# Rolling a Ball on an Inclined Plane – ID: 8550

Time required
*45 minutes*

## Topic: Kinematics

- *Measure and describe one- and two-dimensional position, displacement, speed, velocity, and acceleration over time.*
- *Construct and interpret graphs of position, velocity, and acceleration versus time.*
- *Use appropriate equations to calculate average displacement, velocity, acceleration, and time in two dimensions.*
- *Resolve a vector into perpendicular components using the sine and cosine functions.*

## Activity Overview

*In this activity, students will collect distance and time data for a ball rolling up and down an incline. They will then find the best-fit parabola for the data and construct a tangent to the parabola. They will capture data about the slope of the parabola at various points and graph the results. They will then use those results to explore the relationship between the displacement and velocity functions and between the angle of the inclined plane and the acceleration of the ball.*

## Materials

*To complete this activity, each student or student group will require the following:*

- *TI-Nspire™ technology*
- *Vernier CBR2™ or Go!™Motion sensor*
- *copy of the student worksheet*
- *pen or pencil*
- *inclined plane at least 1 m long*
- *small ball, such as a baseball*
- *safety goggles*
- *meter stick or tape measure*

## TI-Nspire Applications

*Graphs & Geometry, Lists & Spreadsheet*

## Teacher Preparation

*Before conducting this experiment, students should have been introduced to the equations relating displacement, velocity, and acceleration to time.*

- *This activity can be used to introduce students both to the use of the TI-Nspire for data collection and to the concepts of displacement, velocity, and acceleration. You may wish to use part of this activity as an engagement activity at the beginning of a unit on kinematics. Students will probably have learned basic information about displacement, velocity, acceleration, and time in previous classes. However, they may not have been exposed to the equations describing these relationships. If this is the case, you can eliminate question #3 and give students the equations describing the relationships between the quantities mentioned in the question. Students will benefit from a brief discussion of how to obtain these equations, especially the expression representing the component of gravitational force (weight) acting on the ball as a function of angle of the ramp. Even if students have not learned about motion before, encourage them to think critically about what is occurring during this activity. If you wish, you may have students come back to this activity at the end of the unit and discuss how their understanding has changed.*
- *If time permits, you may wish to have students repeat the experiment and vary the angle of the inclined plane.*

- *It make take students several tries before they collect a useable data set. Encourage students to graph their collected data before taking apart the experimental setup.*
- *The screenshots on pages 2–8 demonstrate expected student results. Refer to the screenshots on page 9 for a preview of the student TI-Nspire document (.tns file). Pages 10–13 show the student worksheet.*
- ***To download the .tns file and student worksheet, go to education.ti.com/exchange and enter "8550" in the search box.***

## Classroom Management

- *This activity is designed to be* ***student-centered****, with the teacher acting as a facilitator while students work cooperatively. The student worksheet guides students through the main steps of the activity and includes questions to guide their exploration. Students should record their answers to the questions on a separate sheet of paper.*
- *The ideas contained in the following pages are intended to provide a framework as to how the activity will progress. Suggestions are also provided to help ensure that the objectives for the activity are met.*
- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

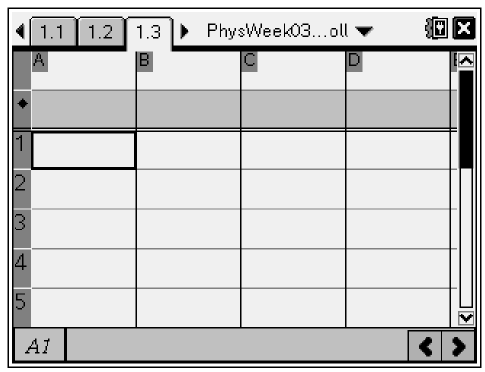
---

The following questions will guide student exploration during this activity:

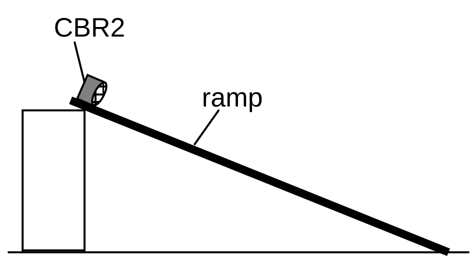
- What does a graph of displacement vs. time look like for a ball rolling up and down an inclined plane?
- How is the slope of a graph of displacement vs. time related to the equation describing velocity as a function of time?

## Part 1: Collecting displacement data

**Step 1:** Students will use a Vernier CBR2™ or Go!™Motion sensor to collect displacement data. When students reach page 1.3, they should insert a data collection box (ctrl D) and then connect the motion sensor to the handheld or computer. This should activate the motion sensor, and a distance display should appear in the data collection box. The motion sensor should start clicking slowly, and the green light on the front should turn on.




