



Math Objectives

- Students will identify a system of equations in augmented matrix form.
- Students will determine which row operation is used that transforms one matrix into another.
- Students will determine which row operations are needed to transform the matrix to its reduced row-echelon form.

Vocabulary

- augmented matrix
- reduced row-echelon form
- scalar multiplier

About the Lesson

- This lesson involves using row operations to reduce an augmented matrix to its reduced row-echelon form.
- As a result, students will:
 - Enter the coefficients of a system into an augmented matrix.
 - Choose the correct row operations that result in the reduced row-echelon form of the matrix.
 - Translate the answer matrix into a solution of the system, algebraically.

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- Use Screen Capture and Live Presenter to demonstrate the process of entering a system into its augment matrix and using row operations to obtain the reduced row-echelon form of the matrix.
- Use Quick Polls to check student understanding of key concepts.

TI-Nspire™ Technology Skills:

- Create a new document
- Insert pages
- Use the **MENU** button to access row operation commands

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.

Lesson Materials:

Student Activity
Solving_Systems_Using_
Row_Operations_1_Student.pdf
Solving_Systems_Using_Row_
Operations_1_Student.doc

Visit www.mathnspired.com for lesson updates.



Discussion Points and Possible Answers

Tech Tip: The calculator page is being used to perform row operations to solve a system of three equations and three unknowns. To ensure exact answers, always multiply a row by integers or fractions. Terminating decimals can also be used. Once a value with a decimal point has been introduced into the calculations, all values will be stored in 13-digit accuracy. This means that $\frac{2}{3}$ will now be stored as 0.6666666666667, which is not exact.

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<p>Use <i>Live Presenter</i> to demonstrate selecting the matrix template and entering the matrices' elements. At any time, use <i>Live Presenter</i> to have students present their solutions to each step.</p>
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Teacher Tip: After solving systems of equations (two equations–two unknowns) algebraically and interpreting the results, you may use this lesson to look at a number of more complicated systems.

The three usual methods of solving a system of equations are graphing, elimination, and substitution. While these methods are excellent, they can be difficult to use when dealing with three or more variables. The row reduction method provides an alternative process. It requires only one matrix and extends the method of elimination.

When solving a system of equations, the system can be written as an augmented matrix. An augmented matrix is a single matrix containing coefficients and a final column for constants.

$$\begin{array}{l}
 2x + 4y - 3z = 29 \\
 \text{For example, } 6x - 2y + 5z = 3 \\
 -7x + 3y + z = -7
 \end{array}
 \quad \text{can be written as} \quad
 \left[\begin{array}{ccc|c}
 2 & 4 & -3 & 29 \\
 6 & -2 & 5 & 3 \\
 -7 & 3 & 1 & -7
 \end{array} \right].$$

The row reduction method, also known as the reduced row-echelon form and the Gaussian Method of Elimination, transforms an augmented matrix into a solution matrix.

The solution of this system can be written as an augmented matrix in reduced row-echelon form.

$$\text{The solution matrix} \left[\begin{array}{ccc|c}
 1 & 0 & 0 & 3 \\
 0 & 1 & 0 & 5 \\
 0 & 0 & 1 & -1
 \end{array} \right] \text{ can be written as} \quad
 \begin{array}{l}
 1x + 0y + 0z = 3 \\
 0x + 1y + 0z = 5 \\
 0x + 0y + 1z = -1
 \end{array}.$$



1. Why is the goal of the row reduction method to have the solution matrix have only 1s in the diagonal of the first 3 columns and zeros for the rest of the numbers in the first 3 columns? What do the constants in the final column represent?

Answer: The diagonal of 1s represent the coefficients of the different variables. The zeros are the coefficients also, but they make that term's value zero. Another way to look at it is to separate the

$$\text{solution matrix: } \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix}$$

Matrix Row Operations

To transform augmented matrices into their reduced row-echelon form, a few rules called **row operations** need to be maintained. When dealing with a matrix, rules allow you to:

- Switch the rows of a matrix
- Multiply a row by a nonzero number
- Multiply a row by a nonzero number and add it to another row

For example, in the matrices listed below, the original matrix becomes the transformed matrix using the row operation $3 \cdot \text{row } 1 + \text{row } 2$.

