Living On The Edge

ID: 9410

Time required
45 minutes

Activity Overview

Students build a solution to a complex problem—finding the edge length of an octahedron given its volume—by solving two simpler problems. First, students find a formula for the edge length of a square given its area. Then, they write a formula with fractional exponents to find the edge length of a particular square. Finally, students solve an equation involving different radical expressions to find a formula for the edge length of an octahedron given its volume.

Topic: Rational & Radical Functions & Equations

- *Use technology to verify the equivalence of radical and fractional exponent representations of expressions.*
- *Graph radical functions and inequalities.*
- *Evaluate a radical function for any real value of its variable.*
- *Solve radical equations and inequalities algebraically and check for extraneous roots.*

Teacher Preparation and Notes

- *Prior to beginning this activity, students should have experience simplifying radical expressions, solving simple radical equations, and applying exponent rules.*
- *This activity requires students to graph functions and measure geometric shapes. If students have not had experience with these functions of the handheld, extra time should be taken to explain them.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "9410" in the keyword search box.***

Associated Materials

- *LivingOnTheEdge_Student.doc*
- *LivingOnTheEdge.tns*
- *LivingOnTheEdge_Soln.tns*

Suggested Related Activities

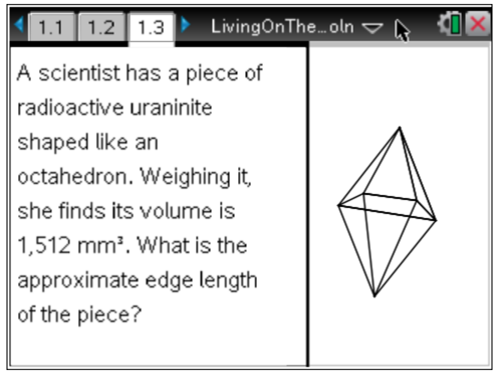
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Rational Exponents (TI-Nspire technology) — 11594*
- *Radicals (TI-84 Plus) —1915*

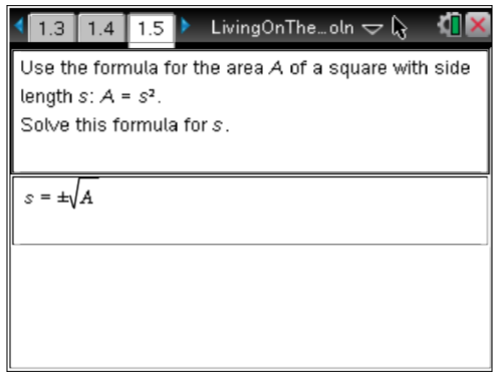
Problem 1 – Edge length of a square

Page 1.2 introduces students to the concept of this activity: working backwards from the area or volume of a regular shape or solid to find its edge lengths. Review the term *regular*. Students should understand why a regular shape or solid has only one edge length (unlike a rectangle for example, which has two) before proceeding.

Page 1.3 presents a real-world situation where this concept might be applied.

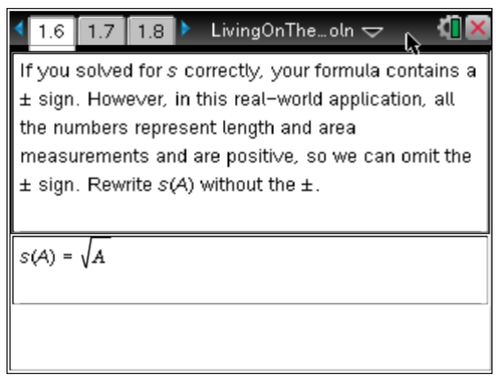


Page 1.4 suggests that students first solve a simpler problem involving the area of squares before tackling the problem about the volume of an octahedron. This helps students connect to material with which they are already familiar and reinforces a powerful problem solving strategy.