**Step 2:** After making sure all students are wearing safety goggles, give each student a baseball or tennis ball and an inclined plane. (The inclined plane can be of any angle and material, as long as the ball will roll up and down it smoothly. A piece of wood or stiff cardboard with one end placed on a stack of books will be fine.) Instruct the students to set up their motion sensors so that the metal mesh on the sensor is facing down the ramp, parallel to its surface, as shown.



**Q1.** Predict the shape of the graph of displacement vs. time for a ball rolling up and then back down the ramp. Sketch it on a blank piece of paper.

**A.** *Students' predictions will vary. Encourage students to discuss their answers and think about other examples of motion they have studied.*

**Q2.** Measure the height and length of your ramp. Use these data to calculate the angle between the ramp and the floor or table. Show your work.

**A.** *Students' answers will vary. Make sure students have used the correct trigonometric function to calculate the angle.*

**Q3.** During this activity, you will roll a ball up and down the ramp. Write the equations describing the ball's position and velocity along the ramp as a function of time. Assume that there is no friction between the ball and the ramp, and that the only acceleration is due to gravity. Use *s* for displacement, *v* for velocity, *g* for acceleration due to gravity, and *t* for time. (Hint: What component of *g* is acting parallel to the ramp?) Show your work.

**A.** *For motion in one direction, the following general equations apply:*

$$s(t) = s_0 + v_i t + \frac{1}{2}at^2$$

$$v(t) = v_0 + at$$

*The only acceleration is due to gravity, which acts at an angle θ to the ramp. In this case, θ is equal to the angle between the ramp and the floor. The component of* g *that is parallel to the ramp is therefore* g·*sin* θ*. Making this substitution into the equations above yields the following:*

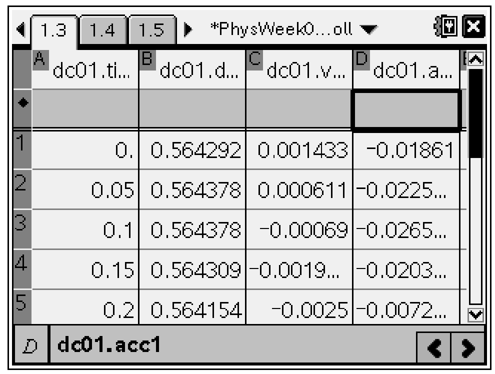
$$s(t) = s_0 + v_i t + \frac{1}{2}gt^2(\sin\theta)$$

$$v(t) = v_0 + gt(\sin\theta)$$

**Step 3:** Have each group of students place their ball at the bottom of the ramp. Have them move the ball slowly up and down the ramp while observing the distance display. The display should change as the students move the ball. This indicates that the motion sensor is functioning correctly and is detecting the ball. Have students practice giving the ball a gentle push from the bottom of the ramp. The push should be enough to make the ball roll up to the top of the ramp, but not so hard that the ball hits the motion sensor. Note: It is important that the ball roll up and down the ramp along a line that is as close to straight as is possible. Have students practice until they can roll the ball smoothly and evenly.

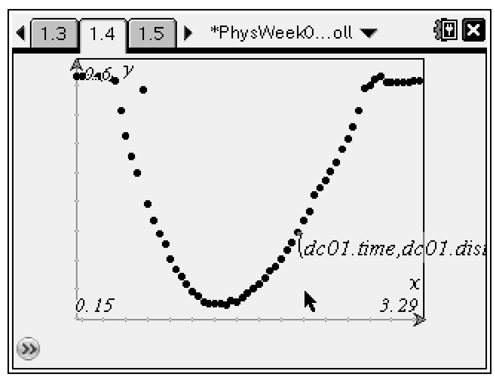
**Step 4:** Next, students will collect distance data using the data collection software. The students should begin data collection a second or so before rolling the ball up the ramp. Once the ball has returned to the bottom of the ramp, students can stop the data collection. After students have collected their data, you can collect the balls. Then, students may remove their safety goggles and disconnect their motion sensors.




| | A dc01.ti... | B dc01.d... | C dc01.v... | D dc01.a... |
|---|---|---|---|---|
| ◆ | | | | |
| 1 | 0. | 0.564292 | 0.001433 | −0.01861 |
| 2 | 0.05 | 0.564378 | 0.000611 | −0.0225... |
| 3 | 0.1 | 0.564378 | −0.00069 | −0.0265... |
| 4 | 0.15 | 0.564309 | −0.0019... | −0.0203... |
| 5 | 0.2 | 0.564154 | −0.0025 | −0.0072... |

D dc01.acc1

**Step 5:** Next, students move to page 1.4 and create a larger scatter plot of the data. Students should use the **Zoom- Fit** or **Zoom- Box** commands to zoom the graph so it shows only the parabolic section of the data set, as shown.




