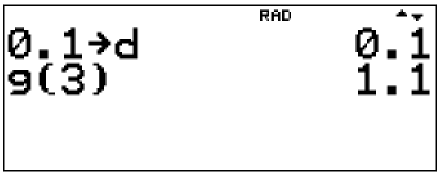
3 Start by exploring the 'limit' as d → 0.
Store 0.1 in d by using the STO and multi-tap variable keys.

reset , A recall clear var
0 . 1 sto→ x^{yzt}_{abc} ... enter



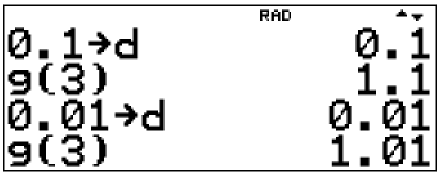
4 To explore the gradient of $f(x)$ at say (3, 0), evaluate $g(3)$.

expr-eval C C op
table 3 3) enter



5 Change the value stored in d and re-evaluate $g(3)$

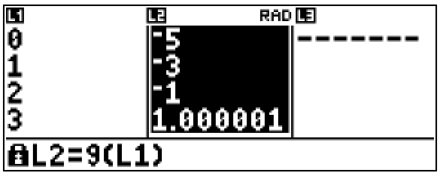
We can see that the gradient appears to be approaching: 1. Note that we could approach from the negative side by using $d \approx$ -0.1



An efficient technique is to use the copy and paste facility. Navigate up to 0.1 → d and press **ENTER** to copy and paste the command to a new entry line. Use the INSERT option [**2nd Delete**] to help edit, then press **ENTER**. Copy and paste g(3) and press **ENTER**.

This process can be repeated, gradually making d smaller and smaller.

6 With d set to a very small quantity, the gradient of the entire function can be computed efficiently by using the calculator's lists. Students can then plot these points to draw a graph of the gradient function.



Sample Problems

Question 1

Let $f(x) = x^3 - 6x^2 + 9x - 6$,

Use the differentiation from first principles to calculate the following: (Let *d* = 0.001)

x	0	1	2	3	4
$f'(x) = \lim\limits_{d \to 0} \dfrac{f(x+d) - f(x)}{d}$	9 (8.994)	0 (-0.003)	-3 (-3.000)	0 (0.003)	9 (9.006)

Given $f'(x)$ is parabolic, use the calculator's lists to help determine the equation for the parabolic function.

Question 2

The function definition tool allows for exploration of functions that would otherwise be too messy on a traditional scientific calculator. Taylor polynomials for e^x, $\sin(x)$ or $\cos(x)$ can be defined as functions.

$$\text{Let } f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \frac{x^8}{8!}$$

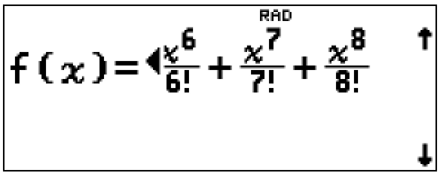
The quantity of terms helps define the accuracy of the output. For the purposes of the table below, there are sufficient terms included in the expression above for students to get a sense of what is happening.

Change the mode setting to Float 3 and set d = 0.0001. Generate a table of values for $f(x)$ and $g(x)$.

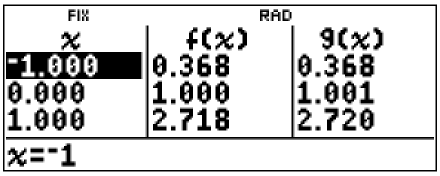
x	-3	-2	-1	0	1	2	3
$f(x)$	0.091	0.137	0.368	1.00	2.718	7.387	20.009
$f'(x) = \lim\limits_{d \to 0} \dfrac{f(x+d) - f(x)}{d}$	-0.071	0.130	0.368	1.001	2.720	7.385	19.856

Notice that $f(x) \approx f'(x)$. Once students know the 'power rule', they can differentiate the expression for $f(x)$ and see that the expression for the derivative is following the same pattern.

This shows $f(x)$ defined as the first 9 terms of the Taylor polynomial for e^x.



The table of values provides a glimpse at the similarity between the original function $f(x)$ and the derivative or gradient function $g(x)$.