<u>original matrix</u>		<u>transformed matrix</u>
$\begin{bmatrix} 7 & 1 & 0 & 1 \\ 5 & 3 & -1 & 9 \\ -2 & -8 & 4 & 6 \end{bmatrix}$	\rightarrow	$\begin{bmatrix} 7 & 1 & 0 & 1 \\ 26 & 6 & -1 & 12 \\ -2 & -8 & 4 & 6 \end{bmatrix}$

2. Write the correct row operation that transforms the original matrix into its transformed matrix. Both the augmented matrix and the corresponding system of equations are listed to aid in this task.

<u>original matrix</u>	<u>transformed matrix</u>	<u>original system</u>	<u>transformed system</u>
a. $\begin{bmatrix} 2 & 4 & -3 & 2 \\ 5 & -2 & -1 & 1 \\ -3 & -1 & 2 & 4 \end{bmatrix}$	\rightarrow $\begin{bmatrix} 2 & 4 & -3 & 2 \\ 10 & -4 & -2 & 2 \\ -3 & -1 & 2 & 4 \end{bmatrix}$	$2x + 4y - 3z = 2$ $5x - 2y - z = 1$ $-3x - y + 2z = 4$	$2x + 4y - 3z = 2$ $10x - 4y - 2z = 2$ $-3x - y + 2z = 4$

Answer: $2 \cdot \text{row } 2$



$$b. \begin{bmatrix} 2 & 4 & -3 & 2 \\ 5 & -2 & -1 & 1 \\ -3 & -1 & 2 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 5 & -2 & -1 & 1 \\ 2 & 4 & -3 & 2 \\ -3 & -1 & 2 & 4 \end{bmatrix} \quad \begin{array}{l} 2x+4y-3z=2 \quad 5x-2y-z=1 \\ 5x-2y-z=1 \rightarrow 2x+4y-3z=1 \\ -3x-y+2z=4 \quad -3x-y+2z=4 \end{array}$$

Answer: Switch rows 1 and 2

$$c. \begin{bmatrix} 2 & 4 & -3 & 2 \\ 5 & -2 & -1 & 1 \\ -3 & -1 & 2 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 4 & -3 & 2 \\ 1 & -10 & 5 & -3 \\ -3 & -1 & 2 & 4 \end{bmatrix} \quad \begin{array}{l} 2x+4y-3z=2 \quad 2x+4y-3z=2 \\ 5x-2y-z=1 \rightarrow x-10y+5z=-3 \\ -3x-y+2z=4 \quad -3x-y+2z=4 \end{array}$$

Answer: $-2 \cdot \text{row } 1 + \text{row } 2$

Teacher Tip: For this method to work, the variables must all be in the same order before being put into the matrix.

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Tell students that you are going to send *Quick Polls (Open Response)* asking for their answer to question 3a and throughout the lesson.

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Use *Live Presenter* for the remaining questions to help demonstrate the menu navigation to students as well as how to enter the correct syntax for each desired row operation. Some students might also see other ways to get to the same result. Encourage this and display it.

Next, solve the system below by transforming it into reduced row-echelon form by following the following steps.

$$2x - 4y + 2z = 14$$

$$3x - 5y + z = 14$$

$$2x - 2y - z = 3$$

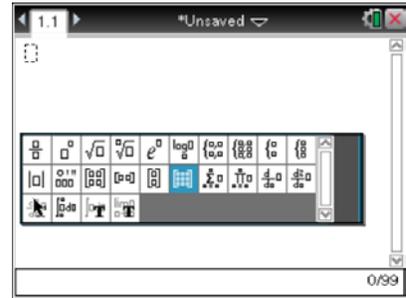
Tech Tip: To move across fields, press tab. To change a value in a field, type the number. To move across elements in a matrix, press the TouchPad arrows or tab.

Teacher Tip: If students enter incorrect values, remind them to use ctrl Z to undo the last action.

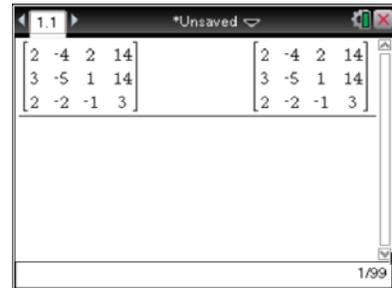


Entering the Matrix

- Press **on** > **New Document** > **Add Calculator**.
- Press **⌘** and select the matrix template.
- Create a matrix with three rows and four columns. Press **tab**, change the number of rows to 3, press **tab**, change the number of columns to 4, press **tab** to select **OK**, and press **enter**.