**Q4.** Describe the shape of the graph.

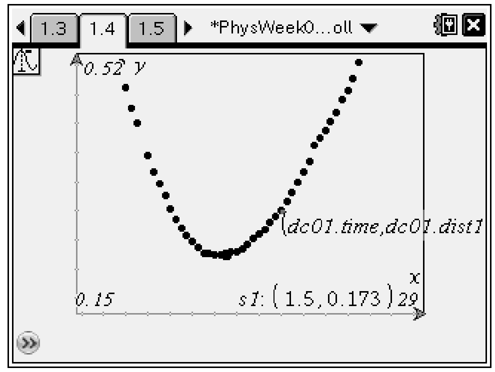
**A.** *The data appear to lie along a parabola.*

**Q5.** Does the graph match your prediction? If not, explain why you made the prediction you did. What assumptions did you make that were incorrect?

**A.** *Students' answers will vary. Encourage metacognitive thinking to help students identify their errors in logic.*

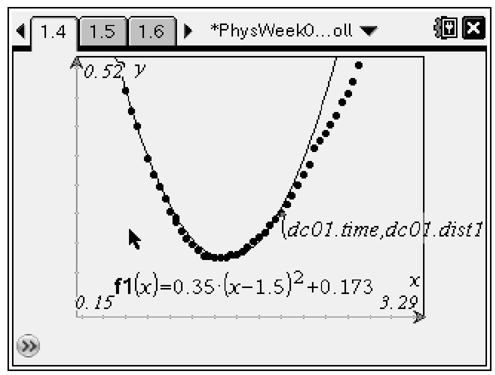
## Part 2: Fitting a curve to the data




**Step 1:** Next, students use the **Trace** tool to locate and mark the vertex of the parabola. Note: The data may flatten out at the bottom of the curve, as shown. In this case, students should mark the center of the lowest point. Then, students will attempt to fit a quadratic curve of the form $y = m(x - n)^2 + q$ to their data.

**Q6.** Use the coordinates of the vertex you marked to predict the values of *n* and *q*.

**A.** *The variable* n *should be close to the* x-*coordinate; the variable* q *should be close to the* y-*coordinate.*




**Step 2:** After students have made their predictions, they change the plot to a **Function** plot and enter their predicted formulae into the formula bar. They then adjust the values of *m*, *n*, and *q* until the parabola matches the data as closely as possible.

**Q7.** Record the equation for the parabola that best fits your data.

**A.** *Students' answers will vary. Encourage students to think logically about how they should vary the values to find the best fit (i.e., they should not just vary the values randomly).*

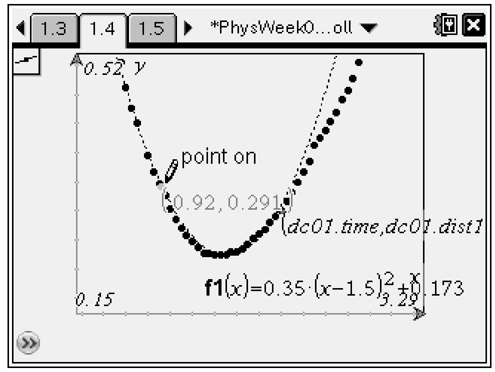
**Q8.** How does the acceleration coefficient in the best-fit equation compare with that in the equation for *s*(*t*) you wrote for question 3? (Hint: Expand the equation for the best-fit parabola to write it in the form $ax^2 + bx + c$).

**A.** *The acceleration coefficient of the best-fit equation should be similar to that in the equation students calculated in question 3. The match will not be exact, but the values should be close.*

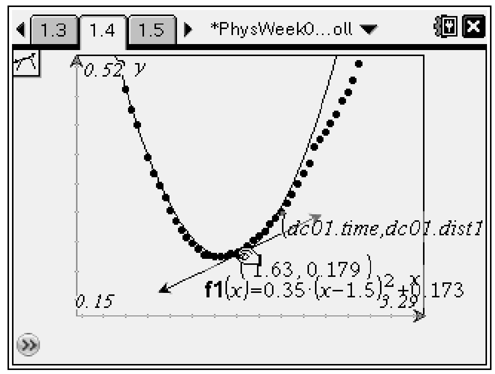
 

## Part 3: Exploring the relationship between slope and equations describing motion

**Step 1:** Next, students will construct a tangent to their best-fit parabolas. First, they place a point on the parabola using the **Point On** tool.



