Teacher Notes

Activity 5

Average Rate of Change, Difference Quotients, and Approximate Instantaneous Rate of Change

Objectives

- Associate a meaning for right, left, and symmetric difference quotients that include rate of change and graphical interpretations
- Use symmetric difference quotients to approximate instantaneous rate of change

Materials

- TI-84 Plus / TI-83 Plus

Teaching Time

- 100 minutes

Abstract

In this activity, the average rate of change of a function between two points is defined and then used as a core concept connecting the ideas of slope, difference quotients, and approximations of instantaneous rates of change. Some questions require the student to use an algebraic approach, some require the student to use the graphing handheld for graphical exploration (tracing, finding intercepts, zooming), and some require the student to write a short paragraph explaining the reasoning behind the solution.

Management Tips and Hints

Prerequisites

Students should be able to:

- plot, trace, zoom, and find intersections on a graphing handheld.
- manipulate linear equations algebraically.

Evidence of Learning

- Students should be able to apply each of the three difference quotients to a wide variety of functions and interpret the results.

Common Student Errors/Misconceptions

- The precision of the terminology (average rate of change between two points, instantaneous rate of change at a point, and so on) is important and should be considered during this activity.

Extensions

Use algebraic reasoning to show that the symmetric difference quotient is the average of the right- and left-sided difference quotients.

Hint: *Look at Questions **9, 10**, and **11** for an example of this.*

Given the general quadratic function, $f(x) = ax^2 + bx + c$, use algebraic reasoning to show that the symmetric difference quotient yields $2ax + b$ regardless of the value of h that you use.

Activity Solutions

1. $\dfrac{9-1}{3-1} = 4$

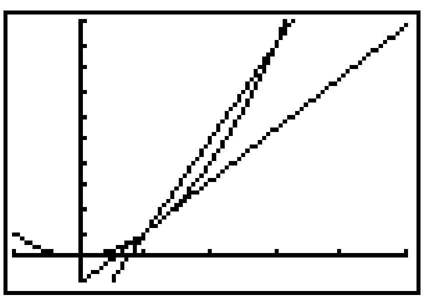
2. $y = 4x - 3$

3. (1, 1) and (3, 9)

4. $\dfrac{(1.2)^2 - 1^2}{1.2 - 1} = 2.2$

5. $y = 2.2x - 1.2$

6.



7. Judging from the graph, the average rate of change found in Question **4** is a better approximation for the instantaneous rate of change because the slope of the secant line connecting the points (1, 1) and (1.2, 1.44) is closer to the slope of the tangent line at (1, 1) than the slope of the secant line connecting the points (1, 1) and (3, 9). If you use the point (1, 1) and an h smaller than 0.2, then the approximation would be more precise (in this case).

Note: *Closer points do not always give a better approximation. For example, examine* $f(x) = x^3 - 2x^2 + 2$. *The average rate of change between (0, 2) and (2, 2) is closer to the instantaneous rate of change than to the average rate of change between (0, 2) and (1, 1).*

8. Right difference quotient

9. $h = 0.2$

10. Left difference quotient with $h = 2$

11. $\dfrac{(1.2)^4 - 1^4}{1.2 - 1} = 5.368$

12. $\dfrac{1^4 - (0.8)^4}{1 - 0.8} = 2.952$

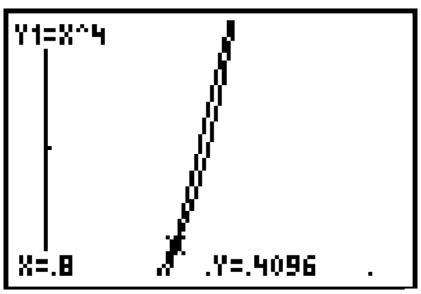
13. $\dfrac{(1.2)^4 - (0.8)^4}{1.2 - 0.8} = 4.16$

14. $y = 4.16x - 2.9184$

15. The symmetric difference quotient computed in Question **13** is the slope of the line found in Question **14**.

16. 0.4096.

The graphing handheld screen looks like this:



17. 2.0736

18. $\dfrac{(1.0001)^4 - (0.9999)^4}{1.0001 - 0.9999} = 4.00000004;\ 4$

19. $\dfrac{\sqrt{1.1 - 1} - \sqrt{1 - 1}}{1.1 - 1} \approx 3.162$

20. No. The domain of f is $x \geq 1$, and in this case, the symmetric difference quotient requires that f be defined for $x < 1$.