Approximate Areas: Part 1

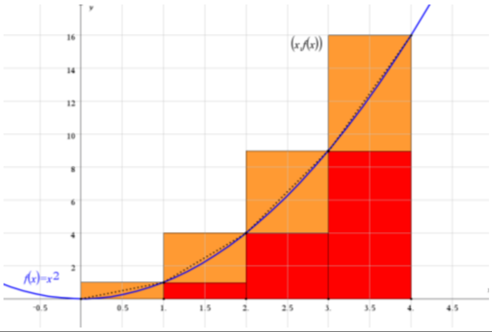
The ability to define functions and perform list calculations make the TI-30XPlus MathPrint a very useful scientific calculator when it comes to integral calculus and approximating the area under a curve.

Formula:

$$\int_a^b f(x)dx \approx \sum_a^b f(x)\Delta x$$

Scan the QR code to watch the video. The video covers aspects both sections on approximate areas.

The graph shows left (lower) and right (upper) bound rectangles used to approximate the area bounded by the curve $f(x) = x^2$, the x axis and the lines x = 0 and x = 4. These results can the be averaged to determine the approximate area using trapezia.

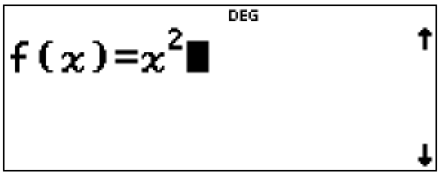


1 The first step is to define the function. Press:

expr-eval table … select **Add/Edit Function**

Enter the function $f(x) = x^2$ and then press:

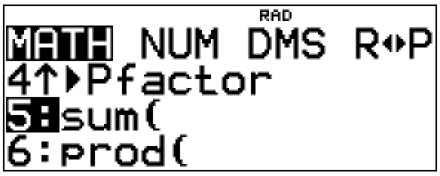
enter … now quit from the function menu.



2 The calculator's summation tool can be used for simple, integer increments for Δx. Press:

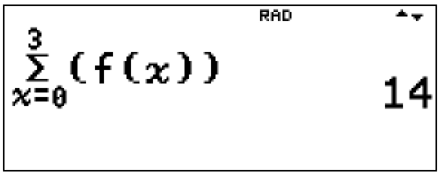
math ▲ 2

Pressing the up arrow will start the selection from the bottom of the menu. Select option **5 – Sum(**



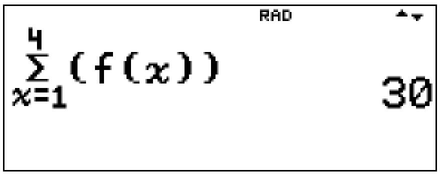
3 To calculate the approximate area using the inner bound rectangles calculate the sum from x = 0 and x = 3.

Use: expr-eval table … option 2 to access $f(x)$ and populate the template.

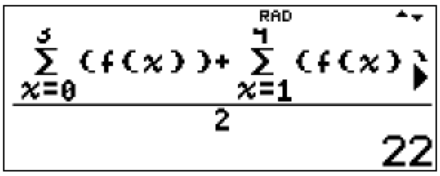


4 Use the arrow keys ▲ to select the previous calculation.

Press enter to paste the calculation. Use the arrow keys to navigate to the parts of the expression that need updating.



5 Averaging the previous two estimates equates to a series of trapezia, and therefore the trapezoidal rule. Use the copy and paste feature (as above).



The summation tool use integer increments. It is possible to manipulate the expression to work for non-integer values. In this example start at x = 0 and go to x = 39, calculate $f(x/10)$ then multiply by the column width 0.1. Similarly, start at x = 1 through to x = 40, however the calculator lists is a better option here. (Refer Part 2)

Simpson's Rule

Simpson's rule fits a quadratic function rather than a straight line for each interval. The 'general' formula for an interval x_0 to x_n is given below:

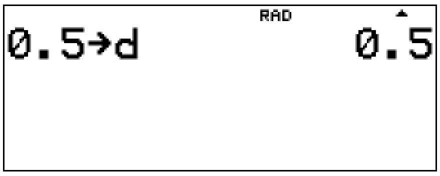
$$S_n = \frac{\Delta x}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \ldots 4f(x_{n-2}) + 2f(x_{n-1}) + f(x_n) \right)$$

It is possible to use a range of calculator commands to ensure that coefficients of the odd terms is a four (4) and the coefficients of the even terms is two (2), however it makes more sense to follow the logic of the formula.