- Enter the matrix element values for the system $3x - 5y + z = 14$.
 $2x - 4y + 2z = 14$
 $2x - 2y - z = 3$



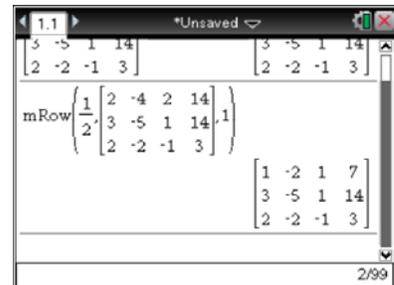
Transforming Column 1

- To solve the system, transform the matrix into reduced row-echelon form by establishing 1s on the main diagonal and zeros in every other position. To do this, transform element (1,1) to 1. Then transform the remaining column values to zero.
 - What multiplication needs to be done to row 1 so that the element in the first row, first column, hereafter using the notation (r,c) is 1? Why do you want this element to be a 1?

Answer: $\frac{1}{2}$; this element needs to be a 1 to represent 1x.

Teacher Tip: In the questions that follow, the **mRow** and **mRowAdd** commands are used. Both of these commands use the term *scalar multiplier*. A scalar multiplier is a value multiplied to some term, expression, or equation. For example, multiplying the equation $2x + 3y = 5$ by the scalar multiplier -2 , results in $-4x - 6y = -10$.

- Multiply row 1 by the scalar multiplier identified above. Press **Menu** > **Matrix & Vector** > **Row Operations** > **Multiply Row**. The correct syntax for the **Multiply Row** command is **mRow**(scalar multiplier, matrix, row number). Enter each value as shown. To copy and paste a previous matrix into a row operation command, use the TouchPad or the arrows to select a matrix in a previous entry line and press **enter**.





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Use *Quick Poll (Open Response)* for students to share their answers to any questions. It will be very helpful to use *Screen Capture* to highlight the correct syntax and common syntax errors.

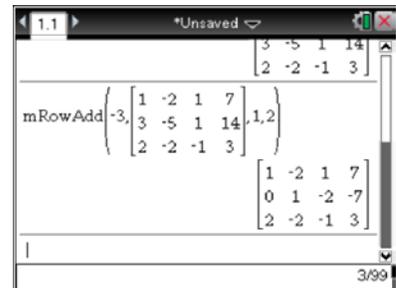
4. After transforming element (1,1) to 1, all other values in the column must be transformed to zero.
 - a. Once a 1 is established in the column, why must all the other values in the column be zero?

Answer: Remember that you are using row operations and augmented matrices to solve a system of equations. If the solution to a variable is in a row, then that variable must not occur in the other rows, thus the zeros in their location.

- b. What multiplication needs to be done to row 1 so when it is added to row 2, element (2,1) is transformed to zero? Explain why this works.

Answer: -3 because $-3 \cdot 1 + 3 = 0$

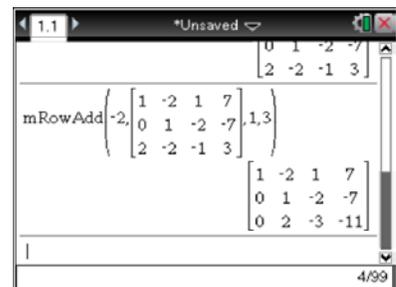
- c. Perform the **Multiply Row & Add** command by pressing **Menu > Matrix & Vector > Row Operations > Multiply Row & Add**. The correct syntax after selecting the **Multiply Row & Add** command is **mRowAdd**(Scalar multiplier, matrix, row being multiplied, row being added to).



- d. What row operation is necessary to transform element (3,1) to zero?

Answer: $-2 \cdot \text{row } 1 + \text{row } 3$

- e. Perform the **Multiply Row & Add** command by pressing **Menu > Matrix & Vector > Row Operations > Multiply Row & Add** and enter the correct syntax.





