The Logarithmic Derivative

ID: 9092

Time required
45 minutes

Activity Overview

Students will determine the derivative of the function y = ln(x) and work with the derivative of both y = ln(u) and y = log_a(u). In the process, the students will show that
$$\lim_{h\to 0}\frac{\ln(a+h)-\ln(a)}{h}=\frac{1}{a}.$$

Topic: Formal Differentiation

- *Derive the Logarithmic Rule and the Generalized Logarithmic Rule for differentiating logarithmic functions.*
- *Prove that $\ln(x)=\ln(a)\cdot\log_a(x)$ by graphing f(x) = log_a(x) and g(x) = ln(x) for some a and deduce the Generalized Rule for Logarithmic differentiation.*
- *Apply the rules for differentiating exponential and logarithmic functions.*

Teacher Preparation and Notes

- *This investigation derives the definition of the logarithmic derivative. The students should be familiar with keystrokes for the* ***limit*** *command, the* ***derivative*** *command, entering the both the natural logarithmic and the general logarithmic functions, drawing a graph, and setting up and displaying a table.*
- *Before starting this activity, students should go to the home screen and select* ***F6:Clean Up > 2:NewProb****, and then press* ENTER*. This will clear any stored variables, turn off any functions and plots, and clear the drawing and home screens.*
- *This activity is designed to be student-centered with the teacher acting as a facilitator while students work cooperatively.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "9092" in the keyword search box.***

Associated Materials

- *LogarithmicDerivative_Student.doc*

Suggested Related Activities

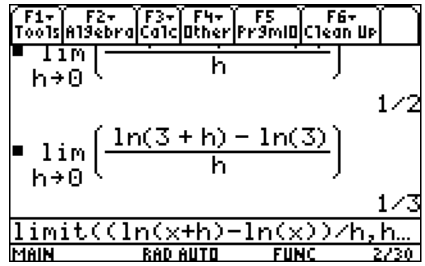
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Logging In (TI-89 Titanium) — 12180*
- *Implicit Differentiation (TI-89 Titanium) — 8969*
- *Investigating the Derivatives of Some Common Functions (TI-84 Plus family) —4368*

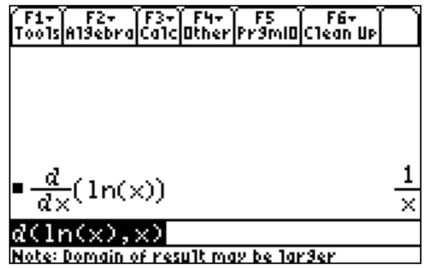
Problem 1 – The Derivative for $y = \ln(x)$

In this problem, students are asked to use the **limit** command (**F3:Calc > 3:limit(**) to find the values. Remind the students to be very careful of their parentheses.

The students should get $\frac{1}{x}$ for the answer to the last problem.

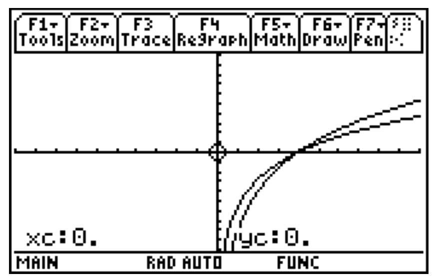


In this portion, the students are asked to use the **derivative** command (**F3:Calc > 1:*d*(differentiate**) to find the derivative.



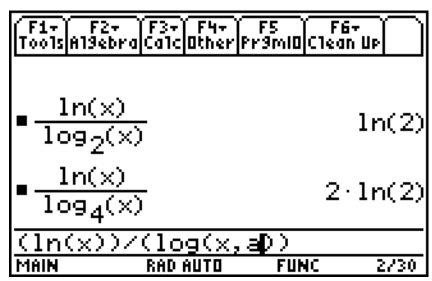
Problem 2 – The Derivative of $y = \log_a(x)$

Students should notice that both functions have a value of zero for $\ln(1)$ and $\log_2(1)$ but that $\log_2(x)$ is a multiple of $\ln(x)$. They both have about the same shape but the positive values of $\ln(x)$ are smaller than those of $\log_2(x)$.