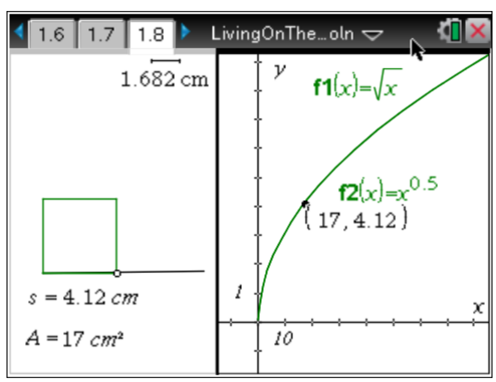


On page 1.5, students are prompted to solve the formula for the area of a square for its side length, *s*. Students should remember to include a ± sign in front of the radical when they take the square root. ($s = \pm\sqrt{A}$)

On page 1.6, students are reminded that they can drop the ± sign from their formulas since the numbers in this application will be nonnegative. (They all represent length and area measurements.)

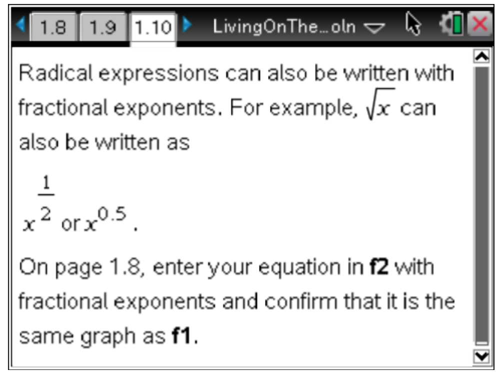


On page 1.8, students measure the side length and area of a model square, and graph their function $s(A)$ side-by-side. They can then compare points on the graph (using the **Graph Trace** feature) with the side length and area of a model square to confirm the accuracy of their model.

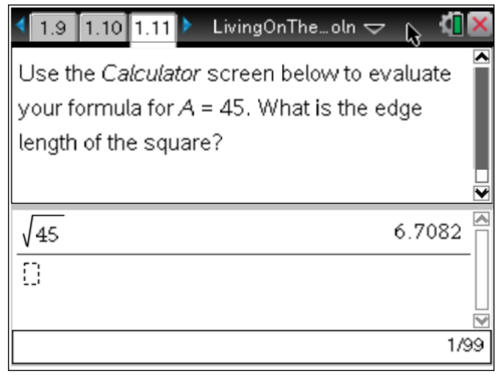


Optionally, you may direct students to graph the original area formula $A(s)$ on this same graph, replacing s with x, to see the inverse relationship between these two functions.

Page 1.10 introduces fractional exponents and prompts students to re-enter their formula with fractional exponents in **f2** to confirm that it is equivalent to the formula written with the radical sign.



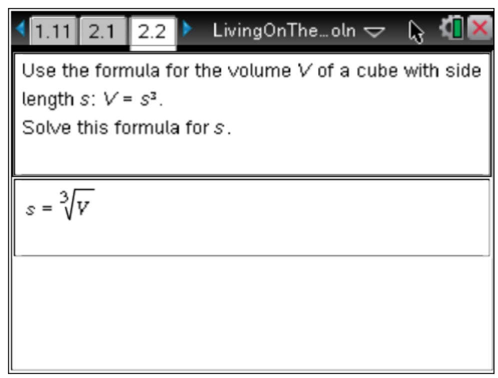
Students evaluate their function for $A = 45$ on page 1.11, arriving at the solution to the simpler problem ($s \approx 6.7802$ cm).

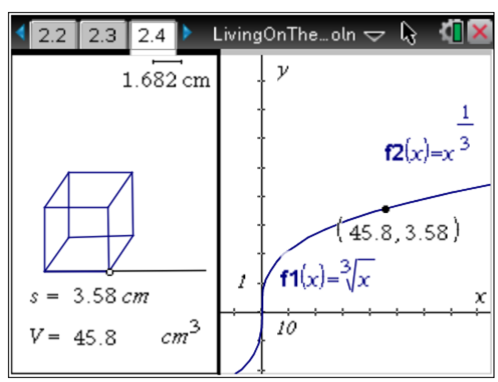


Remind students that this problem is a “proof of concept.” Although all these steps to test the model may seem unnecessary in this simple situation, it is important to make sure that the method used to solve this problem is correct because they will soon be using it in a more complex situation.