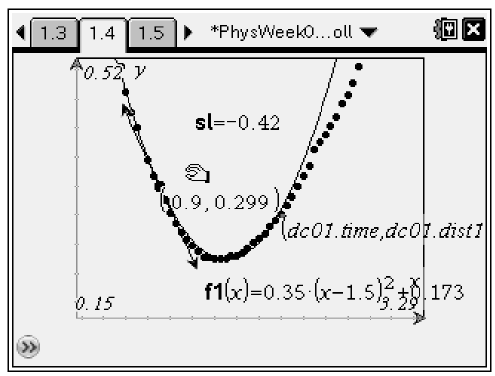

**Step 2:** Next, students construct a tangent to the parabola through the point. After they have constructed the tangent, they can slide the point on the parabola to move the tangent along the curve. Students should practice sliding the tangent back and forth along the curve.



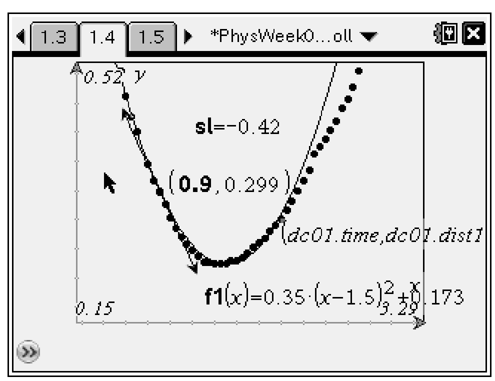

**Step 3:** Next, students use the **Slope** tool to find the slope of the tangent. They assign this value to the variable **sl** using the sto→ var button. Have students drag the point on the *x*-axis and observe how the slope of the tangent changes.

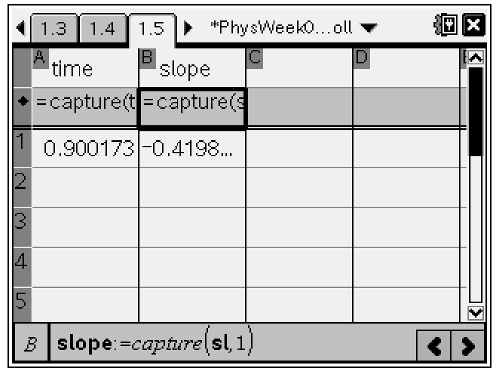
**Q9.** Predict the shape of a graph of the slope of the parabola vs. time. Sketch it on a blank piece of paper.

**A.** *Students' predictions will vary. Encourage students to discuss their answers.*

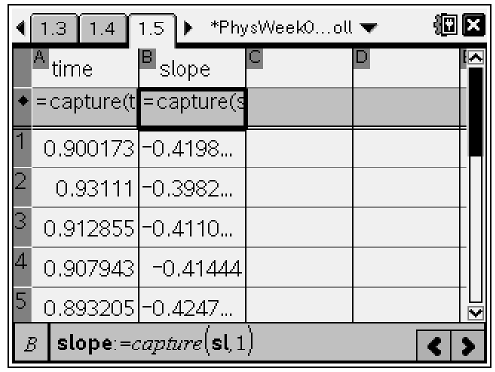



**Step 4:** Then, students move their cursor to the text indicating the *x*-coordinate of the point on the parabola through which the tangent line was constructed. They should assign this point to the variable **ti**. Note: To assign the *x*-coordinate to **ti**, students must click on the *x*-coordinate once, so that only the *x*-coordinate is highlighted in gray, as shown in the student worksheet. Students will know the variable has been successfully assigned when it becomes bold, as shown.

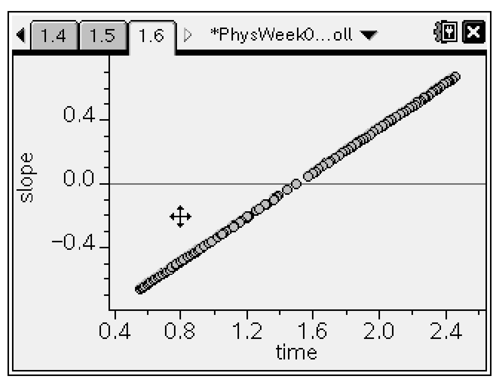





**Step 5:** Next, students use the *Lists & Spreadsheet* application on page 1.5 to capture slope and time data. They assign column A to the variable **time** and column B to the variable **slope**. They use the **capture** command to record slope and time data at each point along the curve. Note: Make sure students have moved the tangent line all the way to the far left-hand side of the parabola before entering the functions in the spreadsheet.

