Simple Harmonic Motion

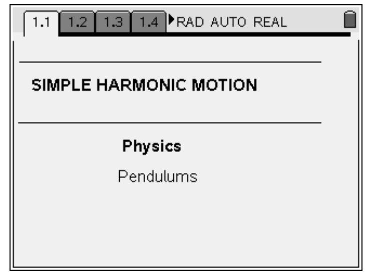
ID: 9461

Name ______________

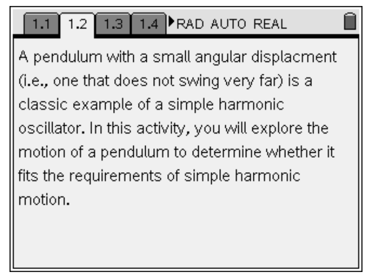
Class ______________

In this activity, you will explore the following:

- *the characteristics of the motion of a pendulum*
- *how well the motion of a pendulum fits the definition of simple harmonic motion*



Open the file **PhyAct17_Simple_Harmonic_Motion_EN.tns** on your handheld or computer, and follow along with your teacher for the first two pages. Move to page 1.2 and wait for further instructions from your teacher.

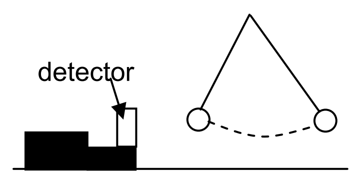


The displacement of a simple harmonic oscillator is directly proportional to the force acting on it. Pendulums are classic examples of simple harmonic oscillators. However, a pendulum acts as a simple harmonic oscillator only when its angular displacement is small (that is, when it does not swing very far).

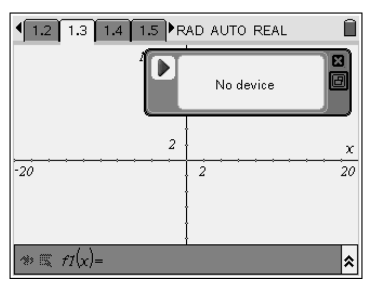
In this activity, you will collect data on the displacement of a pendulum over time. Then, you will analyze the data to determine whether the motion of the pendulum fits the requirements of simple harmonic motion.

Part 1: Collecting displacement data

Step 1: Move to page 1.3. Put on a pair of safety goggles. Insert a new data collection box by pressing ctrl D. Connect a Vernier CBR 2™ or Go!™Motion motion sensor to your handheld or computer. The light on the sensor should come on, and the display in the data collection box on the screen should show the current distance between the sensor and whatever object is in front of it. You will know the motion sensor is functioning correctly when you hear it make a soft, slow clicking sound.

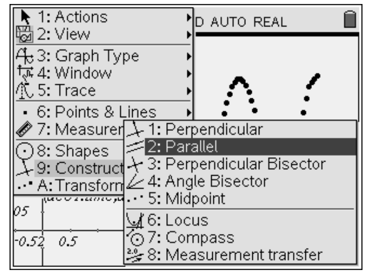


Step 2: Place the CBR 2 on the desk or the floor. Ideally, the motion sensor should about 40 cm from the pendulum when the pendulum is at rest. Open the cover of the sensor, and position the light gray portion so that it is perpendicular to the ground, as shown to the right. Practice swinging the pendulum in front of the sensor until you can swing it without hitting the sensor or getting more than a few centimeters away from it.



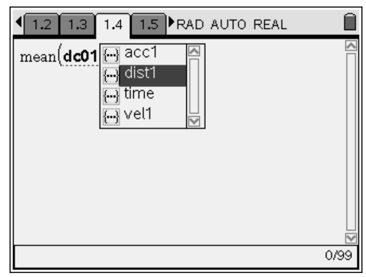
Step 3: Before you begin to collect data, note and record the distance to the pendulum. Pull the pendulum back approximately 20–30 cm and release it. Once it is swinging, press the "play" (►) button to start the data collection. The motion sensor will start to click rapidly. When the sensor stops clicking rapidly, the data collection has finished. View your graph of distance vs. time. (To view your distance graph, press ctrl tab to switch to the *Graphs & Geometry* view. Change the plot type to scatter plot and plot distance vs. time. Adjust the window settings if necessary.) It should look smooth, with no large gaps or horizontal lines. If it does not, repeat the data collection procedure by pressing the ► button again and overwriting the previous set of data. When the data you have collected make a smooth graph, close the data collection box, and disconnect the motion sensor from your handheld or computer.