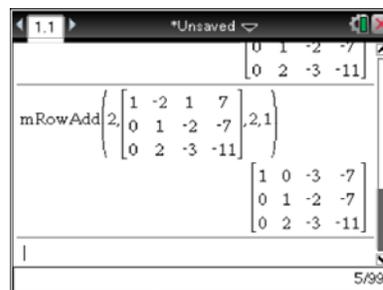
Transforming Column 2

5. Continue establishing 1s along the main diagonal with zeros in every other position. Since the element (2,2) is already 1, transform the other values in the column to zero. After establishing the 1 in the column, all the other values in the column need to be zeros.
- Ann says that in order to transform element (1,2) to zero, she needs to multiply row 2 by 2 and add it to row 1. Pat says that in order to transform element (1,2) to zero, he needs to multiply row 3 by 1 and add it to row 1. Who is correct?

Answer: Both are correct.

Teacher Tip: Depending on the students' choice, students may also want to use the **Row Add** feature instead of multiplying by 1.

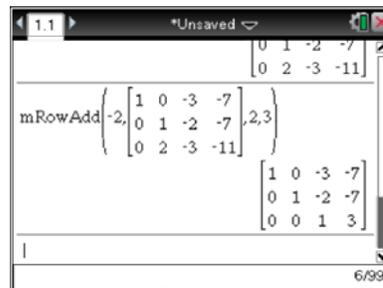
- Perform the **Multiply Row & Add** command by pressing **Menu > Matrix & Vector > Row Operations > Multiply Row & Add** and entering the correct syntax.



- What row operation is necessary to transform element (3,2) to zero?

Answer: $-2 \cdot \text{row } 2 + \text{row } 3$

- Again press **Menu > Matrix & Vector > Row Operations > Multiply Row & Add** and enter the correct syntax.



Transforming Column 3

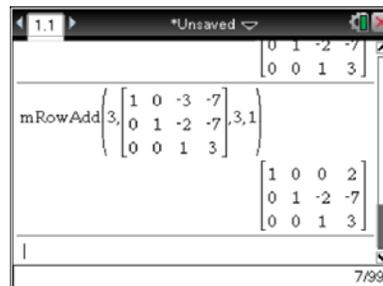
6. Continue establishing 1s along the main diagonal with zeros in every other position. Since the element (3,3) is already 1, transform the other values in the column to zero. After establishing the 1 in the column, all the other values in the column need to be zeros.



- a. What row operation is necessary to transform element (1,3) to zero?

Answer: $3 \cdot \text{row } 3 + \text{row } 1$, or if students chose Pat's answer earlier, the answer could be $2 \cdot \text{row } 3 + 1$.

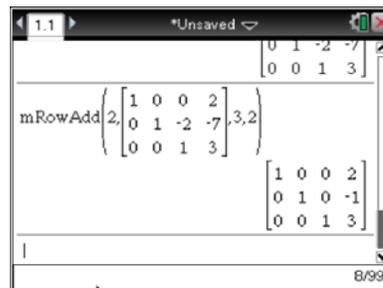
- b. Perform the **Multiply Row & Add** command by pressing **Menu > Matrix & Vector > Row Operations > Multiply Row & Add** and entering the correct syntax.



- c. What row operation is necessary to transform element (2,3) to zero? Is this the only way?

Answer: $2 \cdot \text{row } 3 + \text{row } 2$ to change only the element (2,3) to zero and not change the other 1s or zeros; yes, this is the only way.

- d. Perform the **Multiply Row & Add** command by pressing **Menu > Matrix & Vector > Row Operations > Multiply Row & Add** and entering the correct syntax.



Interpreting the Augmented Matrix

7. Now that the augmented matrix is in its reduced row-echelon form, what is the solution to this system of equations?

Answer: $x = 2$, $y = -1$ and $z = 3$, or the point $(2, -1, 3)$

8. Verify the solution by substituting your solution into the original system of equations manually or using the calculator application.