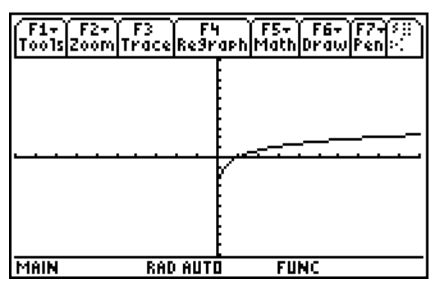


When $\log_4(x)$ is compared to $\ln(x)$, they also intercept at 1 but the positive values of $\ln(x)$ are larger than those of $\log_4(x)$.

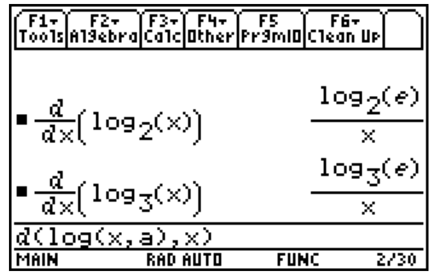
Have the students notice that $\ln(4)$ is listed as $2(\ln(2))$ instead of $\ln(4)$. The answer to the last problem is $\ln(a)$.



Students should graph the functions **y1** = $\ln(x)$, **y2** = $\ln(2) \cdot \log_2(x)$, **y3** = $\ln(3) \cdot \log_3(x)$. They should notice that all three functions yield the same graph. Students can check by graphing each one individually.



Students are asked to find the Generalized Logarithmic Rule for Differentiation. They should find $\frac{dy}{dx} = \frac{\log_a(e)}{x}$. Thus, if $y = \log_a(x)$, then $\frac{dy}{dx} = \frac{1}{(x\ln(a))}$.



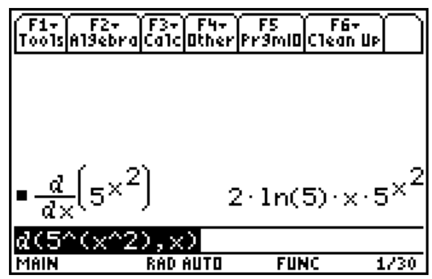
Problem 3 – Derivative of Exponential and Logarithmic Functions Using the Chain Rule

Students are asked to identify $u(x)$ and a for each function and then find the derivative by hand or using the **Derivative** command to find the derivative.

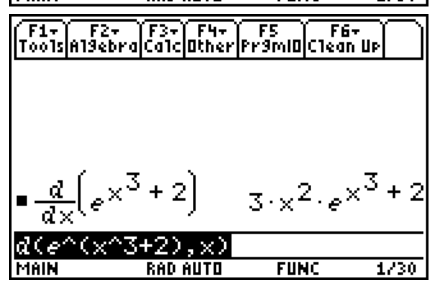
Recall: $y = a^u \rightarrow \frac{dy}{dx} = a^u \frac{du}{dx}$ where u depends on x.

- $y = \log_a(u) \rightarrow \frac{dy}{dx} = \frac{1}{(u\ln(a))} \cdot \frac{du}{dx}$ or $\frac{dy}{dx} = \frac{\frac{du}{dx}}{u(\ln(a))}$

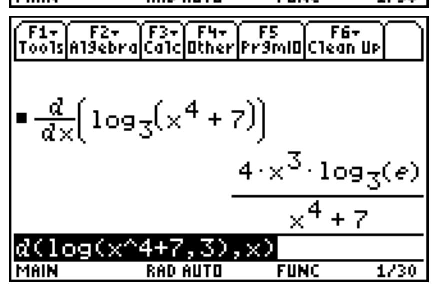
- $f(x) = 5^{(x^2)}$, $u(x) = x^2$, $a = 5$
  $f'(x) = 2 \cdot \ln(5) \cdot x \cdot 5^{(x^2)}$



- $g(x) = e^{(x^3+2)}$, $u(x) = x^3 + 2$, $a = e$
  $g'(x) = 3 \cdot x^2 \cdot e^{(x^3+2)}$



- $h(x) = \log_3(x^4 + 7)$, $u(x) = x^4 + 7$, $a = 3$
  $h'(x) = \frac{4 \cdot x^3 \log_3(e)}{x^4 + 7}$



- $j(x) = \ln\left(\sqrt{x^6 + 2}\right)$, $u(x) = \sqrt{x^6 + 2}$, $a = e$
  $j'(x) = \frac{3 \cdot x^5}{x^6 + 2}$