Part 2: Calculating the force on the pendulum



Step 1: Next, you are to analyze your collected data to find the distance the pendulum is from the CBR2 when the pendulum is at rest (i.e., vertical). First, you will determine the rest position graphically. Your displacement data graph should look like a sine curve. Graphically, the rest position is the horizontal line through the sine curve that the pendulum is above half of the time and below the other half of the time. To determine the rest position graphically, insert a line parallel to the *x*-axis (**Menu > Construction > Parallel**) that runs through the rest position of the curve, as shown to the right. To determine the rest position of the pendulum, draw a point on the parallel line (**Menu > Points & Lines > Point On**) and identify its coordinates using the **Coordinates and Equations** tool (**Menu > Actions > Coordinates and Equations**).

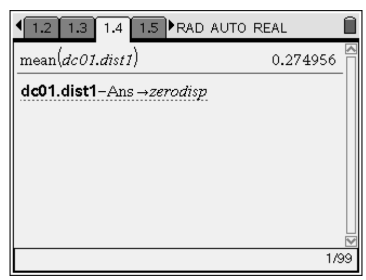
Q1. Based on your graph, what is the rest position of your pendulum?



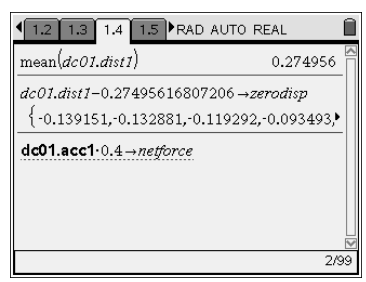
Step 2: Next, you will calculate the rest position numerically. To do this, move to the *Calculator* application on page 1.4. The TI-Nspire has a function, **mean,** that will calculate the average of a set of data. The rest position of the pendulum is equal to the average of its displacement values. Therefore, to calculate the rest position, you must find the mean of the displacement values. Type **mean(dc01.** into the *Calculator* application. A drop-down menu of data in the run0 data set should appear. Select **dist1** from the list and press enter. The mean of the data should be displayed.

Q2. What is the calculated rest position of your pendulum?

Q3. Compare the rest position you obtained graphically with the one you calculated. Comment on any differences. Which value is more accurate? Explain your answer.



Step 3: To use the data you have collected to analyze the motion of the pendulum, you must force the rest position to be zero. To do this, you must subtract the rest position from the displacement values of all the data points you collected. To do this, first type **dc01.** into the *Calculator* application and select **dist1**. Then, press − ctrl (−) to subtract the rest position from the data set. Then, press ctrl ∞β° to open the symbols menu, and select the right arrow (→) from the menu. Finally, type **zerodisp**. Press enter to assign the re-centered data to the variable **zerodisp**.

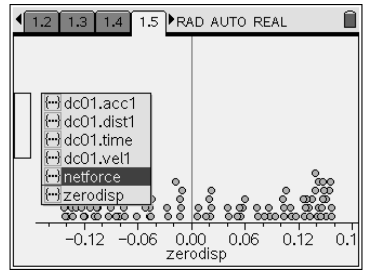


Step 4: The CBR2 recorded data on the acceleration of the pendulum with time. These data are stored in the data set **dc01.acc1**. To convert the acceleration data to force data, you can use the equation $F = ma$, where F is force, m is mass, and a is acceleration. Use the balance to measure the mass of your pendulum. Then, use the *Calculator* application on page 1.4 to define the variable **netforce**. The screenshot to the right shows how to do this. Note that in your calculations, you should substitute the mass of your pendulum (in kilograms) for the value 0.40 in the screenshot.

Q4. What is the mass of your pendulum in kilograms?

Part 3: Analyzing the pendulum's motion

Step 1: Move to page 1.5, which contains a *Data & Statistics* application. Plot **netforce** vs. **zerodisp** on this page. Then, use the **Linear Regression** tool **(Menu > Analyze > Regression > Show Linear (mx + b)**) to find the best-fit equation to the data.