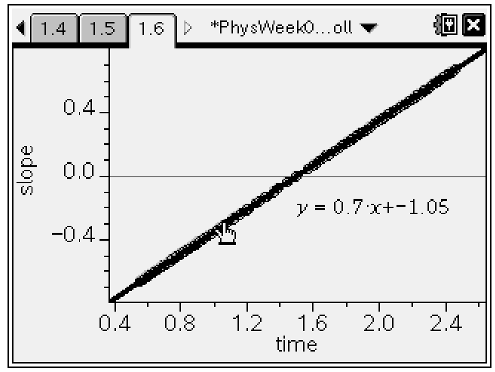



**Step 6:** Next, students move back to the graph and drag the tangent once along the parabola from left to right. Make sure students drag the point slowly—they should not click the Navpad once and hold it down. Instead, they should click the arrow keys several times to move the tangent across the graph. As the spreadsheet collects more data, it may begin to run slowly. Make sure students wait until the clock icon disappears before clicking the arrow key again. Students may think that the speed at which they drag the point will affect their results. Remind them that the variable **time** refers to the *x*-coordinate of the graph, not to the time it takes them to drag the point. After students have dragged the tangent across the graph, the *Lists & Spreadsheet* application on page 1.5 will be populated with data, as shown. Note: Make sure students do not slide the tangent back and forth too many times. This will place too much data in the spreadsheet and cause it to run slowly.




**Step 7:** Next, students make a scatter plot of **slope** vs. **time**. They should observe that the points fall on a straight line. Discuss the significance of this trend with the students.

**Step 8:** Next, students use a linear regression to find the equation of the best-fit line for the slope vs. time data. They then plot this line on the graph of **slope** vs. **time**.




**Q10.** How does the acceleration coefficient for the equation for this line compare with that in the equation for $v(t)$ you wrote for question 3? What does this indicate about the relationship between the equation for displacement and the equation for velocity?

**A.** *The equations should be similar, although they will most likely not be an exact match. This illustrates that the velocity equation is the equation for the slope of the displacement equation. Students may struggle to understand this concept. Make sure to review it with them thoroughly to help them understand the relationship between displacement and velocity.*

**Q11.** What would a plot of acceleration vs. time look like? Explain your answer.

**A.** *Students should be able to predict that the plot of acceleration vs. time would be a horizontal line. They will most likely determine this by reasoning that acceleration is constant throughout the ball's path. Guide students to realize that the slope of the velocity vs. time graph is a constant and that its slope is equal to the acceleration. Emphasize the relationship between velocity and acceleration.*

**Q12.** Suppose the angle of your ramp were larger than it is. How would the acceleration coefficient in these equations change? Explain your answer.

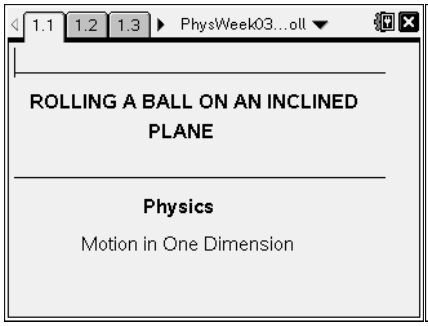
**A.** *A greater angle would increase the component of gravity acting on the ball. Therefore, the acceleration coefficient would be larger.*

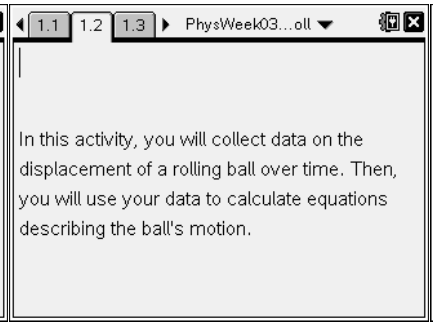
**Q13.** If the ramp were infinitely steep (i.e., vertical), what would the acceleration coefficient in the equations be? Explain your answer.

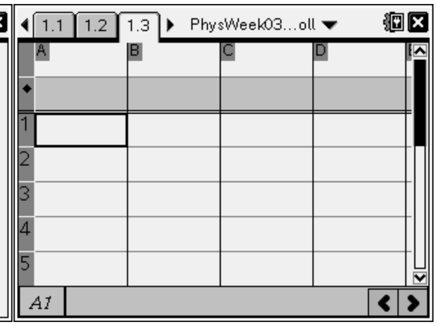
**A.** *If the ramp were vertical, the ball would be moving straight up and down. The acceleration component would simply be acceleration due to gravity, 9.8 $m/s^2$.*

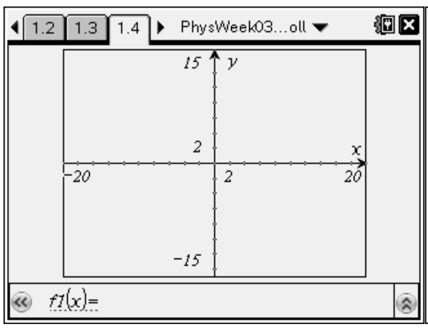
# Rolling a Ball on an Inclined Plane – ID: 8550