On the calculator use *d* to store the value of Δx.

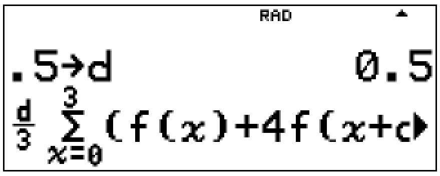
$$\frac{d}{3}\sum_{x=0}^{3}\left(f(x) + 4f(x+d) + f(x+2d) \right) = \frac{d}{3}\left(f(0) + 4f(\tfrac{1}{2}) + f(1) \right) + \left(f(1) + 4f(\tfrac{3}{2}) + f(2) \right) + \ldots$$

1. In this case d = 0.5. Press:

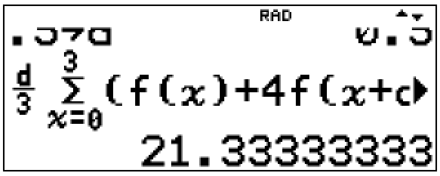


2. Insert the summation template as before, then enter Simpson's rule as shown below (and opposite).

The entire formula is: $\sum_{x=0}^{3}\left(f(x) + 4f(x+d) + f(x+2d) \right)$



3. Since the function in this example is quadratic, Simpson's rule generates the exact area.



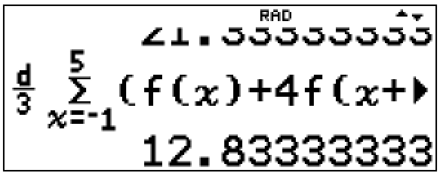
Sample Problem

Determine the area bounded by the curve: $f(x) = (x-1)(x-4)$, the *x* axis and the lines *x* = -1 and *x* = 6.

Sample Problem – Solutions

As the function is quadratic, Simpson's rule will generate the exact answer, however the function is above and below the *x* axis over the nominated interval.

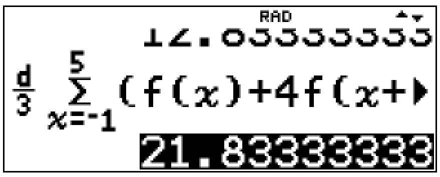
If students do not take the positive / negative regions of the graph into account, they will determine the definite integral: 12.833. The integral could be broken up into three regions, however there is a much more efficient approach



Store the function in $g(x)$ and make $f(x) = |g(x)|$.

The absolute value function is located in the Maths menu:
Math > **Num** > **Abs(**

Now the previous calculation can be retrieved and the answer generated in much less time.



Approximate Areas: Part 2

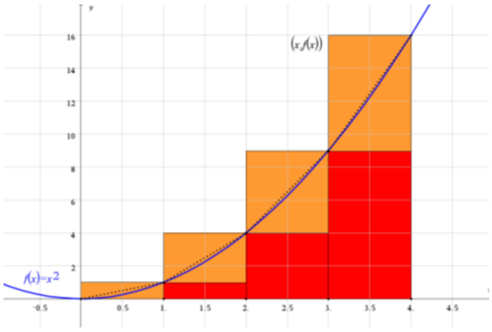
For graphs other than linear and quadratic, it is generally desirable to include more intervals; one of the concepts students need to understand is the notion of the limit, column widths approaching zero (0). To help develop this concept, it is best to incorporate lists.

Formula:

$$\int_a^b f(x)dx \approx \sum_a^b f(x)\Delta x$$

Scan the QR code to watch the video. The video covers aspects of all three sections on approximate areas.

The graph opposite shows left (lower) and right (upper) bound rectangles that can be used to approximate the area under the curve $f(x) = x^2$. The result can the be averaged to determine the approximate area using trapezia.