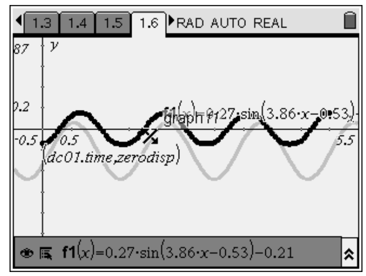


Q5. Is the displacement directly proportional to the net force?

Q6. What is the equation of the best-fit line you found?

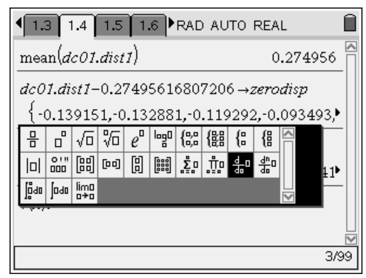
Q7. Based on these data, did your pendulum act as a simple harmonic oscillator (i.e., $F = -kx$) during this activity? Explain your answer.

Step 2: Next, you will attempt to fit mathematical equations to the data to determine whether the pendulum acted as a simple harmonic oscillator. Move to page 1.6. Make a scatter plot of **zerodisp** vs. **dc01.time**. Adjust the window settings so you can see your data clearly. Then, change the graph to a **Function** graph (**Menu > Graph Type > Function**). In the *f1(x)* function bar, enter sin(*x*) and press enter. The function *f1*(*x*) = sin(*x*) should now be displayed on the page. Use the NavPad to manually translate and dilate the function until it matches the data you collected.

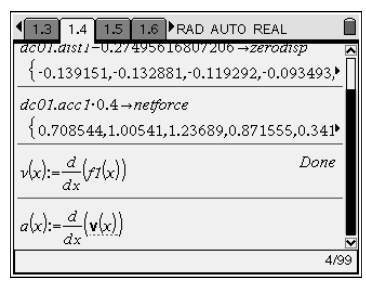


Q8. What is the best-fit equation for displacement vs. time for the data you collected?

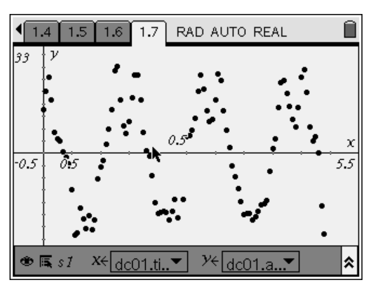
Step 3: Displacement, velocity, and acceleration are related by a calculus function called the derivative. Mathematically, the acceleration of an object as a function of time is equal to the second derivative of the object's displacement as a function of time. The TI-Nspire CAS contains programming that will allow it to calculate the second derivative for you. To use this functionality, move to the *Calculator* application on page 1.4. Type **v(x):=**. Make sure you type **:=**, not just **=.** Press ctrl templates to open the templates menu, and select the $\frac{d}{d\square}\square$ template. Enter **x** in the box in the denominator of the template and **f1(x)** in the box to the right of the template. Press enter. This defines the function **v(x)** as the first derivative of **f1(x)**. The function **v(x)** describes the velocity of the pendulum.



Step 4: Next, define the function **a(x)** as the second derivative of displacement. On page 1.4, type **a(x):=**. Then, press ctrl templates to open the templates menu, and select the $\frac{d}{d\square}\square$ template. Enter **x** in the box in the denominator of the template and **v(x)** in the large box to the right of the template. Then, press enter. This command defines the function **a(x)** as the first derivative of **v(x)** and therefore the second derivative of **f1(x)**. The function **a(x)** describes the acceleration of the pendulum.



Step 5: Move to page 1.7, which contains an empty *Graphs & Geometry* application. Make a scatter plot of **dc01.acc1** vs. **dc01.time**. Then, change the plot type to **Function,** and plot the function **a(x)** along with the scatter plot data.



Q9. Does the second derivative seem to model the acceleration data well?

Q10. Compare the displacement and acceleration equations (**f1(x)** and **a(x),** respectively). Based on these equations, did your pendulum exhibit simple harmonic motion? Explain your answer.