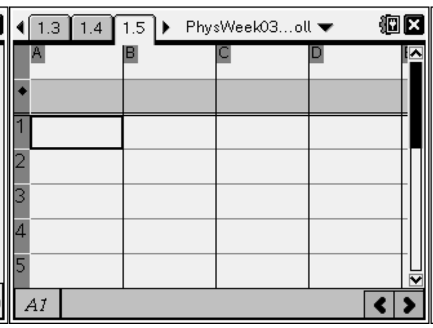
(Student)TI-Nspire File: *PhysWeek03_ballroll.tns*

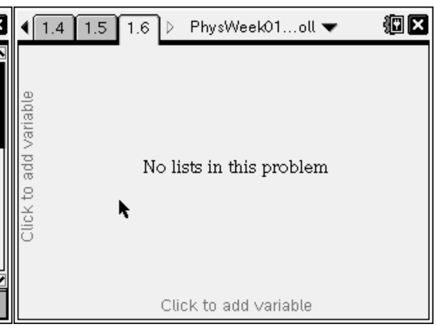


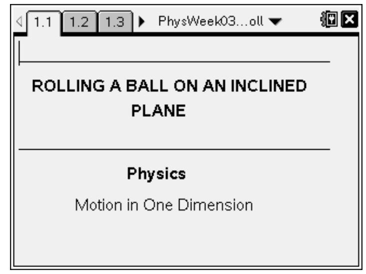

 

# Rolling a Ball on an Inclined Plane

ID: 8550

Name ______________________

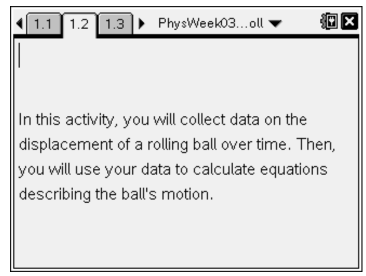
Class ______________________




*In this activity, you will explore the following:*

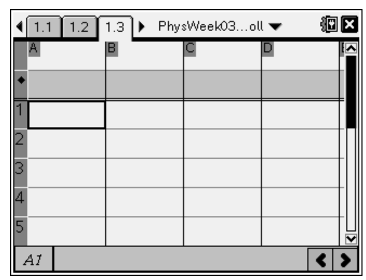
- *the motion of a ball rolling up and down an inclined plane*
- *the relationship between equations describing displacement and velocity as a function of time*

Open the file **PhysWeek01_ballroll.tns** on your handheld or computer and follow along with your teacher for the first two pages. Move to page 1.2 and wait for further instructions from your teacher.



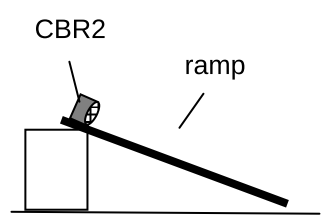

In this activity, you will collect data on the position of a ball rolling up and down an inclined plane. You will then use your data to find a mathematical model for the motion of the ball. You will use your model to explore the relationships between displacement, velocity, and acceleration.

## Part 1: Collecting displacement data




**Step 1:** Move to page 1.3. Insert a data collection box by pressing ctrl D. Put on a pair of safety goggles. Connect a Vernier CBR2™ or Go!™Motion motion sensor to your handheld or computer. The light on the sensor should come on, and the display in the data collection box on the screen should show the current distance between the sensor and whatever object is in front of it. You will know the motion sensor is functioning correctly when you hear it make a soft, slow clicking sound.

**Step 2:** Your teacher will give you a baseball or tennis ball and an inclined plane. Place your motion sensor at the top of the inclined plane, with the mesh part of the sensor facing down the inclined plane and parallel to its surface, as shown to the right.




**Q1.** Predict the shape of the graph of displacement vs. time for a ball rolling up and then back down the ramp. Sketch it on a blank piece of paper.

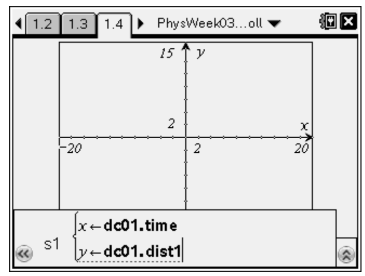
**Q2.** Measure the height and length of your ramp. Use these data to calculate the angle between the ramp and the floor or table. Show your work.

**Q3.** During this activity, you will roll a ball up and down the ramp. Write the equations describing the ball's position and velocity along the ramp as a function of time. Assume that there is no friction between the ball and the ramp and that the only acceleration is due to gravity. Use *s* for displacement, *v* for velocity, *g* for acceleration due to gravity, and *t* for time. (Hint: What component of *g* is acting parallel to the ramp?) Show your work.