Answer: $2(2) - 4(-1) + 2(3) = 14$, $14 = 14$
 $3(2) - 5(-1) + 3 = 14$, $14 = 14$
 $2(2) - 2(-1) - 3 = 3$, $3 = 3$



Solving Another System

9. Use row operations to transform the system of equations below to its reduced row-echelon form. State and verify the solution.

$$5x - 2y + z = 6$$

$$-3x + 4y + 2z = 5$$

$$2x + 3y - 5z = 23$$

Step 1

Calculator screen showing the initial augmented matrix:

$$\left[\begin{array}{ccc|c} 5 & -2 & 1 & 6 \\ -3 & 4 & 2 & 5 \\ 2 & 3 & -5 & 23 \end{array} \right]$$

$1/5 * \text{row 1}$

Step 2

Calculator screen showing the matrix after the first row is scaled by 1/5:

$$\left[\begin{array}{ccc|c} 1 & -2/5 & 1/5 & 6/5 \\ -3 & 4 & 2 & 5 \\ 2 & 3 & -5 & 23 \end{array} \right]$$

$3 * \text{row 1} + \text{row 2}$

Step 3

Calculator screen showing the matrix after row 2 is updated with 3 times row 1:

$$\left[\begin{array}{ccc|c} 1 & -2/5 & 1/5 & 6/5 \\ 0 & 14/5 & 13/5 & 43/5 \\ 2 & 3 & -5 & 23 \end{array} \right]$$

$-2 * \text{row 1} + \text{row 3}$

Step 4

Calculator screen showing the matrix after row 3 is updated with -2 times row 1:

$$\left[\begin{array}{ccc|c} 1 & -2/5 & 1/5 & 6/5 \\ 0 & 14/5 & 13/5 & 43/5 \\ 0 & 19 & -27 & 103 \end{array} \right]$$

$5/14 * \text{row 2}$

Step 5

Calculator screen showing the matrix after row 2 is scaled by 5/14:

$$\left[\begin{array}{ccc|c} 1 & 0 & 4/7 & 17/7 \\ 0 & 1 & 13/14 & 43/14 \\ 0 & 19 & -27 & 103 \end{array} \right]$$

$2/5 * \text{row 2} + \text{row 1}$

Step 6

Calculator screen showing the matrix after row 1 is updated with 2/5 times row 2:

$$\left[\begin{array}{ccc|c} 1 & 0 & 4/7 & 17/7 \\ 0 & 1 & 13/14 & 43/14 \\ 0 & 0 & -125/14 & 125/14 \end{array} \right]$$

$-19/5 * \text{row 2} + \text{row 3}$

Step 7

Calculator screen showing the matrix after row 3 is updated with -19/5 times row 2:

$$\left[\begin{array}{ccc|c} 1 & 0 & 4/7 & 17/7 \\ 0 & 1 & 13/14 & 43/14 \\ 0 & 0 & 1 & -1 \end{array} \right]$$

$-14/125 * \text{row 3}$

Step 8

Calculator screen showing the matrix after row 3 is scaled by -14/125:

$$\left[\begin{array}{ccc|c} 1 & 0 & 4/7 & 17/7 \\ 0 & 1 & 13/14 & 43/14 \\ 0 & 0 & 1 & -1 \end{array} \right]$$

$-13/14 * \text{row 3} + \text{row 2}$

Step 9

Calculator screen showing the matrix after row 2 is updated with -13/14 times row 3:

$$\left[\begin{array}{ccc|c} 1 & 0 & 4/7 & 17/7 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -1 \end{array} \right]$$

$-4/7 * \text{row 3} + \text{row 1}$



Wrap Up

Upon completion of the discussion, the teacher should ensure that students know how to use row operations to transform an augmented matrix to its reduced row-echelon form.

Solving Systems Using Row Operations 2 has students use reduced row-echelon form to continue to explore the properties of systems.

Assessment

$$3x + 2y - 5z = -11$$

1. Use row operations to solve $-x - 3y + z = -10$.

$$4x - y + 2z = -7$$

Answer: $(x = -2, y = 5, z = 3)$

$$2x - 4y + 2z = -2$$

2. Use row operations to solve $3x + 5y + 2z = 20$.

$$5x + 2y - 3z = 27$$

Answer: $(x = 4, y = 2, z = -1)$

$$2w - 3x - 2y - z = 1$$

3. Use row operations to solve $w + 5x - y + 2z = 2$.

$$3w - 3x + 4y + 2z = 17$$

$$-2w + x + 2y - 3z = 5$$

Answer: $(w = 4, x = 1, y = 3, z = -2)$