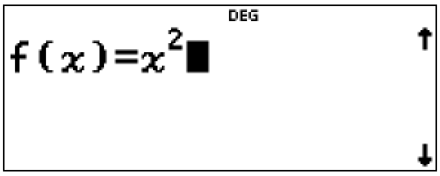


❶ The first step is to define the function. Press:

expr-eval
table … select **Add/Edit Function**

Enter the function $f(x) = x^2$ and then press:

enter … now quit from the function menu.



❷ The calculator's lists can be used to perform multiple calculations quickly and efficiently. Make sure the lists are cleared and ready for the calculations.

stat-reg/distr stat-reg/distr
data data … clear lists as necessary.

❸ The next step is to generate a list of the x values that form the boundary conditions. For small lists, the entries can be done manually, for larger lists, use the sequence command.

stat-reg/distr
data navigate to **OPS** and select **Sequence**

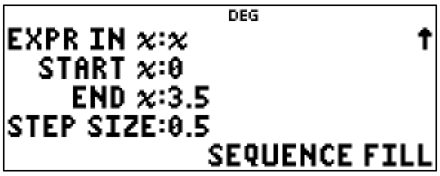
The sequence of numbers will go in List 1 (L1)

❹ The sequence formula is simply: x

To produce left bound rectangles of width 0.5, the x-values need to be: 0, 0.5, 1, 1.5, 2, 2.5, 3 and 3.5

Enter the settings as shown opposite to produce this result, then press:

enter …to generate the sequence (list).

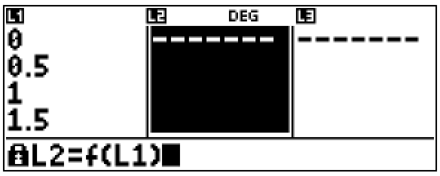


❺ Navigate across to List 2 (L2). Then press:

stat-reg/distr
data … select **Formula** then **Add/Edit Formula**

The **Table** key can be used to access functions and the **Data** key to access list names.

Match the formula shown opposite for the values in List 2.



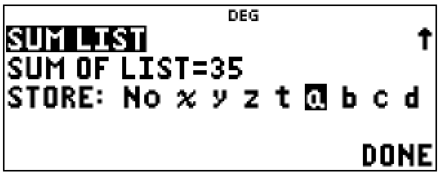
6. To sum all the values in List 2, press:

… select **Ops** then **Sum List**.

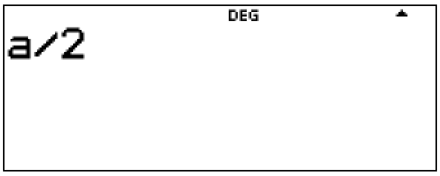
7. Make sure List 2 is selected for the sum.

The sum is displayed on screen (35 in this example) and can be stored in one of the calculator memories.

Store the result in 'a'.



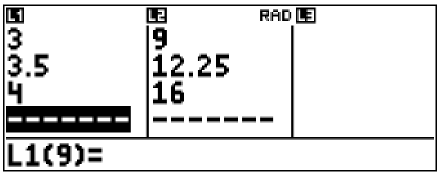
8. Recall that the columns in this example are only 0.5 units wide, so the area is equal to: $a \div 2$.



9. The right bound rectangles can be determined easily by returning to the lists.

Delete the first entry in List 1 (0). Press the up arrow to quickly navigate to the bottom of the list and enter a 4.

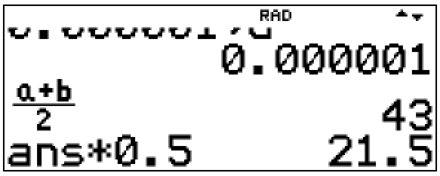
Notice that list 2 is automatically populated.



10. Calculate the sum of the items in List 2. This time, store the result in 'b'.

Once this operation is completed, return to the calculator home screen and evaluate (a + b) ÷ 2, then multiply by the column width, in this case 0.5.

The approximate area under the curve is 21.5, reasonably close to the integral.



Sample Problem

Use inner and outer bound rectangles and trapezia to determine the approximate area bounded by the curve $f(x) = \ln(x)$ between $x = 1$ and $x = 3$ using 20 intervals.

Sample Problem – Solutions

Inner bound = 1.24035; Outer bound: 1.35024; Trapezia: 1.29528. [Actual: 3ln(3) – 2 ≈ 1.2958]

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