**Step 3:** Place the ball at the bottom of the ramp, and move it slowly up and down the ramp. Watch the distance reading while you do this. As you move the ball, the distance reading should change. If it does not, your motion sensor is not detecting the ball. Adjust the position of the motion sensor until it detects the ball along the entire length of the ramp. Then, practice rolling the ball up the ramp by holding it at the bottom of the ramp and giving it a gentle push. Push the ball hard enough that it rolls most of the way up the ramp, but not so hard that it hits the motion sensor or rolls off the ramp.

**Step 4:** Next, you will use the data collection software to collect data on the displacement of the ball as it rolls up and down the ramp. Hold the ball at the bottom of the ramp and click the å symbol to begin collecting data. The motion sensor will start to click rapidly. Wait about one second, and then roll the ball up the ramp as you practiced. When the ball has returned to the bottom of the ramp, stop the data collection by clicking the button in the top left corner of the data collection box. After you have collected your data, your teacher will collect the balls from the class.

**Step 5:** Close the data collection box. You can now disconnect the motion sensor. Move to page 1.4, which contains a *Graphs & Geometry* application. Make a scatter plot of the data you just collected. Use the time data for your *x*-values and the distance data for your *y*-values.

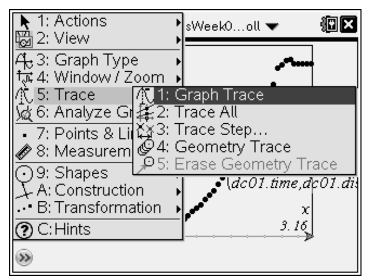



**Q4.** Describe the shape of the graph.

**Q5.** Does the graph match your prediction? If not, explain why you made the prediction you did. What assumptions did you make that were incorrect?

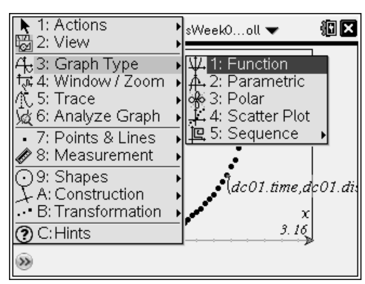
## Part 2: Fitting a curve to the data

**Step 1:** The shape of the graph should be consistent with a parabolic function of the form $y = m(x - n)^2 + q$. In this case, *y* is displacement and *x* is time. Next, you will attempt to find the values of *m*, *n*, and *q* that produce a curve that best fits your recorded data. First, use the **Trace** tool (**Menu > Trace > Graph Trace**) to find the location of the vertex (the lowest point) of the parabola. Use the NavPad to move from point to point until you reach the vertex. Then, click once to mark the coordinates of the point. Press esc to return to the main graph.




**Q6.** Use the coordinates of the vertex you marked to predict the values of *n* and *q*.

**Step 2:** Next, change the plot to a **Function** plot, and enter your prediction for the formula describing the data points in the function bar. Use 1 for the value of *m* to begin with. Adjust the values of *m*, *n*, and *q* to make the parabola fit your data as closely as possible.

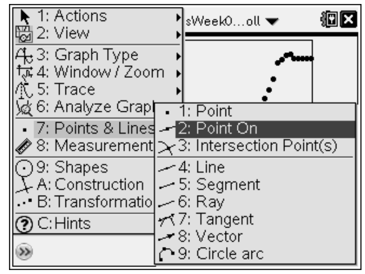



**Q7.** Record the equation for the parabola that best fits your data.

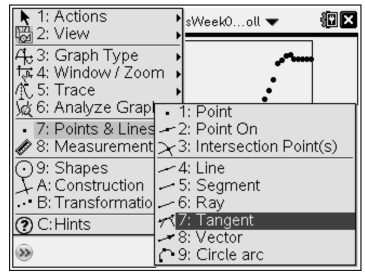
**Q8.** How does the acceleration coefficient in the best-fit equation compare with that in the equation for $s(t)$ you wrote for question 3? (Hint: Expand the equation for the best-fit parabola to write it in the form $ax^2 + bx + c$).

## Part 3: Exploring the relationship between slope and equations describing motion

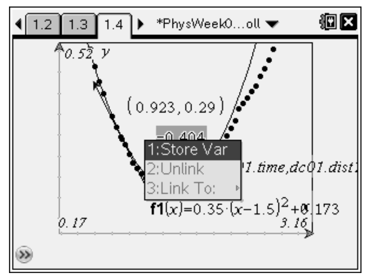
**Step 1:** Next, you will construct a tangent to your parabola. First, place a point on the parabola using the **Point On** tool (**Menu** > **Points and Lines** > **Point On**).




**Step 2:** Then, construct a tangent to the parabola through the point (**Menu** > **Points and Lines** > **Tangent**). After you have constructed the tangent, you can click and drag the point on the parabola to move the tangent line around the curve. Practice sliding the tangent curve back and forth.