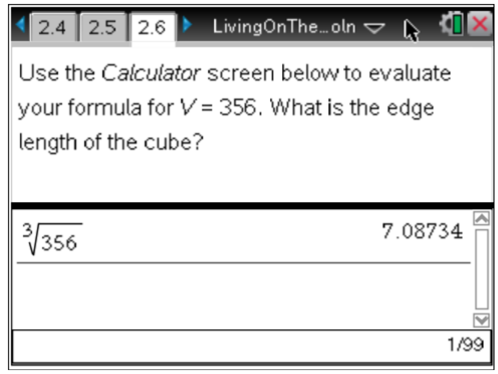
Problem 2 – Edge length of a cube

In this problem, students solve a slightly more complicated problem involving the volume of cubes. Students are given the formula for the volume of a cube and prompted to solve it for the side length, *s*. Again they test their formula by graphing it alongside a model cube and comparing points on the graph with the measurements of the cube. Students will again rewrite their formula with fractional exponents and establish that it is equivalent to the formula written with radical signs.





Finally, they evaluate the formula for $V = 356$ to solve the problem.



Problem 3 – Edge length of an octahedron

Now students apply what they have learned to solve the original problem. Solving the equation on page 3.3 involves manipulating several radical expressions, applying exponent rules, and rationalizing the denominator, as shown.

$$V = \frac{\sqrt{2}}{3}s^3$$

$$(V)^{\frac{1}{3}} = \left(\frac{\sqrt{2}}{3}s^3\right)^{\frac{1}{3}} = \left(\frac{\sqrt{2}}{3}\right)^{\frac{1}{3}} \cdot s$$

$$s = (V)^{\frac{1}{3}}\left(\frac{\sqrt{2}}{3}\right)^{-\frac{1}{3}} = (V)^{\frac{1}{3}}\left(\frac{2^{\frac{1}{2}}}{3}\right)^{-\frac{1}{3}} = (V)^{\frac{1}{3}}\left(\frac{3}{2^{\frac{1}{2}}}\right)^{\frac{1}{3}} = (V)^{\frac{1}{3}}\frac{3^{\frac{1}{3}}}{2^{\frac{1}{2}\cdot\frac{1}{3}}}$$

$$= (V)^{\frac{1}{3}}\frac{3^{\frac{1}{3}}}{2^{\frac{1}{6}}} = \frac{(3V)^{\frac{1}{3}}}{2^{\frac{1}{6}}} = \frac{(3V)^{\frac{1}{3}}}{2^{\frac{1}{6}}} \cdot \frac{2^{\frac{5}{6}}}{2^{\frac{5}{6}}} = \frac{2^{\frac{5}{6}}(3V)^{\frac{1}{3}}}{2} = \frac{\sqrt[6]{2^5}\sqrt[3]{3V}}{2}$$

Rather than graphing and comparing with a model octahedron, students check their algebra by substituting the original expression for V into their equation:

$$s = \frac{\sqrt[6]{2^5}\sqrt[3]{3V}}{2} = \frac{\sqrt[6]{2^5}\sqrt[3]{3\left(\frac{\sqrt{2}}{3}s^3\right)}}{2} = \frac{\sqrt[6]{2^5}\sqrt[3]{\frac{3\sqrt{2}}{3}}s}{2} = \frac{\sqrt[6]{2^5}\sqrt[3]{\sqrt{2}}s}{2} = \frac{\sqrt[6]{2^5}\sqrt[6]{2}s}{2} = \frac{\sqrt[6]{2^6}s}{2} = \frac{2s}{2} = s$$

Finally, students evaluate their equation for $V = 1{,}512$ to obtain the answer, 14.7475 mm.