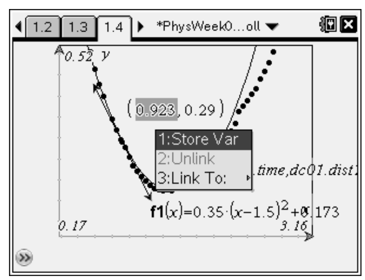



**Step 3:** Next, use the **Slope** tool (**Menu** > **Measurement** > **Slope**) to find the slope of the tangent line. Move the pointer to the tangent line. Click once to select the tangent line and a second time to place the slope value on the page. Press esc to exit the **Slope** tool. Then, select the slope by clicking on it once (it will be highlighted in gray). Press sto→ var, then select **Store Var** to assign the slope to a variable. Type **sl** to assign the slope to the variable **sl**, then press enter.

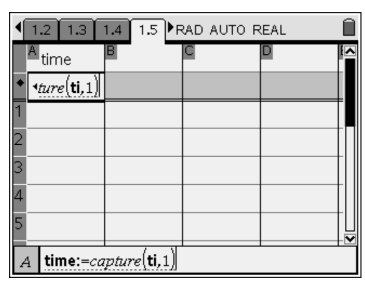



**Q9.** Predict the shape of a graph of the slope of the parabola vs. time. Sketch it on a blank piece of paper.

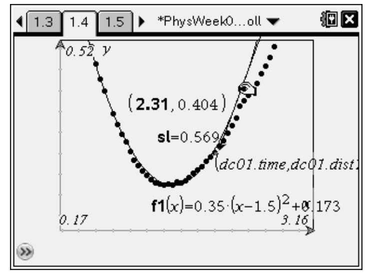
**Step 4:** Now, assign the *x*-coordinate of the tangent point to the variable **ti**. Click once on the *x*-coordinate of the point (click only on the *x*-coordinate). It will be highlighted in gray. Press sto→ var, then select **Store Var** to assign the *x*-coordinate to a variable. Type **ti** to assign the *x*-coordinate to the variable **ti**, then press enter.



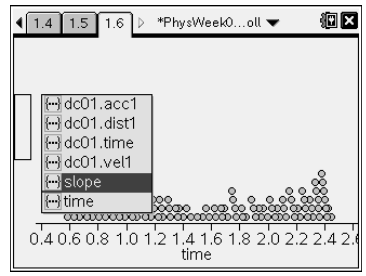

**Step 5:** Next, you will capture the slope and *x*-coordinate values of the parabola at various points. Move to page 1.5, which contains an empty *Lists & Spreadsheet* application. In the title bar (the white bar at the top) of column A, type **time**. This assigns column A to the variable **time**. In the formula bar (the gray bar in the second row) of column A, type **=capture(ti,1)** and press enter. This formula tells the spreadsheet to record the value of the variable **ti**. In the title bar of column B, type **slope**. In the formula bar of column B, type **=capture(sl,1)** and press enter.



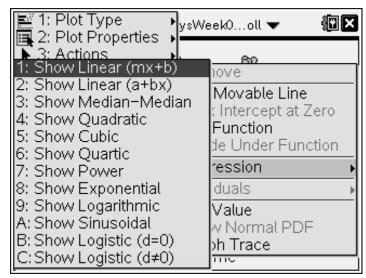
**Step 6:** Next, you will drag the tangent along the parabola so the spreadsheet can record its slope and *x*-coordinates at each point. Move back to page 1.4, and select the point on the *x*-axis. Drag the point slowly from left to right across the *x*-axis. Do not hold down the arrow key to drag the point. Click the arrow key multiple times to move the point across the axis slowly. (Do not drag it more than once; if you do, you may collect too much data and the program will run slowly.)




**Step 7:** Click once to release the point on the *x*-axis, and then move two pages forward to page 1.6. Make a scatter plot of the data you just collected. Use **time** as your *x*-values and **slope** as your *y*-values. Examine the shape of the graph, and discuss it with the rest of the class.




**Step 8:** Next, you will use a linear regression to find the equation describing the data. To use the **Linear Regression** tool, select **Show Linear (mx + b)** from the **Regression** menu (**Menu > Analyze > Regression > Show Linear (mx + b)**). The TI-Nspire will calculate the linear best-fit line for the data and display the line and its equation on the screen.




**Q10.** How does the acceleration coefficient for the equation for this line compare with that in the equation for *v*(*t*) you wrote for question 3? What does this indicate about the relationship between the equation for displacement and the equation for velocity?

**Q11.** What would a plot of acceleration vs. time look like? Explain your answer.

**Q12.** Suppose the angle of your ramp were larger than it is. How would the acceleration coefficient in these equations change? Explain your answer.

**Q13.** If the ramp were infinitely steep (i.e., vertical), what would the acceleration coefficient in the equations be? Explain your answer.